(ii) Higgs mechanism

simplest example for a gauged continuous symmetry: complex scalar field coupled to itself and a $U(1)$ field

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi)$$

where $D_\mu = \partial_\mu + ieA_\mu$ and $V(\phi) = V(|\phi|)$

e.g. $V(\phi) = -\mu^2 \phi\phi^* + \frac{\lambda}{2}(\phi^*\phi)^2$

$\Rightarrow \mathcal{L}$ is invariant under $\phi(x) \longrightarrow e^{i\alpha(x)}\phi(x)$

$A_\mu(x) \longrightarrow A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$

as in previous example: $\boxed{V(\phi=0) \neq \text{minimum} \Rightarrow \text{SSB}}$

real minimum: $V'(\phi) \overset{!}{=} 0 \rightsquigarrow 0 = -\mu^2\phi^* + \frac{\lambda}{2} 2|\phi|^2\phi^*$

$\Leftrightarrow |\phi|^2 = \frac{\mu^2}{\lambda}$

$$\Rightarrow \langle\phi\rangle = \underbrace{\frac{\mu}{\sqrt{\lambda}}}_{\equiv \phi_0} \underbrace{[\cdot e^{i\varphi}]}_{\hookrightarrow 1 \,\equiv\, \text{gauge fixing}}$$

as before: expand $\mathcal{L}$ around true vacuum:

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$$

$$\Rightarrow \quad \bullet\ V(\phi) = \dots = \underbrace{\left[-\frac{1}{2}\mu^4\right]}_{\to\ \text{constant}} + \frac{1}{2}\, 2\mu^2 \phi_1^2 + \underbrace{\mathcal{O}(\phi_i^3)}_{\text{interactions between } \phi_1 \,\&\, \phi_2}$$

$$\left.\begin{aligned} &\Rightarrow m_{\phi_2} = 0 : \text{Goldstone boson} \\ &\quad m_{\phi_1} = \sqrt{2}\,\mu \end{aligned}\right\} \text{ as before...}$$

$$\bullet\ |D_\mu \phi|^2 = (\partial_\mu \phi + ieA_\mu \phi)(\partial^\mu \phi^* - ieA^\mu \phi^*)$$

$$= \left| \frac{1}{\sqrt{2}}\partial_\mu \phi_1 + \frac{i}{\sqrt{2}}\partial_\mu \phi_2 + ieA_\mu \phi_0 + \frac{1}{\sqrt{2}} ieA_\mu (\phi_1 + i\phi_2) \right|^2$$

$$= \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 \to \text{standard kinetic terms for real scalar fields}$$

$$+ e^2 \phi_0^2 A_\mu A^\mu \to \text{"photon } \underline{\text{mass}} \text{ term"!} \quad \frac{1}{2} m_A^2 A^2 \rightsquigarrow m_A = \sqrt{2}\, e\phi_0$$

$$+ \sqrt{2}\, e \phi_0 (\partial_\mu \phi_2) A^\mu \to$$

$$\left.\begin{aligned} &+ \text{cubic terms} \\ &+ \text{quantic} \quad = \end{aligned}\right\} \text{interactions between } A^\mu \text{ and } \phi_1, \phi_2$$

remarks : • "mass term" $\mathcal{L} \supset (\phi_2 \;\; A) \begin{pmatrix} m_2^2 & m_{2A}^2 \\ m_{2A}^2 & m_A^2 \end{pmatrix} \begin{pmatrix} \phi_2 \\ A \end{pmatrix}$

$\rightsquigarrow$ need to diagonalize for mass eigenstates
= physical states

• in this case, the Goldstone boson (ϕ_2) does not appear as an independent physical particle !

E.g. "unitary gauge" : choose $\alpha(x)$ such that $\underbrace{\phi'(x)}_{\text{"}\phi_1 + v\text{"}} = e^{i\alpha(x)} \phi(x)$ is real.

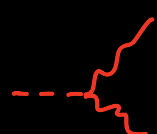
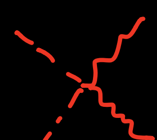
$$\Rightarrow \mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi)$$

$$= -\frac{1}{4}(F_{\mu\nu})^2 + \underbrace{(\partial_\mu \phi' + ie A_\mu \phi')(\partial^\mu \phi' - ie A^\mu \phi')}_{=(\partial \phi')^2 + e^2 \phi'^2 A_\mu A^\mu} - V(\phi')$$

• still : if $\langle \phi' \rangle \equiv v \neq 0$

$\rightsquigarrow$ • mass term for A_μ (same as above!)

• couplings $\propto \phi A_\mu A^\mu$, $\phi^2 A_\mu A^\mu$

• recall d.o.f. of – massless vector = 2 (transverse)
– massive vector = 3 (2 transverse + 1 longitudinal)
$= 2 S_A + 1$ as in QM!

⇝ $\boxed{A_\mu \text{ has acquired its mass, and an additional d.o.f., by "eating" the Goldstone boson!}}$

[general statement!]

