UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: Relativistic quantum field theory (FYS4170 & FYS9170)

Time of exam: 9am – 1pm, November 28, 2018

Exam hours: 4 hours

This examination paper consists of 3 pages. (including the title page)

Appendices: none

Permitted materials: 2 A4 pages (two-sided) with own notes.

Make sure that your copy of this examination paper is complete before answering.

Final exam

Lecture autumn 2018: Relativistic quantum field theory (FYS4170)

$\rightsquigarrow$ **Carefully read** *all questions before you start to answer them! Note that you don't have to answer the questions in the order presented here, so try to answer those first that you feel most sure about. Keep your descriptions self-contained, but as short and concise as possible! Answers given in English are preferred; however, feel free to write in Scandinavian if you struggle with formulations! Maximal number of available points:* **45**.

Good luck!

Problem 1

In the **path integral formalism**, the n-point correlation function for any type of bosonic field ϕ can be written as

$$\langle\Omega|\phi(x_1)...\phi(x_n)|\Omega\rangle = \frac{\int \mathcal{D}\phi\, \phi(x_1)...\phi(x_n)\exp\left[i\int d^4x\,\mathcal{L}\right]}{\int \mathcal{D}\phi\, \exp\left[i\int d^4x\,\mathcal{L}\right]}\,.$$

a) Re-express this correlation function in terms of the generating functional, and demonstrate explicitly that, to any order in perturbation theory, it is sufficient to know the generating functional for the *free* theory. For the concrete example where ϕ describes a real scalar field, state this generating functional in a form that no longer involves path integrals! (*4 points*)

b) What changes in the above discussion when *fermionic* fields are involved? State the generating functional (after functional integration) also in this case! (*3 points*)

c) What is the most important conceptual difference between quantum and classical fields, and how do you think this would manifest itself in the path integral formalism? As a (new) application, consider QED and show explicitly that the interaction vertex for electrons and positrons with an *external* (classical) electromagnetic field A_μ is given by simply replacing the QED vertex rule $-ie\gamma^\mu \to -ie\gamma^\mu \tilde{A}_\mu$! (*6 points*)

Problem 2

As a follow-up to 1c), let us now describe the scattering of an electron, or positron, in a **time-independent classical electromagnetic field**. In the vertex rule just 'derived', we thus use the Fourier transform $\tilde{A}_\mu(\mathbf{q})$ of the classical electromagnetic potential, where $q \equiv p_f - p_i$ is the difference between incoming and outgoing *fermion* momenta.

a) For an external potential that is not only time-independent but also localized in space, the scattering cross section can be written as

$$d\sigma = \frac{1}{2|\mathbf{p}_i|} \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \left|\mathcal{M}(p_i \to p_f)\right|^2 (2\pi)\, \delta(E_f - E_i)\,.$$

Argue why this expression makes sense, and show that it is equivalent to

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2} \left|\mathcal{M}(p_i \to p_f)\right|^2$$

(*3 points*)

b) Compute the scattering amplitude $\mathcal{M}$ for the scattering of an electron in the Coulomb potential created by a nucleus of charge Z, i.e. $A^\mu = (Ze/4\pi r, \mathbf{0})$. How does this expression look like for the scattering of a positron? (*5 points*)
(Hint: The Fourier transform of the Coulomb potential is most easily calculated by adding a regulating factor $e^{-\mu r}$ to the potential, and then sending the 'photon mass' μ to zero at the end of the calculation.)

c) Using the above expressions, calculate the spin-averaged differential cross-section for the scattering of an electron in a Coulomb potential, as a function of the scattering angle θ. The result is known as the *Mott formula*. Take the non-relativistic limit of this expression to obtain a well-known expression for the scattering of charged particles obtained earlier by *Rutherford*. (*5 points*)
(Hint: You can simplify the resulting expression by using the trigonometric identity $1 - \cos\theta = 2\sin^2(\theta/2)$.)

Problem 3

We now turn to aspects of the full **standard model** of particle physics (SM).

a) What are the two main theoretical frameworks – beyond QED – that form the basis for the construction of the SM? Give a *brief* outline of the general ideas behind them! (*6 points*)

b) State the full gauge group of the SM, and describe in words how the concrete realizations of the two frameworks mentioned above explain the observed bosonic field content of the SM (i.e. the number and types of *bosons* in the SM)! Why do some of the SM *fermion* fields appear in doublets and triplets (and with respect to what?)? State all SM fermion fields fo which this applies (and whether they belong to a doublet or a triplet)! (*7 points*)

c) Draw all Feynman diagrams for the QED process $\ell\bar{\ell} \to \gamma\gamma$. Now consider instead the QCD process $q\bar{q} \to gg$, and draw the *additional* tree-level diagram(s) (i.e. those that do *not* have a QED analogue). At next order in perturbation theory, how many diagrams that contain ghosts contribute to the amplitude? Draw one of those, and write down the amplitude! (*6 points*)