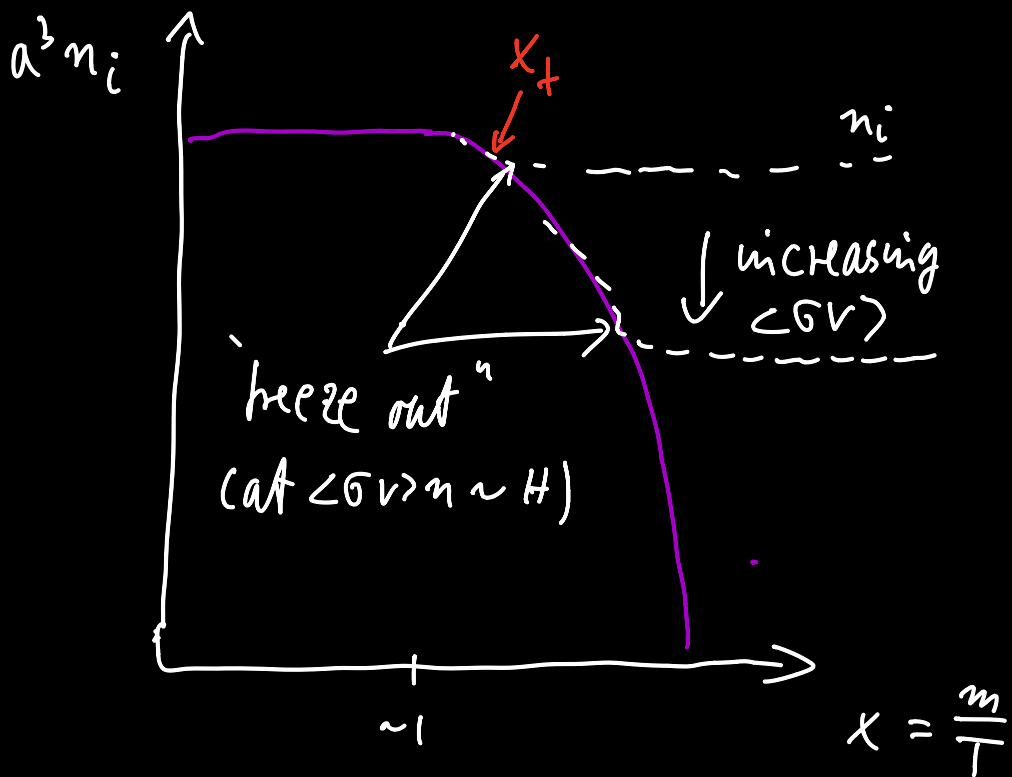
$$\boxed{\frac{dn_i}{dt} + 3Hn_i = -\langle\sigma v\rangle\left[n_i^2 - (n_i^{eq})^2\right]}$$



hot thermal relics

$(T_f \gtrsim 3\, m_i)$

$$\Rightarrow n_i^0 = \underbrace{n_i^{eq}}_{[\simeq n_i(t_f)]}(t_f)\cdot a^3(t_f) \qquad \Big|\ a^3 s = \text{const.} \Rightarrow a \propto g_{eff}^{-1/3}\, T^{-1}$$

$$= \underbrace{g_i \frac{\zeta(3)}{\pi^2} \cancel{T_f^3}}_{n_i^{eq}(t_f)} \left[\left(\frac{g^s_{eff}(T_f)}{g^s_{eff}(T_0)}\right)^{-1/3}\left(\frac{\cancel{T_f}}{T_0}\right)^{-1}\right]^3$$

$\Rightarrow$ very weak dependence on T_f!

$$\Rightarrow \Omega_i h^2 = \begin{cases} \xrightarrow{\text{NR today}} h^2 \dfrac{\rho = m_i n_i^0}{\rho_c} = 7.8 \cdot 10^{-2} \dfrac{g_i}{g_{eff}^s(T_f)} \dfrac{m_i}{eV} & a) \\ \xrightarrow{\text{rel. today}} h^2 \dfrac{\rho = T_i^0 n_i^0 \frac{\pi^4}{30\zeta(3)}}{\rho_c} = 1.8 \cdot 10^{-5} \dfrac{g_i}{g_{eff}^s(T_f)} \dfrac{T_i^0}{K} & b) \end{cases}$$

