FASIT / ANSWERS EXAM FYS4130

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1.1 Proportionality with N or V for extensive.

1.2 $$H_{MF} = -J\sum_{\sigma_x \sigma_{x'}} \sigma_x \overline{\sigma_{x'}} - B\sum_{\sigma_x} \sigma_x$$

$$= -B_{eff}\sum_{\sigma_x}\sigma_x \,, \quad B_{eff} = B + zJm$$

State of interacting neighbor replaced by average.

1.3 Distinguishable / localized particles

1.4 $$m\frac{dv}{dt} = -\alpha v + F(t)$$

1 Friction law is the same as in non-fluctuating case.

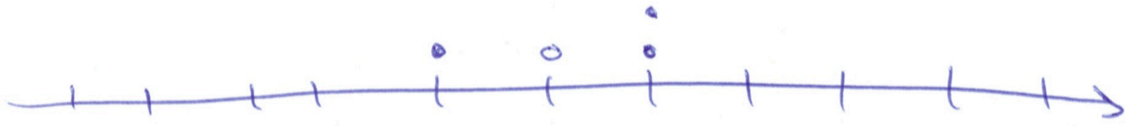
2 Only $v(t)$, not $v(t'<t)$ plays a role (Markovian)

3 δ-correlated in time

4 uncorrelated from v

1.5 $$\mu = \left.\frac{\partial F}{\partial N}\right|_{TV} = 0$$ since F does not depend on N.

2 N_s siter



$n_i = 0, 1, 2$

2.1 In equilibrium $N_s(V)$, μ & T are given $\rightarrow$ Landau pot. Ω.

2.2 $N = \sum_i n_i$

Grand canonical state prob

$$P_{sN} \propto e^{-\beta(\underset{0}{\underbrace{E_s}} - \mu N)} = \prod_i e^{\beta\mu n_i}$$

$$\underline{\underline{p(n_i) = \frac{e^{\beta\mu n_i}}{Z}}}, \quad Z = \sum_{n_i=0}^{2} e^{\beta\mu n_i}$$

2.3 $\langle N \rangle = N_s \langle n_i \rangle$, $\langle n_i \rangle = \sum_{n_i=0}^{2} n_i \, p(n_i) = \frac{\partial \ln Z}{\partial(\mu\beta)}$

$$\langle n_i \rangle = \frac{1}{\mu}\frac{\partial}{\partial\beta} \ln\left(1 + e^{\beta\mu} + e^{2\beta\mu}\right)$$

$$\underline{\underline{\langle n_i \rangle = \frac{e^{\beta\mu} + e^{2\beta\mu}}{1 + e^{\beta\mu} + e^{2\beta\mu}} = \frac{\langle N \rangle}{N_s}}}$$

2.4 $\mu = \left.\frac{\partial F}{\partial N}\right|_{TV} = kT\left(\ln\frac{N\Lambda^3}{V - Nb} - \cancel{1}\right) + NkT\left(\cancel{\frac{1}{N}} + \frac{b}{V - Nb}\right) + \frac{2aN}{V}$

~~since~~ $\cancel{\frac{\partial \Lambda}{\partial T}} = \cancel{\frac{1}{2T}}\Lambda =$

2.4 continued.. $\mu = 2an + kT\left(\frac{nb}{1-nb} + \ln\left(\frac{n\lambda^3}{1-nb}\right)\right)$

2.5 b = particle volume

a = average pair interaction energy ($a<0$ for attraction)

3 2D fermions $\varepsilon = p^2/2m$

3.1 $N = \sum_{\vec{p}, s=\pm 1/2} \langle n \rangle_{\vec{p} s} = \underset{\substack{\uparrow \\ \#\ \text{spin values}}}{2} \sum_{\vec{p}} \langle n_{\vec{p}} \rangle = 2 \sum_{\vec{p}} \frac{1}{e^{\beta(\varepsilon_p - \mu)} + 1}$

where $\sum_{\vec{p}} \ldots \simeq \frac{A}{(2\pi)^2} \int d^2k \ldots = \frac{A}{2\pi\hbar^2} \int_0^\infty dp\, p \ldots$

