


Physics of adherent cells, Part 1

Force & movement are central to the behaviour of biological systems.

Animal cells are very sensitive to environmental cues

Differentiation of stem cells can be guided by the mechanical or adhesive properties of the substrate

- exciting regenerative medicine
- fundamentally not well understood

Physics :  How can the cell know the properties of the surroundings?

- * actively generate a force
- * transmit force to surroundings
- * sense response

* Soft matter

- liquid crystals
- colloidal dispersions
- emulsions
- fluid membranes
- gels

"Soft" interaction energies $\sim kT$
 \Rightarrow sensitive to thermal fluctuations

Biology : * active remodelling controlled by

- genetics
- signalling networks

\Rightarrow active matter



Cell response to mechanical perturbations

Short timescale^(s): elastic (passive)
 Long —————^(mic →): active reorganization of CSK

Stress & strain $\sigma = E \epsilon$ Stiffness / rigidity
 $\frac{1}{2} \epsilon$ $\frac{1}{L_0}$ E - Yocings modulus
(spring constant)
 $[\sigma] = [E] = \text{Pa}$

typical $E \sim 10 \text{ kPa}$


length scale of deformation \sim membrane thickness $l \sim 10 \text{ nm}$
 macromolecule

\Rightarrow Strain energy $\sigma \cdot l \sim 10 \text{ nm} \cdot 10^{-16} \cdot 10^4 \text{ N} \sim 10 \text{ nm pN}$
 $kT = 4.1 \text{ nm pN}$
 \approx Thermal noise

In addition: "biological noise"

For example force generation of x molecular motors
 * actin polymerization

Soft matter

Liquid crystals  - They move around like fluid
 - They can align

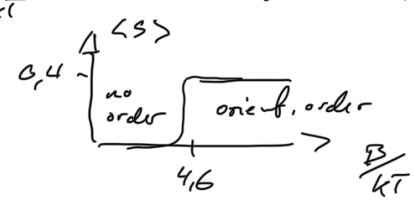
No direction \Rightarrow order parameter is even function of angle

$$S_i = \frac{1}{2} (3 \cos^2 \theta_i - 1)$$


Energy of element $U_i = B \langle S \rangle S_i$ $\langle S \rangle$ is mean orient.

$$\langle S \rangle = \frac{1}{Z} \int d\Omega S_i e^{-U_i/kT}$$

B interaction const



Polymers single polymers DNA, actin, proteins, microtub.

Flexible: No energy penalty for bending \Rightarrow perfectly isotropic
 random walk  $R^2 \propto N$ (MSD $\propto t$)
 (synthetic polymers) — number of monomers

Semiflexible

Actin, DNA ... bend on longer length scales L
 \Rightarrow Energy \propto curvature² $H_b = \frac{\kappa}{2} \int ds \left(\frac{\partial^2 R}{\partial s^2} \right)^2$

Persistence length $\ell \sim \frac{\kappa}{kT}$ R -bending modulus $\ell \sim \sqrt{\frac{\kappa}{kT}}$
 $\ell < \{$: stiff, rod like

$\ell > \{$ excluded volume RW: $R \sim N^{3/5}$ (coef $N^{1/2}$)

Polymer gels

Actin & microtubules crosslink in CSK

Timescale $t <$ days of sec
 crosslinks are rigid \Rightarrow elastic gel

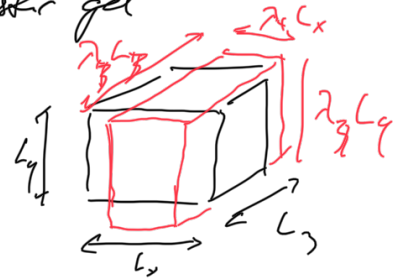
Flexible polymers (RW, cubic)

$$\epsilon_i = \lambda_i - 1 \quad (i = x, y, z)$$

incompressible: $\lambda_x \cdot \lambda_y \cdot \lambda_z = 1$

Free energy $F \propto \frac{kT}{2} (\lambda_x^2 + \lambda_y^2 + \lambda_z^2 - 3)$

small ϵ $\sigma \propto gkT \cdot \epsilon$
 large ϵ nonlinear



$$\left(P \propto g \frac{N}{V} kT \right)$$

$$\Delta P \propto g kT (\Delta V + \dots)$$

Semiflexible polymers

small $\epsilon \propto \frac{\sigma}{\zeta}$, large nonlinear

Longer time-scales: Polymer networks continuously remodel
 \Rightarrow crosslinks disappear
 \Rightarrow creep / flow

