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http://www.uio.no/studier/emner/matnat/fys/FYS4160/v24/

Lecture spring 2024:

General Relativity

Problem sheet 4

⇝ These problems are scheduled for discussion on **Thursday, 22 February 2024**.

Problem 12

Show that the flat two-dimensional torus, $T^2 = S^1 \times S^1$, is a manifold by explicitly constructing an appropriate atlas! (This is problem 2.3 in the book.)

Problem 13

In the lecture we just learned that the coordinate basis vectors for the tangent space T_p are given by $\hat{e}_{(\mu)} = \partial_\mu$.

- For *any* given coordinate system, what are the *components* of each of these 4-vectors?
- What are the components of the Euclidian coordinate basis $\{\partial_\mu\}$ (in Minkowski space) expressed in spherical coordinates, and vice versa?
- In the coordinate basis, any vector A takes the form $A = A^\mu \partial_\mu$. Based on how we introduced general elements of T_p, try to give an intuitive / visual interpretation of the components A^μ.

Problem 14

The commutator, or Lie bracket, of two vector fields X and Y is defined by

$$[X, Y](f) \equiv X(Y(f)) - Y(X(f)),$$

where f is any function that takes values from a manifold to the real numbers. Show that

- $[X, Y]$ is itself a vector field.
- Its components, when expressed in the coordinate basis, are given by

$$[X, Y]^\mu = X^\lambda \partial_\lambda Y^\mu - Y^\lambda \partial_\lambda X^\mu$$

- the commutator of any pair of coordinate basis vectors ($\hat{e}_{(\mu)} = \partial_\mu$) vanishes.