Solution to exercise in FYS4715: A simple model of neuronal calcium dynamics - November 19, 2019

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The python file 'Exercise_solution.py' contains the complete model (after problems 1-3 were solved). You may turn off the calcium channel (to simulate problem 1) by setting the parameter 'p['gbar_Ca'] = 0'.

Problem 1: Speed up the Hodgkin-Huxley model:

Problem 1i: When the stimulus was 10 μA/cm^2 and lasted in 25 ms, the original HH model produced 2 broad action potentials (Fig. 1A). When all rates were speeded up by a factor 2 (and no calcium current: 'p['gbar_Ca'] = 0'), it responded to the same stimulus by eliciting three action potentials that were briefer in duration (Fig 1B).

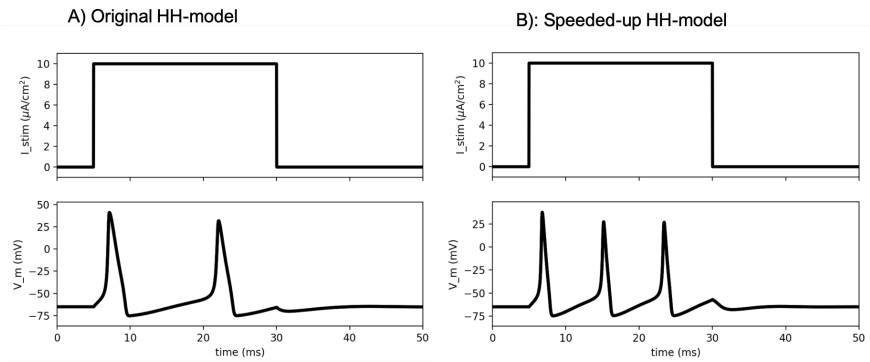

Figure 1: Response of original (A) and speeded-up version (B) of the HH-model.

Problem 1ii: The $f - I$ curve (firing rate as function of input current) of the modified HH-neuron should be something like in Figure 2. The threshold for sustained firing is about 6.5 μA/cm^2. Note that model has a lowest sustained firing rate of almost 100 Hz (i.e., it can not have an arbitrarilly low sustained firing rate). Neurons with this property are often called Type 2 or Class 2 neurons.

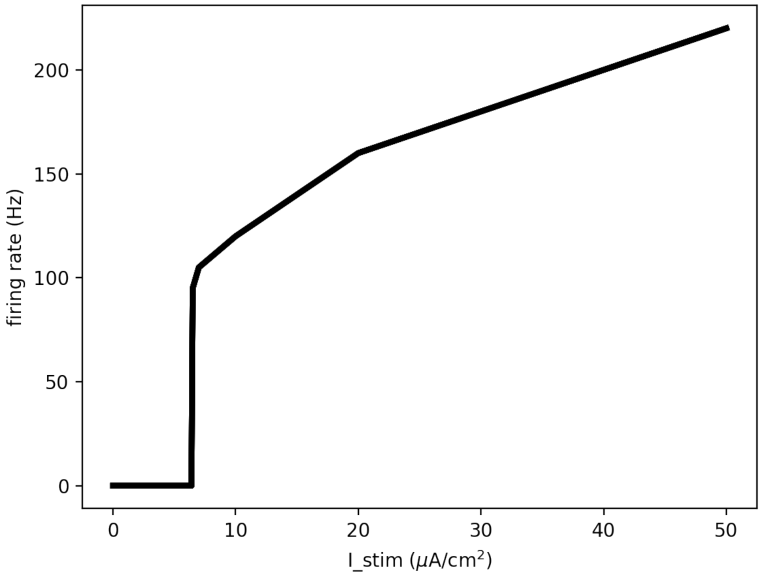

Figure 2: f-I-curve of speeded-up HH-model.

1. **Problem 2: Add a calcium channel to the (speeded-up) model:**

Problem 2i-ii The solution code has I_{Ca} and the resulting Ca2+ dynamics added. With the original stimulus conditions, and with a calcium conductace set to p['gbar_Ca'] = 10, the final model produces a response as in Fig 3. Comparing the first spike (red vs black), we see that the addition of I_{Ca} (blacks) drags out its duration slightly. This, in turn, makes the neuron repolarize more slowly, so that it takes a bit longer for it before it is able to fire its next action potential.

