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http://www.uio.no/studier/emner/matnat/fys/FYS4160/v24/

Lecture spring 2024:

General Relativity

Problem sheet 3

⇝ These problems are scheduled for discussion on **Friday, 15 February 2024**.

Problem 9

In the lecture we derived the geodesic equation as given in problem 8 also directly from the variational principle, $\delta\left(\int d\tau\right) = 0$.

a) Briefly repeat the derivation as it was given in the lecture. Why is it not valid for photons? Also: what is the most general choice of λ that leads to the geodesic equation in this form (this is called an 'affine parameter')?

b) Start instead from $\delta\left(\int g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu d\lambda\right) \equiv \delta\left(\int \mathcal{K} d\lambda\right) = 0$, where now $\dot{x}^\mu = dx^\mu/d\lambda$. Show that this leads to the same geodesic equation even for photons, and determine $\mathcal{K}$ for the solution to this equation.

Problem 10

In order to get some praxis with coordinate transformations, solve problem 2.7 in the book (prolate spherical coordinates)!

Problem 11

GPS satellites orbit the earth at an altitude of about 20 000 km, with a speed of roughly 10 000 km/h. Assume that the clocks on board of these satellites have been synchronized with identical clock on earth at launch (featuring a few nanoseconds' accuracy). By how much will the time on the clocks at earth and in the satellites differ *per day* a) due to the time-dilation effect of special relativity and b) due to the additional effect of Earth's gravitational field according to general relativity?
[To get an answer in seconds, be careful to use the values for the Earth's gravitational field and radius in the right units!].
In order to appreciate the answer, imagine that your GPS unit tells you today that the nearest "Peppe's pizza" is 0.5 km away. Taking into account that the satellites send their signals to earth with the speed of light, by how much would this estimate be off *after only one day* if either of those effects was not taken into account?