$= \frac{Am}{2\pi\hbar^2} \int_0^\infty d\varepsilon \ldots$ so that

$N = \frac{Am}{2\pi\hbar^2} 2 \int_0^\infty d\varepsilon \frac{e^{-\beta\varepsilon}}{e^{-\beta\mu} + e^{-\beta\varepsilon}} = \frac{-Am}{\pi\hbar^2\beta} \Big| \ln\left(e^{-\beta\mu} + e^{-\beta\varepsilon}\right) \Big|_0^\infty$

$N = \frac{AmkT}{\pi\hbar^2} \ln\left(1 + e^{\beta\mu}\right)$

$$= \frac{Am}{\pi\hbar^2}\int_0^\infty -\frac{1}{\beta}\ln\left(e^{-\beta\mu}+e^{-\beta\varepsilon}\right)$$

$$= \frac{Am}{\pi\hbar^2}\,kT\left(\ln\left(1+e^{-\beta\mu}\right)-\ln\left(e^{-\beta\mu}\right)\right)$$

$$= \frac{Am\,kT}{\pi\hbar^2}\ln\left(1+e^{\beta\mu}\right)$$

3.2 $\rho = \frac{N}{A} \doteq \frac{m\,kT}{\pi\hbar^2}\ln\left(1+e^{\beta\mu}\right)$

$$1+e^{\beta\mu} = e^{\beta\rho\frac{\pi\hbar^2}{m}}$$

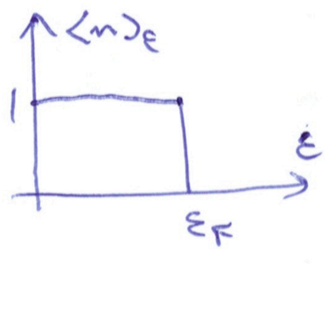
$$\mu = kT\ln\left(e^{\beta\frac{\pi\hbar^2\rho}{m}}-1\right)$$

$$\approx kT\ln\left(e^{\beta\frac{\pi\hbar^2\rho}{m}}\right) \text{ when } \beta\to\infty$$

$$\mu = \frac{\pi\hbar^2}{m}\rho\,(=\varepsilon_F) \text{ as } T\to 0$$

3.3 At $T=0$ $\quad U = \frac{Am}{\pi\hbar^2}\int_0^\infty d\varepsilon\,\varepsilon\,\langle n\rangle_\varepsilon$

$$U = \frac{Am}{\pi\hbar^2}\int_0^{\varepsilon_F} d\varepsilon\,\varepsilon = \frac{Am}{2\pi\hbar^2}\varepsilon_F = \frac{A\pi\hbar^2}{2m}\rho^2$$



3.4 In 2D $V \to A$ and $PV = -\Omega = kT \ln \Xi$ becomes

$$PA = kT \sum_{\vec{p}\, s=\pm 1/2} \ln\left(1 + e^{-\beta(\varepsilon_p - \mu)}\right)$$

$$= 2kT \frac{Am}{2\pi\hbar^2} \int_0^\infty d\varepsilon \ln\left(1 + e^{-\beta(\varepsilon - \mu)}\right) \cdot 1$$

Partial integration:

$$= \frac{Am}{\pi\hbar^2} \int_0^\infty d\varepsilon\, \varepsilon \frac{e^{-\beta(\varepsilon-\mu)}}{e^{-\beta(\varepsilon-\mu)} + 1}$$

$$= \sum_{\vec{p}\, s=\pm 1/2} \varepsilon \langle n \rangle_{\varepsilon_{\vec{p}}} = U = A \frac{\pi\hbar^2}{2m} \rho^2 \text{ at } T=0$$

$$\boxed{PA = U}$$

so that $P = \frac{\pi\hbar^2}{2m} \rho^2$ as $T \to 0$