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Lecture autumn 2020:

Relativistic quantum field theory

Problem sheet '0'

⇝ These problems are scheduled for discussion on **Friday, 21 August 2020**

Problem 1

This problem serves as a reminder to practice the use of tensor notation.

a) Write the following in index notation:

- ∇S (where S is a scalar).
- $\nabla \cdot \mathbf{A}$, $\nabla \times \mathbf{A}$ (where $\mathbf{A}$ is a 3D vector).
- Trace and Transpose of a matrix M.

b) Prove the following 3D identitites, using index notation:

- $\nabla \cdot (\nabla \times \mathbf{A}) = 0$, where $\mathbf{A}$ is a 3D vector.
- $\nabla \times (\nabla S) = 0$, where S is a scalar.

c) Are these equalities valid? Correct where necessary!

- $\partial_\mu x^\nu = \delta_\mu^\nu$
- $\partial_\mu x^\mu = 1$
- $\partial^\mu x^\nu = g^{\mu\nu}$
- $T_\alpha{}^\beta{}_\gamma = g^{\beta\mu} T_{\alpha\mu\gamma} = g^{\mu\beta} T_{\alpha\mu\gamma}$
- $T_\alpha{}^\beta{}_\beta = g_{\alpha\mu} g^{\beta\alpha} T^\mu{}_{\alpha\beta}$

d) Construct (as many as possible)

- independent Lorentz scalars from two four-vectors A and B
- independent Lorentz scalars from a rank-2 tensor T
- independent Lorentz scalars involving one (copy of a) rank-2 tensor T and some combination of two four-vectors A and B

(NB: 'independent' here does not necessarily mean 'linearly independent', so any pair (x, y) that can be written as $y = f(x)$ is not independent.)

Problem 2

Write the Schrödinger wave function $\psi(\mathbf{x})$ in Dirac ('bra-ket') notation, by expanding the states $|\psi\rangle$ in the real space basis and then projecting it onto a basis vector $|\mathbf{x}\rangle$. Then change from the real space (or position) basis to the basis of momentum eigenstates and show that this recovers the Fourier representation of $\psi(\mathbf{x})$.

Problem 3

These problems are meant to (re-)familiarize you with delta functions, Fourier tranforms and complex integration. Prove the following identities:

a)

$$\int d^3x \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} k_j f(\mathbf{x}) = i\frac{df}{dx_j}(\mathbf{y})$$

Hint: *Get rid of* k_j *before you perform the* $\int d^3k$ *integration!*

b)

$$\int_{-\infty}^{\infty} dx \, \frac{e^{ikx}}{a^2 + x^2} = \frac{\pi}{a} e^{-|k|a} \qquad (a > 0)$$

Hint: *Rewrite this integral as a closed countour integral in the complex plane; choose the upper or lower half-circle as contour such that the integral over the complex part of the contour vanishes. This means that you need to consider the cases* $k < 0$ *and* $k > 0$ *separately.*