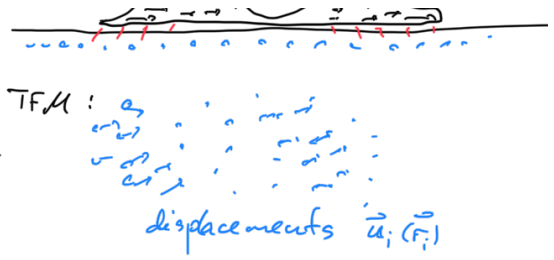
Elasticity

smooth displacement field $\vec{u}(\vec{r})$



strain tensors :

$$u_{ij}(\vec{r}) = \frac{1}{2} \left(\frac{\partial u_i}{\partial r_j}(\vec{r}) + \frac{\partial u_j}{\partial r_i} \right) + \text{NL}$$



Free energy per unit volume

$$f_e = \frac{K}{2} \left(\sum_i u_{ii} \right)^2 + \mu \sum_{ij} \left(u_{ij} - \frac{1}{3} \delta_{ij} \sum_l u_{ll} \right)^2$$

Young's modulus: $E = \frac{9K\mu}{3K+2\mu}$

Poisson's ratio $\nu = \frac{3K-2\mu}{6K+2\mu}$ tissue: $\nu = \frac{1}{3} \rightarrow \frac{1}{2}$

Incompressible materials

$$\frac{K}{\mu} \rightarrow \infty, \quad \left[\nu = \frac{1}{2} \right]$$

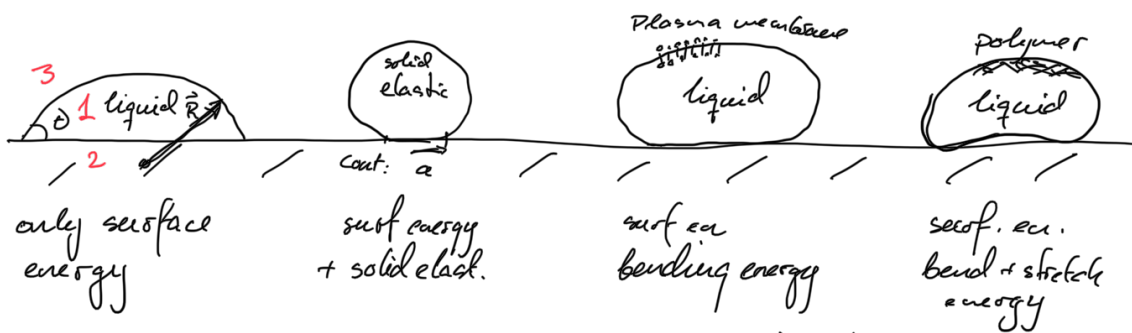
water
Bulk modulus
 $K = 2.1 \text{ GPa}$
 $= -V \frac{dP}{dV}$
 $E_v = \frac{dV}{V} = -\frac{dP}{K}$
 $\frac{1 \text{ atm}}{K} = 10^{-9} = 0.1 \text{ e}$
 $10^4 \cdot 10^{-5} = 10^{-1} \text{ e}$
strain
1 atm

local Force density: $f_i(\vec{r}) = - \sum_j \frac{\partial \sigma_{ij}}{\partial r_j} = - \frac{E}{1+\nu} \left[\sum_j \frac{\partial u_{ij}}{\partial r_j} + \frac{\nu}{1-2\nu} \frac{\partial u_{ii}}{\partial r_i} \right]$

$\sigma_{ij} = \frac{\partial f_e}{\partial u_{ij}}$, $f_e = \frac{1}{2} \sum_{ij} \sigma_{ij} u_{ij}$

Adhesion on surfaces

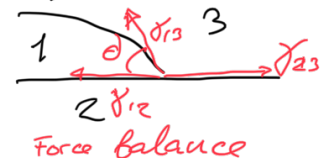
Vesicles



Young-Laplace equation $\Delta p = -\gamma H = -\gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ > mean curvature

Surface energy $\gamma = \frac{W}{\Delta A}$ ($= \frac{F}{2L}$ = surface tension)

Young's law: $\cos \theta = \frac{\gamma_{23} - \gamma_{12}}{\gamma_{13}}$



Elastic deformation counteracts adhesion energy gain W :
contact radius a : $a^3 = \frac{9\pi(1-\nu^2)}{2E} R^2 W$ JKR theory

Bending energy $U_b = 2K \int H^2 dA$ K - bending rigidity

Biological bilipid membranes: $K = 20kT$

Stretching + bending \Rightarrow buckling, coupling ...
No buckling in cells, but shape is modified