

## FYS 4130 Statistical Mechanics

### Homework 11 April 23, 2010

#### 1) 1-D, 3 particle Ising model

The Hamiltonian for a chain of 3 Ising spins  $\sigma_i = \pm 1$  is given by:

$$H = -J \sum_{i=1}^2 \sigma_i \sigma_{i+1}$$

- Calculate the partition function.
- Calculate the internal energy and specific heat.
- Find the  $\langle \sigma_2 \rangle$  and the spin correlation function  $\langle \sigma_1 \sigma_3 \rangle$ .

$$Z = 4(1 + \cosh 2\beta J)$$

$$U = -2J \tanh \beta J$$

$$\langle \sigma_2 \rangle = 0$$

$$\langle \sigma_1 \sigma_3 \rangle = \tanh^2 \beta J$$

#### 2) Ising model

A lattice has  $N + 1$  Ising spins  $\sigma_i = \pm 1$  in an external magnetic field  $B$ . Each spin in the lattice has a coupling to a single spin  $\sigma_0$ . The Hamiltonian is:

$$H = -B \sum_{i=1}^N \sigma_i - J \sum_{i=1}^N \sigma_i \sigma_0$$

- Calculate the partition function
- Find  $m = \langle \sigma_i \rangle$  and the correlation  $\langle \sigma_0 \sigma_i \rangle$ .
- Calculate the average energy  $E$ . What is  $E$  in the limit where  $B \rightarrow 0$  and  $J \rightarrow 0$ ?

$$Z = e^{\beta B} (2 \cosh(\beta B + \beta J))^N + e^{-\beta B} (2 \cosh(\beta B - \beta J))^N$$

$$\langle E \rangle = -B \langle \sigma_i \rangle - J \langle \sigma_0 \sigma_i \rangle$$