$H_0 = h \cdot 100 \frac{km}{s \cdot Mpc}$

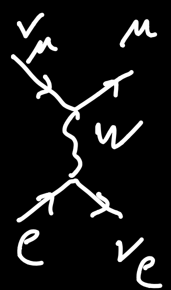
a) e.g. massive neutrinos:

$$\Omega_{\nu\bar{\nu}} h^2 \overset{!}{<} \underbrace{\Omega_m h^2}_{\sim 0.3} \Leftrightarrow \boxed{\sum m_{\nu_i} < 94\, eV\, \Omega_m h^2}$$

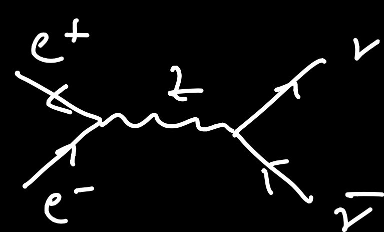
b) temperature of relativistic ν today?

i) rel. species @ $T \lesssim 100\, MeV$: $\nu, \gamma, e^\pm$

e.g. through



,



, etc.

ii) $\Gamma \sim H$ @ $T \sim 4\, MeV$

$\rightsquigarrow$ decoupling of ν

iii) $T \lesssim 1\, MeV$ $\quad e^+ e^- \xrightarrow{\cancel{\longleftarrow}} \gamma\gamma$

$$\Rightarrow g_{eff}^s(\text{before}) = \underset{(\gamma)}{2} + \frac{7}{8} \cdot \underset{(e^\pm)}{2 \cdot 2} = \frac{11}{2} \quad (T_\gamma = T_\nu)$$

g_{eff}^{s} (after) $= 2$ (T_γ & T_ν = "unchanged")

$g_{eff}^{s}(aT)^3 = \text{const.} \Rightarrow \left(\frac{11}{2}\right)^{1/3} \underbrace{aT|_{before}}_{aT_\nu} \stackrel{!}{=} 2^{1/3}\, aT|_{after}$

$$\Rightarrow \boxed{T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma}$$

today : $T_\gamma = 2.73$ K (observed!)

