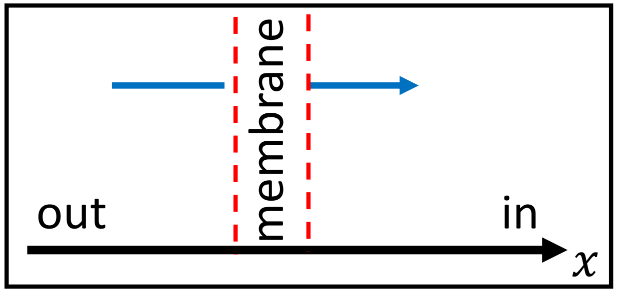
11.1-2 Ion pumping -> sodium anomaly in all animal cells



Fluxes, $j_i, j_{p,i}$ of ion species i, **out -> in** positive

Forces, $\Delta\mu_i = kT\Delta \ln c_i + q_i \Delta V + v_i \Delta P$, **in-out** positive

Linear transport: $j_i = j_{p,i} - g_i \Delta\mu_i$

$$= j_{p,i} - g_i \Delta \ln c_i + g_i q_i \Delta V + g_i v_i \Delta P$$

steady state $= 0$

$j_{p,i} = \alpha_i j_p$ ion pump flux, $\alpha_{Na+} = 3, \quad \alpha_{K+} = -2, \quad j_p > 0$

j_i flux through membrane

g_i conductance through membrane

q_i charge of ion i

ΔV (electrical) membrane potential

$v_i = c_i^{-1}$ (partial) volume of ion i (ideal mixture)

$\Delta P = 0$ assume no pressure difference

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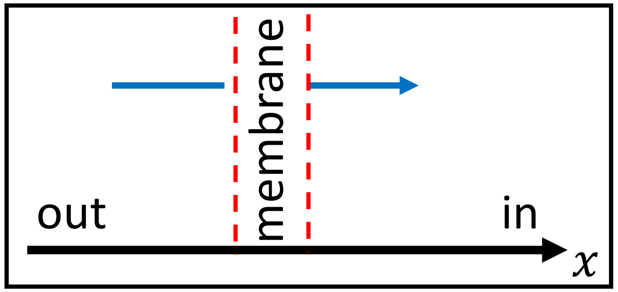

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Linear transport: $j_i = j_{p,i} - g_i \Delta\mu_i$

$= j_{p,i} - g_i \Delta \ln c_i + g_i q_i \Delta V + g_i v_i \Delta P$

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charge neutrality inside: $c_{Na+,in} + c_{K+,in} - c_{Cl-,in} - \frac{\rho_m}{e} = 0$

flux charge neutrality: $j_{Na+} + j_{K+} + j_{Cl-} = 0$ (1)

steady state:

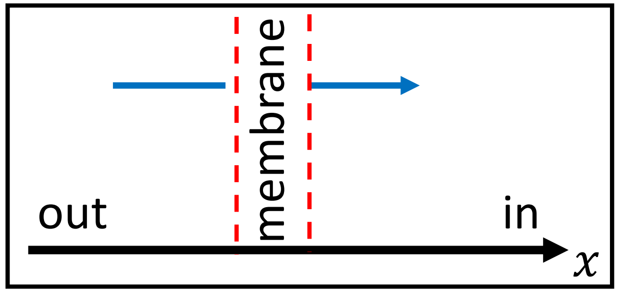
$$\frac{3 j_p}{g_{Na+}} = kT \ln \frac{c_{Na+,o}}{c_{Na+,i}} + e\Delta V \quad (2)$$

$$-\frac{2 j_p}{g_{K+}} = kT \ln \frac{c_{K+,o}}{c_{K+,i}} + e\Delta V \quad (3)$$

$$0 = kT \ln \frac{c_{Cl-,o}}{c_{Vl-,i}} - e\Delta V \quad (4)$$

4 equations, 4 unknowns: $c_{Na+,i}, c_{K+,i}, c_{Cl-,i}, \Delta V$

11.1-2 Ion pumping -> sodium anomaly in all animal cells



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flux charge neutrality: $j_{Na+} + j_{K+} + j_{Cl-} = 0$ (1)

steady state:

$$\frac{3j_p}{g_{Na+}} = kT \ln\frac{c_{Na+,o}}{c_{Na+,in}} + e\Delta V \quad (2)$$

$$-\frac{2j_p}{g_{K+}} = kT \ln\frac{c_{K+,o}}{c_{K+,in}} + e\Delta V \quad (3)$$

$$0 = kT \ln\frac{c_{Cl-,o}}{c_{Vl-,in}} - e\Delta V \quad (4)$$

4 equations, 4 unknowns: $c_{Na+,in}, c_{K+,in}, c_{Cl-,in}, \Delta V$

Only permeating ions not pumped $\Delta V \approx V_i^N = -\frac{kT}{q_i} \ln\frac{c_{i,o}}{c_{i,i}}$