Problem 3: Add calcium dynamics to the model:

Problem 3i: The solution code shows how Ca^2+ dynamics was added to the model.

Problem 3ii: With a brief (2 ms) stimulus injection (still 10 μA/cm^2), the neuron responded with a single action potential. Figure 4 shows the calcium and voltage response. With the chosen parameters, [Ca^{2+}] increases rapidly from its basal level (50 nM) and up to about 200 nM during an action potential. This is a fairly realistic response magnitude (although this varies quite a lot between different neurons). The calcium response has about the same time-course as an action potential, and is thus a quite good indicator.

Problem 3iii: Figure 5 shows the effect of varying different parameters by multiplying them, one by one, by a factor 2 (black curve is default). By increasing k_{Ca} (red), you increase

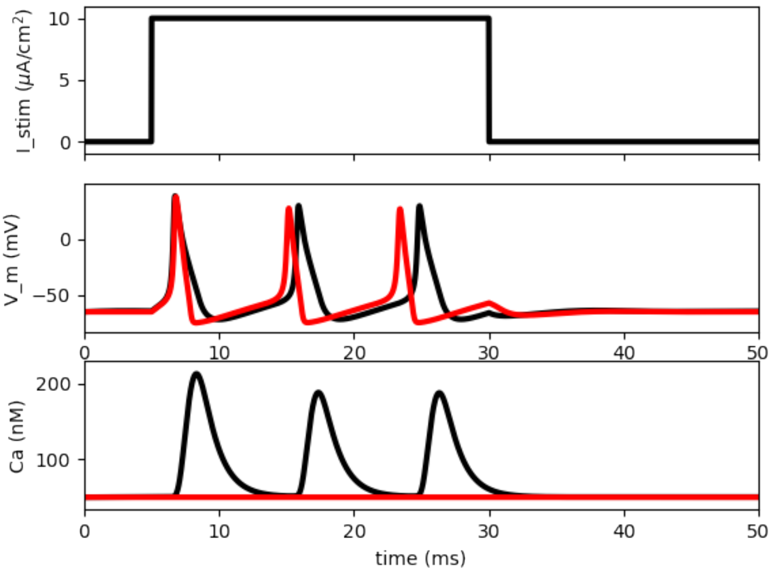

Figure 3: HH-model without (red) and with (black) I_{Ca}.

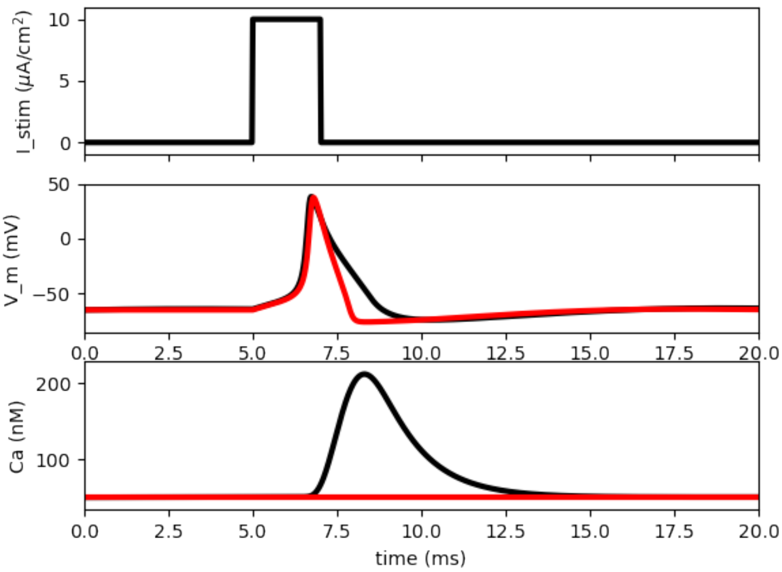

Figure 4: HH-model without (red) and with (black) I_{Ca}.

the [Ca^{2+}] responses without affecting anything else in the model, since the current stays the same, and nothing in the model depends on [Ca^{2+}] as such. If you instead increase g_{Ca} (blue) you will increase both the calcium response and I_{Ca}, and thus its effect on the action potential. If you increase τ_{Ca} (green), the calcium response amplitude gets slightly larger, but the main effect is that it falls back to baseline more slowly.