$\Rightarrow T_\nu = 1.95$ K

$= 1.7 \cdot 10^{-4}$ eV

$\Rightarrow \Omega_r h^2 = \Omega_{\gamma+\nu} h^2 = 4.2 \cdot 10^{-5}$

$\Downarrow$

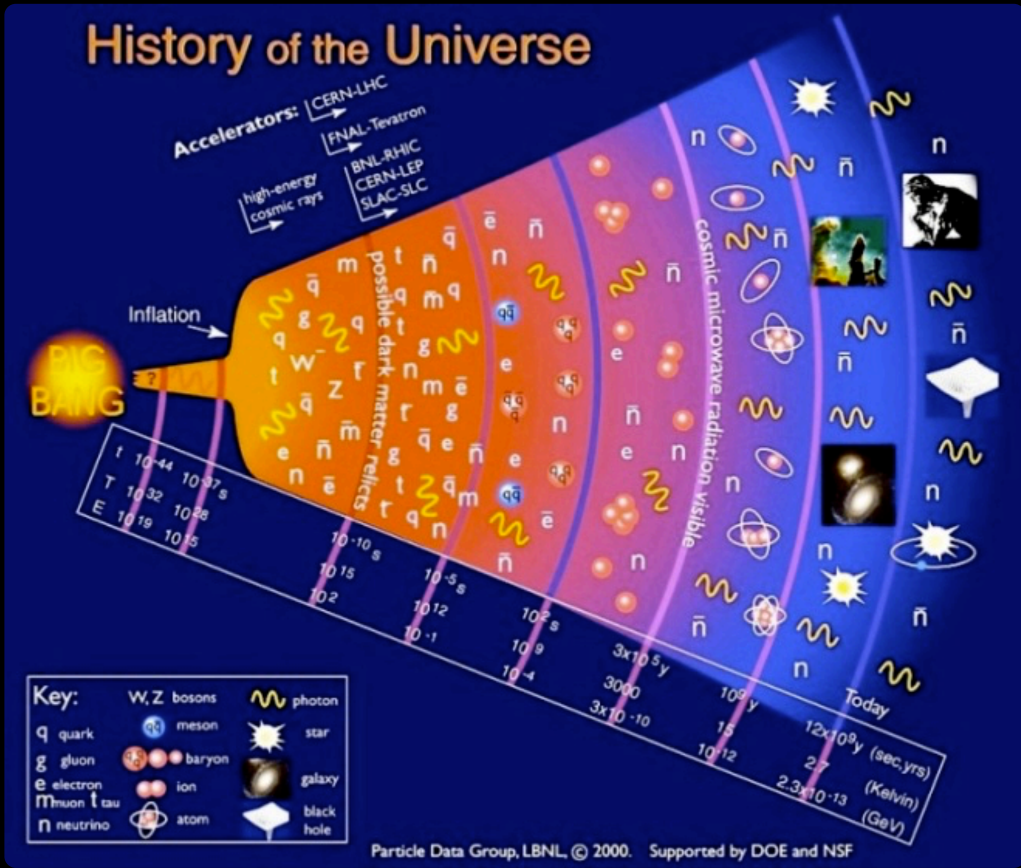
$n_\gamma \sim \frac{410}{cm^3}$

$1 + z_{eq} = \frac{\Omega_m h^2}{\Omega_r h^2} \simeq 3100$

$n_\nu \sim \frac{360}{cm^3}$

$\Rightarrow T_{eq} = T_\gamma^0 (1+z_{eq}) \simeq 8500$ K

$= 0.7$ eV


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relics
cosmic microwave radiation visible
Today
Key:
q quark
g gluon
e electron
m muon
t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
(sec,yrs)
(Kelvin)
(GeV)
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8.4. Evolution of density perturbations

so far $\rho = \rho(t)$

now allow small perturbations:

$$\rho(t,\vec{x}) = \bar{\rho}(t)[1+\delta(t,\vec{x})]$$

Newtonian analysis

$\rightsquigarrow p \ll \rho$, scales $\lesssim (aH)^{-1}$

1) continuity equation: $\dot{\rho} + \vec{\nabla}(\rho\vec{v}) = 0$

$\ulcorner \partial_\mu j^\mu = 0;\ j^\mu = \rho u^\mu = \rho(1,\vec{v}) \lrcorner$

2) Euler equation: $\dot{\vec{v}} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\nabla\phi_N - \frac{1}{\rho}\nabla p$

$\ulcorner f(t+dt, \vec{x}+\vec{v}dt, \vec{v}+\underbrace{a}_{-\nabla\phi_N}dt) = f(t,\vec{x},\vec{v})$

$\Rightarrow \frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla}f - \nabla\phi_N \cdot \frac{\partial f}{\partial \vec{v}} = 0 \quad \Big| \times \int d^3v\, v_i$ $\lrcorner$

3) Poisson eq.: $\Delta\phi_N = 4\pi G\rho$

now use: • $\vec{v} = \frac{d}{dt}(a\vec{x}) = \dot{a}\vec{x} + a\vec{u}$

peculiar velocity $\rightsquigarrow$ small

• comoving frame: $\nabla_i = a^{-1}\partial_i$

- $\bar{\rho} \propto a^{-3}$
- FT

$\downarrow$ $\vdots$

"Jeans equation"

first order : $$\boxed{\ddot{\delta}_{\vec{k}} + 2H\dot{\delta}_{\vec{k}} + \left(\frac{c_s^2 k^2}{a^2} - 4\pi G\bar{\rho}\right)\delta_{\vec{k}} = 0}$$

$c_s^2 = \frac{\partial p}{\partial \rho} = w$: sound velocity

$$\left(\frac{k_J}{a}\right)^2 \equiv \frac{4\pi G\bar{\rho}}{c_s^2} \rightarrow \lambda_J \equiv 2\pi\frac{a}{k_J}$$ "Jeans length"

a) $\lambda_{phys} \lesssim \lambda_J$: damped harmonic oscillator

$\Rightarrow (\ ) > 0$ $\rightarrow$ perturbations oscillate and decay (slowly)

"acoustic oscillations"

b) $\lambda_{phys} \gg \lambda_J$: $\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta \simeq 0$

$\Rightarrow (\ ) \ll 0$

- $2H = 2\frac{\dot{a}}{a} \overset{a\sim t^{2/3}}{=} \frac{4}{3t}$
- $4\pi G\bar{\rho} = \frac{3}{2}H^2 = \frac{2}{3t^2}$

$$\Leftrightarrow \ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0 \quad | \text{ ansatz: } \delta \propto t^n$$

$$\Leftrightarrow n(n-1) + \frac{4}{3}n - \frac{2}{3} = 0$$

$$\Leftrightarrow n^2 + \frac{1}{3}n - \frac{2}{3} = 0$$

$$\Leftrightarrow n = -\frac{1}{6} \pm \sqrt{\frac{1}{36} + \frac{24}{36}} = -\frac{1}{6} \pm \frac{5}{6}$$

$\Rightarrow$ • growing solution : $\boxed{\delta \propto t^{2/3} \propto a(t)}$

will soon dominate

• decaying solution $\delta \propto t^{-1}$