University of Oslo

FYS4170/9170 — Relativistic Quantum Field Theory

Problem set 13

Problem 1 Quantization and energies (2019 final exam, slightly modified) (T. Bringmann)

Let us start by revisiting one of the important **conceptual aspects** of quantum field theory.

a) Demonstrate that a naive quantization of relativistic particles, along the lines of how Schrödinger quantised a non-relativistic particle (*i.e. taking* $p^\mu \to i\partial^\mu$ *and enforcing the relativistic energy-mass relation* $p^2 = m^2$), leads to the existence of negative energies! Why is this not acceptable for a physical theory?

b) When adopting the canonical quantization prescription that we learned about in this course, we still encountered the same relativistic wave equation as one would expect from the argument in a). What is the fundamental difference, both conceptually (in the procedure) and in terms of interpretation (of the resulting equation of motion)? Explain (in words) how to obtain the Hamiltonian H of a scalar quantum field, and state it in a form where it becomes manifest that its eigenvalues are bounded from below!

c) Explain how the requirement of $H > 0$ enforces particles described by the Dirac equation to be fermions! *[Following the same procedure as in b), introducing commutation relations for the fermion fields, would lead to the expression*

$$H = \int d^3x \mathcal{H} = \int \frac{d^3k}{(2\pi)^3} \sum_{s=1,2} \omega_{\mathbf{k}} \left(a_{\mathbf{k}}^{s\dagger} a_{\mathbf{k}}^s - b_{\mathbf{k}}^{s\dagger} b_{\mathbf{k}}^s \right).$$

Identify the problem with this expression and argue that it is fixed by replacing the commutation relations with anti-commutation ones. Then show that the resulting creation/annihilation operators create/annihilate fermions.]

Problem 2

a) What is a scalar? A pseudo-scalar? A vector? Pseudo-vector? Give examples.

b) Is the Lagrangian scalar? The Hamiltonian?

c) Is the three-dimensional delta function a scalar? If yes, why? If no, how can you scale it such that it becomes scalar? Is $\int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}}$ scalar? Why/why not?

d) What are the conserved quantities associated with translational invariance (in space and time)?

e) What is the advantage of using the interaction picture in perturbation theory?

f) What does Wick's theorem tell you about the vacuum expectation value of products of fields?

g) Why do not the disconnected diagrams contribute to the correlation function?

h) The S matrix is defined as the overlap between in- and out-states. Are the in-state and/or out-state eigenstates of the Hamiltonian H? Of the free-field Hamiltonian H^0?

i) How can you obtain the S matrix elements from correlation functions?

j) What are the similarities and differences between Yukawa theory and QED?

k) Why can we not quantize the photon field in the same way as fermion and scalar fields, and how is this dealt with?

l) Can you build a laser producing longitudinal or scalar photons?

m) What is Ward's identity? Mention one application of it.

n) Why does the photon remain massless at all orders in perturbation theory, without the need for renormalization?

o) Explain how infrared divergences occur, and how they are canceled.

p) Explain the main idea behind renormalization. Specialize the discussion to the case of mass and electric charge.