Steady state = resting potential $\Delta V = V^0$

Book notation $\mathscr{g}_i = g_i q_i$

11.1-2 Ion pumping -> sodium anomaly in all animal cells

$$V_i^N = -\frac{kT}{q_i}\ln\frac{c_{i,o}}{c_{i,i}}, \quad \mathcal{g}_i = g_i e$$

flux charge neutrality: $$j_{Na+} + j_{K+} + j_{Cl-} = 0 \quad (1)$$

steady state: $$\frac{3j_p}{\mathcal{g}_{Na+}} = -V_{Na+}^N + \Delta V \quad (2)$$

$$-\frac{2j_p}{\mathcal{g}_{Na+}} = -V_{K+}^N + \Delta V \quad (3)$$

$$0 = V_{Cl-}^N - \Delta V \quad (4)$$

(2)+(3) $$2\mathcal{g}_{Na+}(\Delta V - V_{Na+}^N) = 3\mathcal{g}_{K+}(\Delta V - V_{K+}^N)$$

$$\Delta V = \frac{2\mathcal{g}_{Na+}V_{Na+}^N + 3\mathcal{g}_{K+}V_{K+}^N}{2\mathcal{g}_{Na+} + 3\mathcal{g}_{K+}}$$

Pumping =>

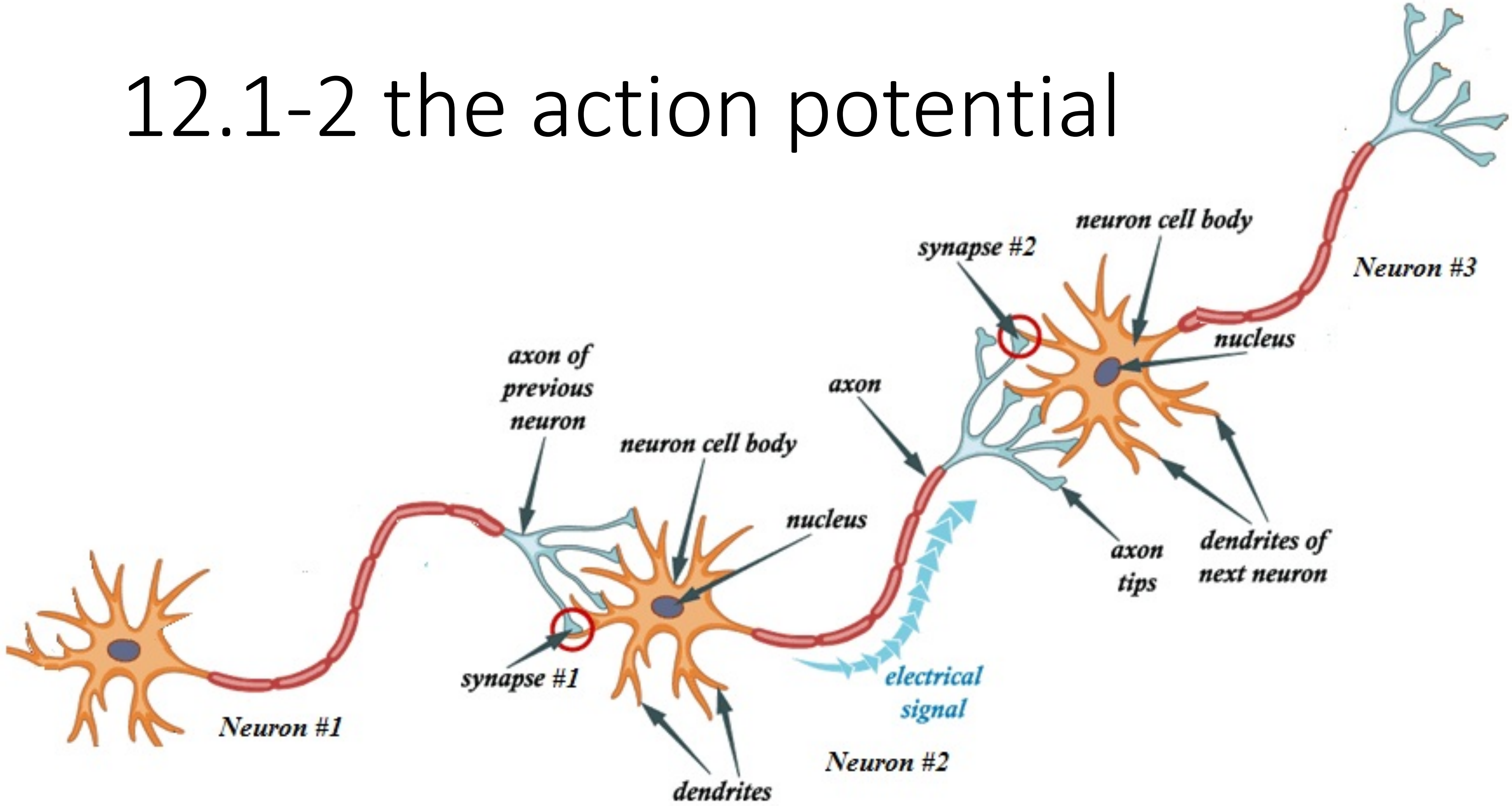
ion imbalance

	$c_{i,o}$ [mM]	$c_{i,in}$ [mM]	V_i^N [mV]	$\mathcal{g}_i/\mathcal{g}_{K+}$
K^+	20	400	-75	1
Na^+	440	50	54	$\frac{1}{25}$ (* 8)
Cl^-	560	52	-59	$^1/_2$

=> $\Delta V = -72$ mV, but eq. (4): $\Delta V = -59$ mV

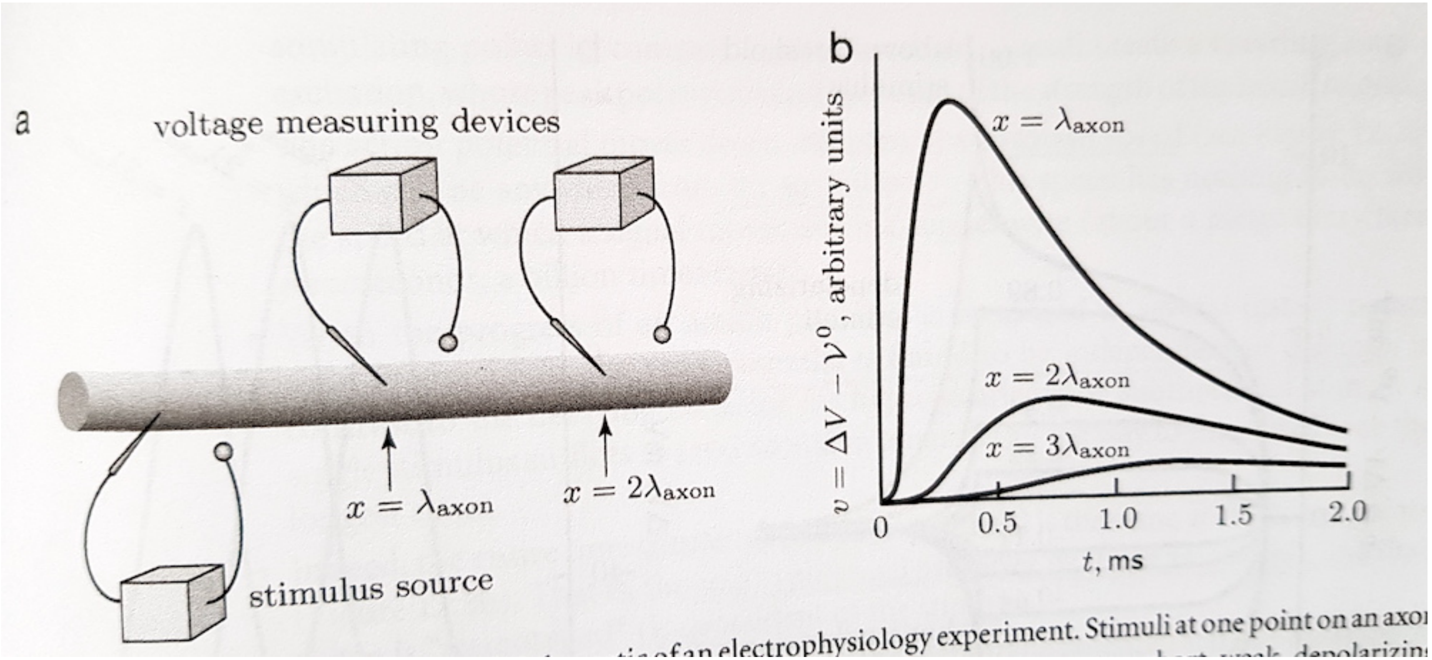
(1) can be used to correct (4), but (1) is not complete because other ions present ++

