Torsten Bringmann (torsten.bringmann@fys.uio.no)
http://www.uio.no/studier/emner/matnat/fys/FYS4160/v24/

Lecture spring 2024:

General Relativity

Problem sheet 0

⇝ These problems are scheduled for discussion on **Thursday, 22 January 2024**, along with discussing questions related to necessary background knowledge for this course ('FYS4160: preliminaries').

Problem 1

This problem serves as a reminder to practice the use of index notation.

a) Write the following in index notation:

- ∇S (where S is a scalar).
- $\nabla \cdot \mathbf{A}$, $\nabla \times \mathbf{A}$ (where $\mathbf{A}$ is a 3D vector).
- Trace and Transpose of a matrix M.

b) Prove the following 3D identities, using index notation:

- $\nabla \cdot (\nabla \times \mathbf{A}) = 0$, where $\mathbf{A}$ is a 3D vector.
- $\nabla \times (\nabla S) = 0$, where S is a scalar.

Problem 2

And another fresh-up...

a) Are these equalities valid? Correct where necessary!

- $\partial_\mu x^\nu = \delta^\nu_\mu$
- $\partial_\mu x^\mu = 1$
- $\partial^\mu x^\nu = g^{\mu\nu}$
- $T_\alpha{}^\beta{}_\gamma = g^{\beta\mu} T_{\alpha\mu\gamma} = g^{\mu\beta} T_{\alpha\mu\gamma}$
- $T_\alpha{}^\beta{}_\beta = g_{\alpha\mu} g^{\beta\alpha} T^\mu{}_{\alpha\beta}$

b) Construct

- all independent Lorentz scalars from two four-vectors A and B
- all independent Lorentz scalars from (up to two copies of) a rank-2 tensor T, as well as from invoking one rank-2 tensor T and two four-vectors A and B
- all independent Lorentz four-vectors from a scalar S and two four-vectors A and B