Problem set 12
FYS4130 at UiO Spring 2014

April 7, 2014

1 Weiss mean field theory for magnets

Consider a system of N Ising-spins with $\sigma_i = \pm 1$. Each spin interacts with its z nearest neighbours (n.n.) with coupling J and with an external field B, so the Hamiltonian is

$$H = -B\sum_i \sigma_i - J\sum_{ij}^{j\in \text{n.n.}} \sigma_i\sigma_j.$$

a) The units in the Hamiltonian above are inconsistent (it says joule = tesla). How could this be fixed, and would the fix change the rest of our calculations?

b) Use Weiss mean field theory to show that the average magnetization $m = \langle\sigma_i\rangle$ is

$$m = \tanh(\beta B^{\text{eff}}),$$

with $B^{\text{eff}} = B + Jzm$.

c) For $J = 0$, find the average magnetization and the magnetic susceptibility χ. Evaluate the results in the limit of weak field, $B\beta \ll 1$.

d) For $J > 0$, find m and χ.

e) Find the critical temperature T_c where χ diverges.

f) For $B = 0$, $J > 0$ and $T < T_c$ find an approximate solution for the temperature dependence of m.