Torsten Bringmann (`torsten.bringmann@fys.uio.no`)
`http://www.uio.no/studier/emner/matnat/fys/FYS4160/v24/`

Lecture spring 2024:

General Relativity

Problem sheet 10

⇝ These problems are scheduled for discussion on **Thursday, 18 April**

Problem 32

Solve problem 5.3 in the book, i.e. calculate the maximal time it takes you to reach the singularity once you have passed the event horizon of a Schwarzschild black hole.

Problem 33

Assume a spaceship that was supposed to study the environment of a Schwarzschild black hole, hovering at a distance $R_{\rm ship}$ slightly above the horizon. Now it is time to return home, but since it has used up almost all its fuel the only chance for the crew to escape is by using the recoil gained from ejecting part of the rest mass of the ship into the black hole.

a) In order for the crew to escape with the maximum rest mass, they will want to escape radially (why?). Which (minimal) values of the conserved quantities E and L describe hence the resulting geodesic, and what is the corresponding minimally required 4-velocity?

b) Use momentum conservation to derive the 4-momentum of the ejected fragment. From the result, derive the largest fraction f of the initial rest mass of the space craft that can escape to infinity. What happens to this fraction as the radius $R_{\rm ship}$ of the mission approaches the Schwarzschild radius of the black hole?

Problem 34

Calculate the area of the horizon for a) a Schwarzschild and b) a Kerr black hole by first deriving the induced metric on the horizon! Bonus question: How could you immediately have obtained the answer to a) ? *[Hint: Does the size of a light-like surface depend on the choice of coordinates?]*