Torsten Bringmann (torsten.bringmann@fys.uio.no)
http://www.uio.no/studier/emner/matnat/fys/FYS4160/v24/

Lecture spring 2024:

General Relativity

Problem sheet 11

⇝ These problems are scheduled for discussion on **Thursday, 25 April**

Legend

* If pressed for time, make sure to try solving the other problem(s) before attempting this one

† You need to wait for the lecture on Monday to be able to address this one

Problem 35

In the lecture we derived the linearized version of Einstein's equations,

$$G^{(0)}_{\mu\nu} = 8\pi G T_{\mu\nu}\,,$$

where $G^{(0)}_{\mu\nu}$ is given by Eq. (7.8) in the book.

a) By deriving the transformation properties of $h_{\mu\nu}$ directly from the way it was introduced, $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$, show explicitly that this describes a Lorentz-invariant theory of a symmetric rank-2 tensor field (h) on flat spacetime.

b)* Show that this theory follows from the Lagrangian given in Eq. (7.9) in the book, after adding a matter part $\mathcal{L}_M$!

Problem 36

Discuss in what sense the theory introduced in the previous problem is invariant under the replacement $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$, and relate this to the situation of gauge transformations in electrodynamics! Show explicitly that, for a metric decomposition as in (7.16, 7.17), the gauge transformations of linearized gravity are given by (7.33).

Problem 37*,†

The *helicity* of a particle is defined as its spin along the direction of motion. To measure this spin, one can rotate the polarization vector by an angle θ around the axis defined by the 3-momentum $\mathbf{k}$. A polarization vector with helicity λ is then an eigenstate of the rotation matrix with eigenvalue $\exp[i\lambda\theta]$.

Consider now a gravitational wave propagating in x_3 direction which, as we will shortly see in the lecture, can be described by two polarizations (h_+ and $h_\times$). Introduce circular polarizations $h_{R,L} \equiv \frac{1}{\sqrt{2}}(h_+ \pm i h_\times)$. Now transform to a new coordinate system that is related to the original one via a rotation by an angle θ in the $x_1 - x_2$ plane. How do the polarization vectors $h'_{R,L}$ in the new system look like, and what does this imply for the helicity of gravitational waves?