12. Electroweak theory (Glashow - Weinberg - Salam)

ingredients: i) gauge group $SU(2) \times U(1)$ → "standard YM"

NB: $\neq U(1)_{EM}$! ⇝ fermion multiplets

ii) Higgs field

= scalar field ϕ in spinor rep. of G,

i.e. a complex doublet:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^2 + i\chi^3 \\ H - i\chi^3 \end{pmatrix}$$

, i.e. 4 real scalar degrees of freedom

$$\phi \xrightarrow[\text{trafo}]{\text{gauge}} e^{i\alpha^a(x)\tau^a} e^{iY_\phi \beta(x)} \phi$$

$\tau^a = \frac{\sigma^a}{2}$ for $SU(2)$

Y: U(1) charge "hypercharge"

choose $Y_\phi = \frac{1}{2}$ for ϕ

iii) SSB, i.e. $\langle \phi \rangle \neq 0$

in general, the following parameterization is always possible:

$$\phi = U(x) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H = v + h(x) \end{pmatrix}$$

general $SU(2)$ trafo

physical Higgs field

$$\longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} \Rightarrow \bullet \langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

"unitary gauge": $U(x) \equiv \mathbb{1}$

$\circ \ \langle h \rangle = 0$

gauge boson masses

expectation from SSB ?

$Y_\phi = \frac{1}{2}$

$$\phi \longrightarrow e^{i\alpha^a \tau^a} e^{iY\beta} \phi = e^{i\alpha^1 \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + i\alpha^2 \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + i\alpha^3 \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i\beta \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \phi$$

$\rightsquigarrow \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ is left invariant by $\alpha^1 = \alpha^2 = 0$,
$\alpha^3 = \beta$

$$\left[\Rightarrow \left\{ i\alpha^3 \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i\beta \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right]$$

$\Rightarrow$ • before SSB: 4 continuous symmetries (α^i, β)

• after " : 1 " " $(\beta = \alpha^3)$

$$\boxed{\begin{array}{l} \Rightarrow \underline{\text{expect}}: \blacktriangleright \text{ 3 massive gauge bosons} \\ \qquad (\rightsquigarrow \text{"eat" components } \chi^1, \chi^2, \chi^3) \\ \qquad \longrightarrow W^\pm, Z \\ \blacktriangleright \text{ 1 massless gauge boson} \rightsquigarrow \gamma \text{ (photon)} \\ \blacktriangleright \text{ 1 real (physical Higgs boson } h) \end{array}}$$

now the details...

$$\boxed{D_\mu \phi = (\partial_\mu - i g A_\mu^a \tau^a - i Y_\phi g' B_\mu)\phi}$$

$Y_\phi = 1/2$

g: SU(2) coupling strength; A_μ^a: SU(2) gauge bosons; g': U(1) coupling strength; B_μ: U(1) gauge boson

$$\mathcal{L}_\phi = |D_\mu \phi|^2 - V(\phi) \quad [+ \text{couplings to fermions}]$$

$|D_\mu\phi|^2$: this term gives gauge boson masses for $\phi = \langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$

$h(x)=0$

$$\rightsquigarrow |D_\mu \phi|^2 \supset \frac{1}{2}(0 \;\; v)\left(+igA_\mu^a \frac{\sigma^{a\dagger}}{2} + \frac{i}{2} g' B_\mu\right)\left(-igA^{\mu a}\frac{\sigma^a}{2} - \frac{i}{2} g' B^\mu\right)\begin{pmatrix}0\\v\end{pmatrix}$$

$\times 1/2 \times 2$ $1/2 \times 2$

$\downarrow$ use explicit representations of σ^a

...

$$= \frac{1}{2}\left\{ \underbrace{\frac{v^2 g^2}{4}(A_\mu^1)^2 + \frac{v^2 g^2}{4}(A_\mu^2)^2}_{\text{2 vector bosons w/ mass } \frac{vg}{2}} + \underbrace{\frac{v^2}{4}(g A_\mu^3 - g' B_\mu)^2}_{\text{1 vector boson}} \right\}$$

$$\rightsquigarrow \boxed{W^\pm \equiv \frac{1}{\sqrt{2}}(A_\mu^1 \mp i A_\mu^2) \qquad m_W = g\frac{v}{2}}$$

$$\boxed{Z_\mu^0 \equiv \frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^3 - g'B_\mu)^2 \qquad m_Z = \sqrt{g^2+g'^2}\,\frac{v}{2}}$$

convention to ensure correct normalization of mass + kin. terms

+ massless field orthogonal to Z^0_μ :

$$\boxed{A_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(g' A_\mu^3 + g B_\mu\right) \; ; \; m_A = 0}$$

$\Rightarrow \mathcal{L} \supset -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2$

$\rightsquigarrow$ as expected ✓

now introduce standard conventions :

- $$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix}$$

  "weak mixing angle" / "Weinberg angle"

  w/ $\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$ $\rightsquigarrow$ $\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$

  $\rightsquigarrow$ $\boxed{m_W = m_Z \cos\theta_W}$

- $$\boxed{e \equiv g \cdot \sin\theta_W = g' \cos\theta_W}$$

- $\boxed{Q \equiv T^3 + Y}$ $\Rightarrow Q_h = -\frac{1}{2} + \frac{1}{2} = 0$

T^3: eigenvalue of $\tau^3 = \frac{\sigma^3}{2}$

= projection of "[weak] isospin"

Y: U(1) charge : "hypercharge" [NB: sometimes conventional factor of 2!]

- $t^{\pm} \equiv \tau^1 \pm i\tau^2 = \frac{1}{2}(\sigma^1 \pm i\sigma^2)$ $\rightsquigarrow$ $t^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$; $t^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$\Downarrow$$

$$\boxed{D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}\underbrace{\left(W_\mu^+ t^+ + W_\mu^- t^-\right)} - i\frac{g}{\cos\theta_w} Z_\mu \left(t^3 - \sin^2\theta_w Q\right) - \underbrace{ie A_\mu Q}}$$

↳ $W^{\pm}$ couples only to lower / upper component of doublet

as expected for a photon (!) coupling to a particle w/ EM (!) charge Q