Torsten Bringmann (torsten.bringmann@fys.uio.no)
http://www.uio.no/studier/emner/matnat/fys/FYS4160/v22/

Lecture spring 2024:

General Relativity

Problem sheet 12

⇝ These problems are scheduled for discussion on **Thursday, 2 May 2024**.

Legend

* If pressed for time, make sure to try solving the other problem(s) before attempting this one

Problem 38

Consider linearized gravity in the transverse traceless gauge. Show that the time dependence of the metric is such that any (2D) *area* perpendicular to the direction of motion of the gravitational wave stays constant (while, as we have seen in the lecture, *lengths* change as gravitational waves pass through)!

Problem 39

As we have discussed, linearized gravity takes the form of a symmetric rank-2 tensor field $h_{\mu\nu}$ propagating in Minkowski space, where the Lagrangian is given in Eq. (7.9) in the book.

a) Use the transverse traceless gauge. Show that for a free gravitational wave, propagating in x_3 direction, this Lagrangian simplifies to an expression that is proportional to $(\partial^\mu h_{11})(\partial_\mu h_{11}) + (\partial^\mu h_{12})(\partial_\mu h_{12})$.

b)* Calculate the canonical energy momentum tensor from the Lagrangian obtained in a), i.e. the one that is obtained from Noether's theorem:

$$t_\mu^{\ \nu} = -h_{\rho\sigma,\mu} \frac{\partial \mathcal{L}}{\partial h_{\rho\sigma,\nu}} + \delta_\mu^\nu \mathcal{L}\,.$$

What is the energy flux in direction x^i?

c) Now apply this result to a gravitational wave source with quadrupole tensor $I_{ij} \equiv \int d^3x\, x_i x_j\, \rho$. What is the energy flux as measured far away from the source?

Problem 40

Estimate the number of gravitons detected by LIGO in the first BH binary merging event ever observed! How much more sensitive would LIGO thus have to be in order to detect individual gravitons?

Hint *From the answer to the previous problem, you can get the energy density of a GW with frequency* ω *and strain amplitude* f *to equal* $\omega^2 f^2/(32\pi G)$*. Compare that to the energy density of a single graviton (based on the fact that it cannot be constrained to a region smaller than its wavelength).*