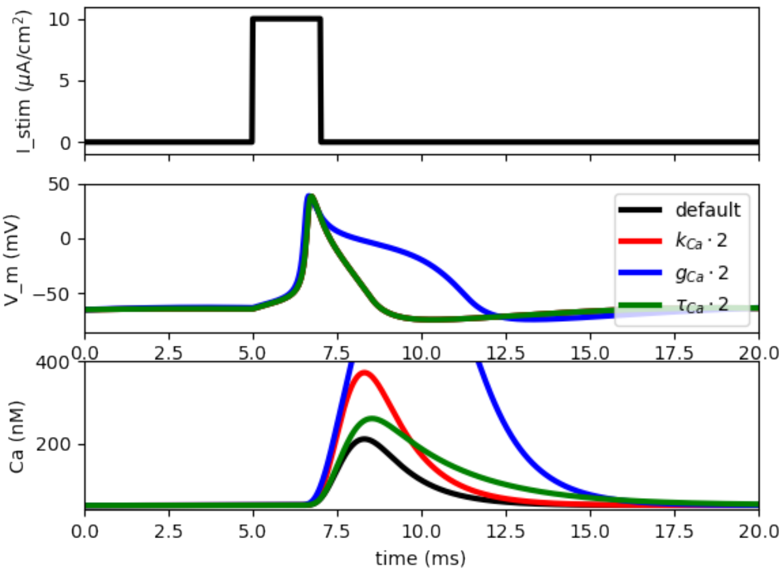

Figure 5: Effects on action potential and calcium response by varying selected model parameters.

Problem 3iv: If τ_{Ca} is slow, [Ca^{2+}] does not have time to return to the basal level between two consecutive action potentials. It will then increase gradually until the neuron reaches a dynamic steady state where the calcium entering per action potential equals the decay between two action potentials. This is illustrated in Figure 6, where τ_{Ca} was set to the rather extreme value of 5000 ms. The figure illustrates how the dynamic steady state levels for[Ca^{2+}] depends on firing rate, and is higher for the neuron that fires fastest. The neuron can in principle use the dynamic steady state concentration to "read off" its own firing rate.

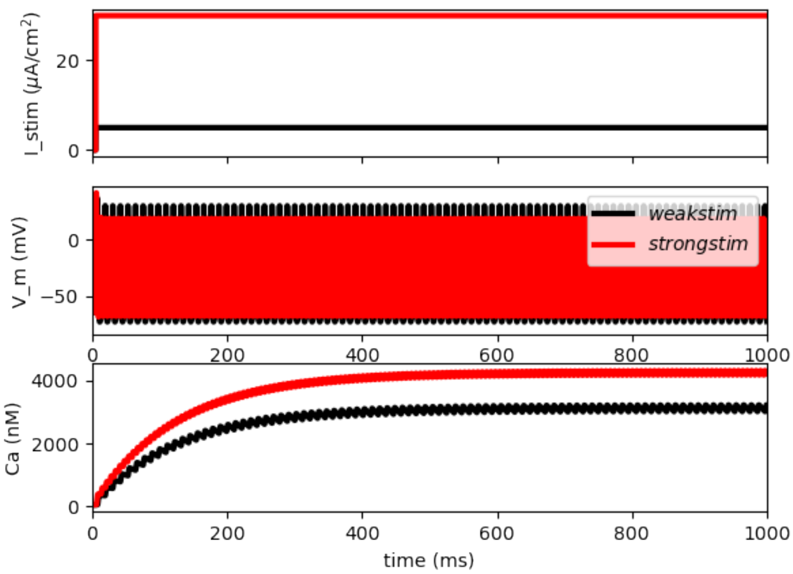

Figure 6: Calcium dynamics for fast and slow firing rates with slow calcium decay.