

Lecture spring 2024:

General Relativity

## Problem sheet 9

↪ These problems are scheduled for discussion on **Thursday, 4 April 2024**.

### Problem 29

Consider an observer in the Schwarzschild geometry that is stationary with respect to the standard Schwarzschild coordinates, e.g. because she is sitting on the surface of a planet. Calculate the acceleration this observer experiences, as given by the magnitude of  $d^2x^i/d\tau^2$ . (*Hint: First re-write this as the covariant directional derivative of the 4-velocity. Why?*). Discuss the result in light of the Newtonian expectation!

### Problem 30

In the (online) lecture we just introduced Eddington-Finkelstein coordinates to describe the Schwarzschild solution,

$$v \equiv t + r^*,$$

$$u \equiv t - r^*,$$

where  $r^* = r + R_s \log(r/R_s - 1)$ . Here,  $R_s = 2GM$  is the Schwarzschild radius, and  $r, t$  are the radial and time coordinates of the Schwarzschild metric. Show explicitly that ingoing radial null geodesics are characterized by  $v = \text{const.}$ , while outgoing radial null geodesics are given by  $u = \text{const.}$  Determine (again) the form of radial light cones for the different coordinate choices – i.e.  $(t, r)$  vs.  $(v, r)$  vs.  $(u, r)$  – and discuss qualitatively the interpretation of the behaviour of the light cones around  $r = R_s$ .

### Problem 31

Do the very instructive exercise 5.5 in the book by Carroll, i.e. study how an object falling into a black hole appears to a distant observer!