UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: Relativistic quantum field theory (FYS4170)

Day of exam: October 15, 2020

Exam hours: 4 hours

This examination paper consists of 4 pages. (including this title page)

Appendices: none

Permitted materials: 2 A4 pages (two-sided) with own notes.

Make sure that your copy of this examination paper is complete before answering.

Midterm exam

Lecture autumn 2020: Relativistic quantum field theory (FYS4170)

$\rightsquigarrow$ **Carefully read** *all questions before you start to answer them! Note that you don't have to answer the questions in the order presented here, so start with those that you feel most sure about. Keep your descriptions self-contained, but as short and concise as possible! Answers given in English are preferred; however, feel free to write in Scandinavian if you struggle with formulations!*
Maximal number of available points: **50**.

Good luck!

Problem 1 *(6 points)*
Let us start by revisiting one of the important **conceptual aspects** of quantum field theory.

a) Demonstrate that a naive quantization of relativistic particles, along the lines of how Schrödinger quantised a non-relativistic particle, leads to the existence of negative energies! Why is this not acceptable for a physical theory? (*3 points*)

b) When adopting the canonical quantization prescription that we learned about in this course, we still encountered the same relativistic wave equation as one would expect from the argument in *a)*. What is the fundamental difference, both conceptually (in the procedure) and in terms of interpretation (of the resulting equation of motion)? Explain briefly (in words) how to obtain the Hamiltonian H of a scalar quantum field, and state it in a form where it becomes manifest that its eigenvalues are bounded from below! (*3 points*)

Problem 2 *(9 points)*
Compute the following expressions involving **Dirac matrices** and spinors (p refers to the 4-momentum of a physical, i.e. on-shell particle):

a) $\gamma_\mu \gamma^\rho \gamma^\mu$ *(2 points)*

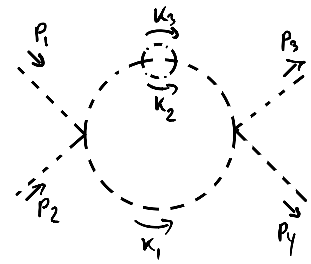
b) $\mathrm{Tr}\left[\gamma^\mu \not{p} \gamma^\nu \not{p}\right]$ *(3 points)*

c) $(\bar{v}^r P_L \gamma^\mu u^s)^*$ *(2 points)*

d) $(\bar{u}^r \gamma^5 \gamma^\mu \gamma^\nu u^s)^*$ *(2 points)*

Problem 3 *(14 points)*
We have learned how to use **Feynman rules** to compute both n-point correlation functions and the invariant matrix element.

a) The figure shows an example diagram for ϕ^4 theory. What is the contribution to the *invariant matrix element* from this diagram? *(4 points)*
[Please use the same momentum conventions as indicated. Don't try to evaluate non-trivial integrals (i.e. those not just over delta functions).]



b) Consider the QED process $e^+e^- \to \gamma\gamma$. Draw all Feynman diagrams that contribute to the corresponding *n-point correlation function* to fourth order in e. Indicate those that also contribute to the invariant matrix element, and argue in each case why the others do not contribute! *(8 points)*
[You may save time time by not actually drawing every single *diagram – but then you must explain well how to obtain the missing ones, based on those that you have drawn.]*

c) Calculate the cross section for $e^+e^- \to \gamma$, to fourth order in $\alpha_{\rm em} \equiv e^2/(4\pi)$! *(2 points)*
[Hint: A good argument for the result may not require a detailed calculation...]

Problem 4 *(21 points)*
Let us consider a theory containing Dirac fermions ψ and a **pseudoscalar** ϕ:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{1}{2}\left(\partial_\mu\phi\right)^2 - \frac{1}{2}m_\phi^2\phi^2 - ig\bar{\psi}\gamma^5\psi\phi\,.$$

We focus on the process $\psi\bar{\psi} \to \phi\phi$, i.e. the production of pseudoscalars (with outgoing four-momenta k_1^μ and k_2^μ) from fermion anti-fermion annihilation (with incoming four-momenta p_1^μ and p_2^μ).

a) Show that the vertex rule for this theory is given by $-g\gamma^5$, by computing the 3-point function $\left\langle\Omega|T\left\{\psi\bar{\psi}\phi\right\}|\Omega\right\rangle$ to leading order (i.e. to lowest non-vanishing order in g)! *(4 points)*

b) Draw all leading-order diagrams that contribute to this process (i.e. to $\mathcal{M}$), and state their values by applying Feynman rules. Simplify these expressions as far as possible, by using the Dirac equation! *(5 points)*

c) Now compute the absolute value of the invariant amplitude (squared), for *unpolarized* particles in the external states. Show that, under the assumption that the produced particles are very light (i.e. $m_\phi \ll m$), this expression can be simplified to *(6 points)*

$$-\frac{g^4}{2}\frac{(k_1\cdot k_2)^2}{(p_1\cdot k_1)(p_1\cdot k_2)} + g^4\left(\frac{p_1\cdot k_2}{p_1\cdot k_1} + \frac{p_1\cdot k_1}{p_1\cdot k_2}\right)$$

d) Finally, compute the differential, unpolarized cross section, $d\sigma/d\cos\theta$, in the center-of-mass frame (again in the limit of $m_\phi \ll m$)! Convince yourself that the result has the expected behaviour at high energies! *(6 points)*
[Plus a bonus of 3 points if you are able to explain the behaviour for very low energies as a consequence of a discrete symmetry!]

Formulas that you might find useful:

$$\gamma^{0\dagger} = \gamma^0\,, \quad \gamma^{i\dagger} = -\gamma^i \qquad \rightsquigarrow \qquad \gamma^{\mu\dagger}\gamma^0 = \gamma^0\gamma^\mu \tag{1}$$

$$\gamma^{5\dagger} = \gamma^{5T} = \gamma^5\,, \quad P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma^5), \quad (\gamma^5)^2 = 1\,, \quad \{\gamma^5, \gamma^\mu\} = 0 \tag{2}$$