The kinetic terms

Again: follow Faddeev-Popov approach to account for gauge freedom of A^i_μ

$$\to \int \mathcal{D}A \, \exp\Big[i\int d^4x \, \underbrace{\tfrac{1}{4}(F^a_{\mu\nu})^2}_{\equiv -\mathcal{L}_0}\Big]$$

insert $1 = \int \mathcal{D}\alpha \, \delta(G(A^\alpha)) \det\left(\frac{\delta G(A^\alpha)}{\delta \alpha}\right)$

where • $(A^\alpha)^a_\mu \equiv A^a_\mu + \frac{1}{g}\partial_\mu \alpha^a + f^{abc} A^b_\mu \alpha^c + \mathcal{O}(\alpha^2)$

$$= A^a_\mu + \frac{1}{g} D_\mu \alpha^a$$

↑ in adjoint rep

• gauge fixing condition = generalized Lorentz gauge

$$G(A) \equiv \partial^\mu A^a_\mu(x) - \omega^a(x) \overset{!}{=} 0$$

$$= \int \underbrace{\mathcal{D}A}_{\mathcal{D}A^\alpha} \underbrace{\int \mathcal{D}\alpha}_{\propto \text{ prefactor}} \exp\Big[i\int d^4x \, \underbrace{\mathcal{L}_0[A]}_{\mathcal{L}_0[A^\alpha]}\Big] \, \delta(G(A^\alpha)) \det\left(\frac{\delta G(A^\alpha)}{\delta\alpha}\right)$$

$$\xrightarrow{\int \mathcal{D}\omega} \exp\Big[i\int d^4x \Big\{\mathcal{L}_0[A^\alpha] - \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2\Big\}\Big]$$

($\int \mathcal{D}\omega$: linear combination of all possible $\omega^a(x)$, as before)

$\det\left(\frac{\delta G(A^\alpha)}{\delta\alpha}\right)$: $\frac{1}{g}\partial^\mu D_\mu$ = **not independent of A!**

recall : $\left(\prod_i \int d\theta_i^* d\theta_i\right) e^{-i\theta_i^* B_{ij} \theta_j} = \det B$

for any $B_{ij} = B_{ji}$
+ Graßmann numbers θ_i, θ_j^*

$$\Rightarrow \det(\partial^\mu D_\mu) = \int \mathcal{D}\bar{c} \int \mathcal{D}c \, \exp\left[i \int d^4x \, \bar{c}(-\partial^\mu D_\mu) c\right]$$

$\bar{c} \,\hat{=}\, \theta^*$, $c \,\hat{=}\, \theta$, $(-\partial^\mu D_\mu) \,\hat{=}\, B$

<u>interpretation</u> : c are
- anti-commuting fields (in adjoint representation)
- scalars under Lorentz transformations

$\Rightarrow$ "wrong" spin-statistics relation !!!

"Faddeev - Popov ghosts"

⇒ effect of general gauge fixing:

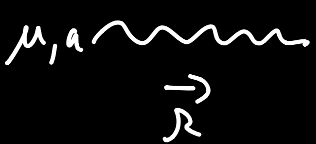
$$\mathcal{L}_{YM} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar\psi(i\not{D} - m)\psi$$

$$\Rightarrow \boxed{\mathcal{L}^{g.f.}_{YM} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar\psi(i\not{D}-m)\psi - \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 + \bar c^a(-\partial^\mu D^{ab}_\mu)c^b}$$

$$= \partial_\mu \delta^{ab} + g f^{abc} A^c_\mu$$
(in adjoint rep.!)

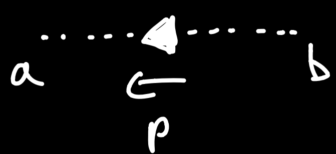
⇒ Feynman rules:

- gauge bosons:



$$= \frac{-i}{k^2 + i\varepsilon}\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}$$

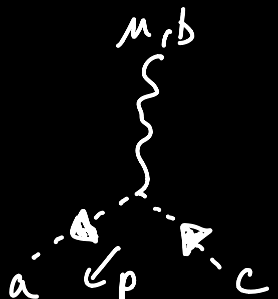
- ghosts:



$$= \frac{i\delta^{ab}}{p^2}$$

NB: arrow must be attached to propagator
(flow of "ghost number", like "fermion number" for fermions)

- interactions:



$$= -g f^{abc} p^\mu$$

Γ sign: $-\bar c^a \partial^\mu (g f^{abc} A^b_\mu c^c)$

$= +\bar c^a \overleftarrow{\partial}^\mu g f^{abc} A^b_\mu c^c$

| for outgoing ghost:

- factor of (−1) for each ghost loop!

  ⌈as for fermions, because of anticommutation⌋

$\partial_\mu = +i p_\mu$

$\Rightarrow$ vertex $= i \times (+i p_\mu) g f^{abc}$ ✓ ⌋

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ghosts:
- are <u>not</u> real particles
  $\Rightarrow$ cannot appear as external states!
- still appear in Feynman-diagrams
  ($\Rightarrow$ only at loop level!)
- technically: serve to cancel unphysical timelike and longitudinal degrees of freedom of gauge bosons

  ⌈recall $p \cdot \varepsilon = 0 \Rightarrow A$ is transverse.
  only 2 d.o.f. allowed for spin 1
  $\Rightarrow$ timelike d.o.f. also unphysical⌋