12.1-2 the action potential



- How can a leaky cable carry a sharp signal over long distances?
- Nonlinearity in cell membrane's conductance => excitable medium => regenerates signal

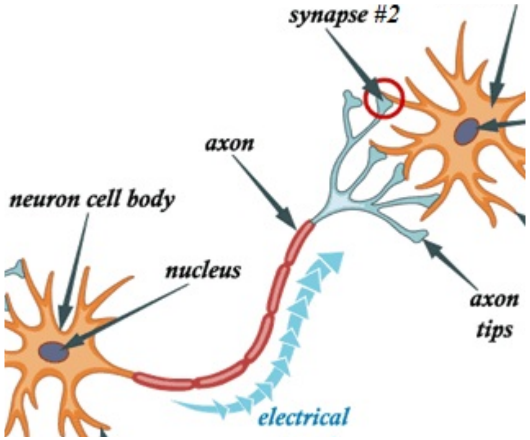
Nerve impulses???

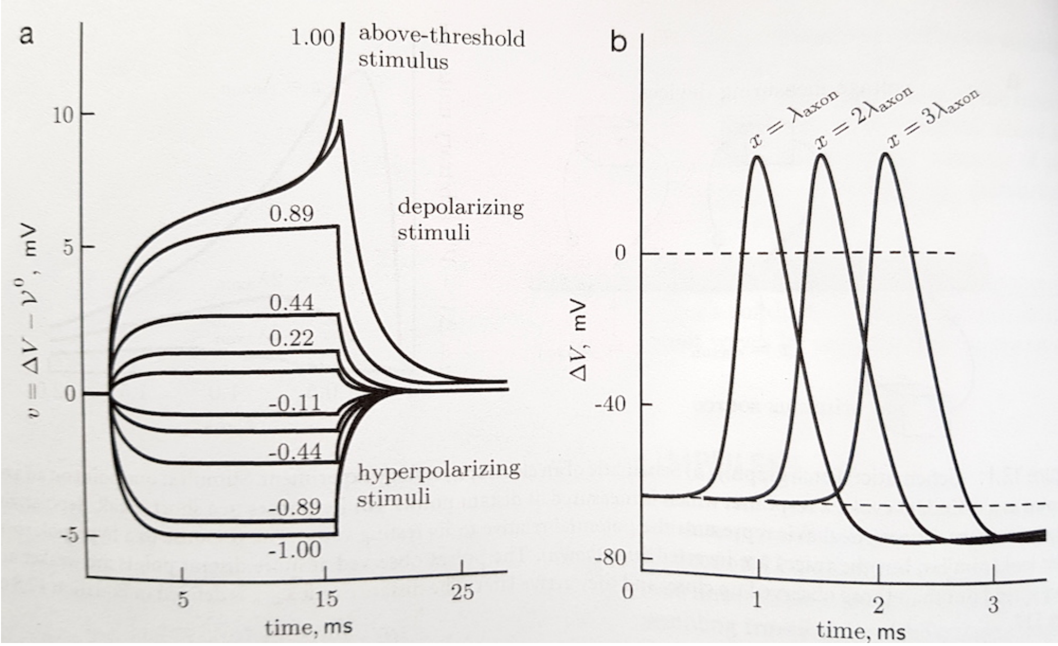


- We are looking for propagating waves of $\Delta V - V^0$
- We only have a dissipative equation:

$$j_i = j_{p,i} - g_i \Delta\mu_i$$

Observations of action potential in axon





- When stimulated beyond a threshold the axon changes polarization for a short while
- This pulse travels along the axon at constant speed (0.1-120 m/s)
- Peak potential and pulse shape are independent of distance
- afterhyperpolarization
- harder to stimulate during a refractory period
- The peak and the shape is independent of the exact triggering pulse

Squid giant axon

Numerical example:

	$c_{i,o}$ [mM]	$c_{i,in}$ [mM]	V_i^N [mV]	g_i/g_{K+}
K^+	20	400	-75	1
Na^+	440	50	54	$\frac{1}{25}$ (* 8)
Cl^-	560	52	-59	$^1/_2$