Torsten Bringmann (torsten.bringmann@fys.uio.no)
http://www.uio.no/studier/emner/matnat/fys/FYS4160/v24/

Lecture spring 2024:

General Relativity

Problem sheet 8

⇝ These problems are scheduled for discussion on **Thursday, 21 March 2024**.

Legend

***** If pressed for time, make sure to try solving the other problem(s) before attempting this one

Problem 26*

As we have seen in the lecture, the field equations follow from varying the Einstein-Hilbert action,

$$S_H = \int d^4x \sqrt{-g} R \,.$$

a) The Lagrangian thus contains up to *second* derivatives of the fields (i.e. the metric). Why does one typically not encounter this situation in field theories? Why is this not a worry in this particular case? *[Hint: Show that the* action *actually only depends on first derivatives of the metric]*

b) In deriving the field equations from the above action, we used that the *variation* of the Christoffel symbols, $\delta \Gamma^\mu_{\rho\sigma}$, transforms like a tensor (unlike the Christoffel symbols themselves). Show that this is indeed the case!

Problem 27

The Lagrangian for a scalar field ϕ is given by

$$\mathcal{L}/\sqrt{-g} = -\frac{1}{2} \left(\nabla^\mu \phi\right) \left(\nabla_\mu \phi\right) - V(\phi) \,.$$

Calculate the stress-energy tensor, as it would appear on the right-hand side of Einstein's equations – by varying the action with respect to $g_{\mu\nu}$ – and show that it is covariantly conserved! Compare this result to the one that follows from Noether's theorem in Minkowski space, i.e.

$$T^{\mu\nu} = -\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)} \partial^\nu \phi^a + \eta^{\mu\nu} \mathcal{L} \,.$$

(If time allows, do make sure to look up – and understand – the derivation of this expression, and that of Noether's theorem in general. The importance of Noether's theorem in physics cannot be understated.)

Problem 28

In the last lectures we just shortly revisited the (Newtonian) Kepler problem, i.e. the motion of two celestial bodies around each other due to the effect of gravity. Fill in the gaps by deriving

a) the orbit equation, $r(\phi) = r_0/(1 - \epsilon \cos \phi)$

b) the conditions on the eccentricity ϵ, and equivalently the energy E, to give hyperbolic, parabolic, elliptic or circular orbits (in terms of the quantities appearing in the effective potential).