Gauge boson coupling to Higgs doublet:

$$\boxed{D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+ t^+ + W_\mu^- t^-\right) - i\frac{g}{\cos\theta_w} Z_\mu \left(t^3 - \sin^2\theta_w Q\right) - ie A_\mu Q}$$

Coupling to fermions

NB: Higgs field in same representation as fermion fields

⤳ can use <u>same</u> expression for D_μ

⤳ only need to determine
- T^3 → upper ($+\frac{1}{2}$) or lower ($-\frac{1}{2}$) doublet component of $SU(2)$
- $Y = Q - T^3$

recall: $\psi_L \equiv \frac{1-\gamma^5}{2}\psi$, $\psi_R \equiv \underbrace{\frac{1+\gamma^5}{2}}_{\equiv P_{L/R}}\psi$ ⇒ $\psi = \psi_L + \psi_R$

- $\bar{\psi}_L = (P_L\psi)^\dagger\gamma^0 = \bar{\psi}P_R$
- $P_R P_L = 0$

⇒ • $\bar{\psi}i\partial\!\!\!/\psi = \bar{\psi}_L i\partial\!\!\!/\psi_L + \bar{\psi}_R i\partial\!\!\!/\psi_R$

⇒ no mixing of ψ_L and ψ_R

• $\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$

⇒ mixing of ψ_L and ψ_R!

⇒ massless fermions are <u>"chiral"</u>, they transform in different representations of Lorentz group

[$\psi_R \to e^{i\beta_R}\psi_R$; $\psi_L \to e^{i\beta_L}\psi_L$]

$\Rightarrow$ can assign different $SU(2)$, $U(1)$ charges!

$\boxed{\text{GWS model: let } \underline{\text{only}} \text{ left-handed fields couple to } W \text{ boson!}}$

(experimental input!)

i.e. $\boxed{G_{SM} = SU(2)_L \times U(1)_Y \times SU(3)_c}$

left-handed fields = $SU(2)$ doublets $\Rightarrow T^3 = \pm 1/2$

right-handed fields = $SU(2)$ singlets $\Rightarrow T^3 = 0$

only quarks are charged under $SU(3)$

$\rightarrow$ $SU(3)$ triplets (three different colours)

$\rightsquigarrow$ different values of Y, follows from $Y = Q - T^3$!

e.g. left-handed fields:

$$E_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \Rightarrow Y = \underbrace{0 - \left(+\tfrac{1}{2}\right)}_{\nu_e} = \underbrace{-1 - \left(-\tfrac{1}{2}\right)}_{e^-} = -\tfrac{1}{2}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \Rightarrow Y = \underbrace{\tfrac{2}{3} - \left(\tfrac{1}{2}\right)}_{u} = \underbrace{-\tfrac{1}{3} - \left(-\tfrac{1}{2}\right)}_{d} = +\tfrac{1}{6}$$

corresponding right-handed fields:

$\cancel{\nu_e^R}$, e_R^-, u_R, d_R $\Rightarrow Y = Q$

$$\Rightarrow \mathcal{L} \supset \bar{E}_L (i\not{D}) E_L + \bar{e}_R (i\not{D}) e_R + \bar{Q}_L (i\not{D}) Q_L + \bar{u}_R (i\not{D}) u_R + \bar{d}_R (i\not{D}) d_R$$

e.g.: $D^\mu = \partial^\mu + ie(-1)A^\mu + ig\frac{\sin^2\theta_W}{\cos\theta_W}(-1)Z^\mu$

both terms from $U(1)_Y$

[no coupling to SU(2)!]

→ would be only one term if expressed as coupling to B^μ!

→ $i g' \underset{= -\frac{1}{2}}{Y_{E_L}} B^\mu$

BUT: fermion masses are now forbidden by gauge invariance!

e.g. $\mathcal{L} > -m(\bar{e}_L e_R + \bar{e}_R e_L)$ cannot exist!

– different SU(2) reps of e_L, e_R!

– " $U(1)_Y$ charges " " "

Fermion masses

There is (exactly) one further gauge-invariant combination of fields, compared to what we have considered so far!

"Yukawa coupling"

spinor indices

e.g. for electrons: $\mathcal{L} \supset - \lambda_e \left(\bar{E}_L^i \cdot \phi\right) e_R^i + h.c. \quad (*)$

• SU(2) charges: $\underbrace{\bar{E}_L \cdot \phi = \bar{E}_L^a \phi^a}_{\text{invariant under } SU(2)}$

doublet in conjugate spinor rep. of SU(2) — doublet in spinor rep. of SU(2)

invariant under SU(2)

• U(1) charges: $Y_{\bar{E}_L} = -\left(-\frac{1}{2}\right)$

$Y_\phi = \frac{1}{2}$

$Y_{e_R} = -1$

$\Sigma = 0 \Rightarrow$ invariant under U(1)!

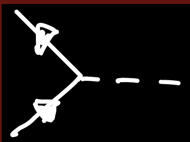
SSB: replace $\phi \to \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$ (in unitary gauge)

$$\overset{(*)}{\Rightarrow} \mathcal{L} \supset - \frac{\lambda_e}{\sqrt{2}} v \, \bar{e}_L e_R + h.c.$$

$\Rightarrow$ mass term with $\boxed{m_e = \frac{\lambda_e}{\sqrt{2}} v}$

$$-\frac{\lambda_e}{\sqrt{2}} h \, \bar{e}_L e_R + h.c$$

$\Rightarrow$ interaction of Higgs boson with fermions



$$= -i \frac{m_t}{v} \qquad \propto m_t$$

v: 246 GeV

$\Rightarrow$ extremely predictive (no free parameter)!

coupling to upper components of doublets (only for quarks!)

$$Q_L \cdot \phi \longrightarrow Q_L \cdot \tilde{\phi} \equiv \varepsilon_{ij} Q_L^i \phi^j$$ [$\rightsquigarrow$ see exercises]

The Higgs boson

$$\mathcal{L}_\phi \supset |D_\mu \phi|^2 - V(|\phi|) = |D_\mu \phi|^2 + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

only dim-4 possibility with $\langle \phi \rangle \neq 0$

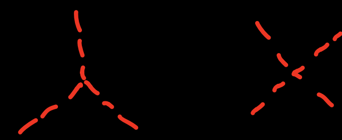
$$\Rightarrow \frac{\partial V}{\partial \phi^\dagger} = \mu^2 \phi - 2\lambda (\phi^\dagger \phi) \phi \overset{!}{=} 0$$

$$\Rightarrow \phi_0^\dagger \phi_0 = \frac{\mu^2}{2\lambda} \equiv \frac{1}{2} v^2$$

$$\Rightarrow \lambda = \frac{\mu^2}{v^2}$$

unitary gauge: $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

$\Rightarrow$ • $V(\phi) = \dots = \frac{1}{2} \underbrace{2\mu^2}_{m_H^2} h^2 - \sqrt{\frac{\lambda}{2}}\, m_h h^3 - \frac{1}{4}\lambda h^4$

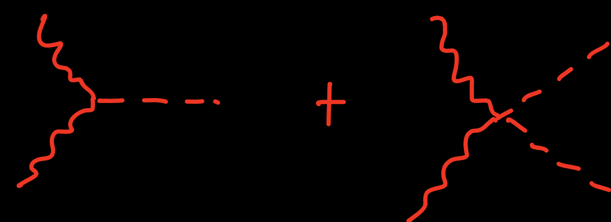


uniquely determined relation between 3- and 4-point vertices!

• $|D_\mu \phi|^2 = \dots = \frac{1}{2}(\partial_\mu h)^2$

$+ \left[m_W^2 W^{\mu +} W^-_\mu + \frac{1}{2} m_Z^2 Z^\mu Z_\mu \right] \times$

$\times \left(1 + \frac{h}{v}\right)^2$

$\Rightarrow$ 

again with uniquely defined relation between 3- and 4-point vertices (no free parameters!)