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Brownian motion—a laboratory experiment

Haym Kruglak

Robert Brown, a British botanist, reported in 1828 that plant pollen particles in water viewed through a microscope were in constant, helter-skelter motion. The phenomenon is known as *Brownian motion* and can be observed with various microscopic particles in fluids.

In 1877, Delsaux, a French physicist, suggested that the Brownian motion was caused by the bombardment of particles by the molecules of the fluid. However, this was only a *qualitative* explanation at a time when some scientists had reservations about the atomic–molecular theory.

Between 1905 and 1908, A Einstein (1956) developed an elegant and comprehensive theory of Brownian motion. J Perrin, a French physicist, confirmed by experiment the formulas derived. One of these was for calculating Avogadro's number

$$N = RTt/3\pi\eta rs^2$$

where N is Avogadro's number, R is the universal gas constant, T is the Kelvin temperature of H_2O, t is a constant, observation period, η is the viscosity of H_2O, r is the radius of particle and s is the root mean square displacement. The theory also predicts that $s^2 = 2Dt$, where D is the diffusion coefficient. This equation is called the Einstein–Smoluchowski Diffusion Law (Atkins 1982).

Haym Kruglak is Professor Emeritus, Western Michegan University, where he was from 1954–78 Professor of Physics. He is also their consultant on lecture demonstrations and has been a Visiting Scholar at the University of Arizona since 1982. A Wisconsin graduate, he obtained his PhD from the University of Minnesota and has been Visiting Assistant Professor of Physics at Princeton University and Assistant Professor of Physics, Astronomy and Natural Science at the University of Minnesota. He has published widely in the fields of the teaching of physics, mathematics, astronomy and general science.

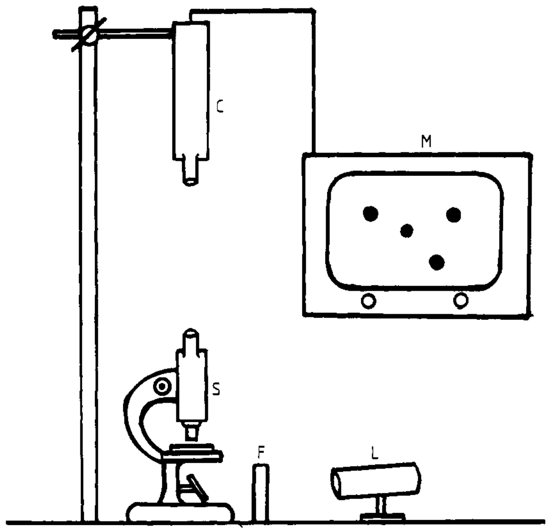

Figure 1 Schematic of the experimental set-up. C, black and white television camera; M, monitor, with 48 cm (diagonal) screen; L, collimated light source; F, heat filter; S, microscope

The laboratory project on Brownian motion developed by J le P Webb (1980) stimulated me to devise an experiment which could be completed in a three-hour period. The experiment was tried with students in a third-semester physics course for majors in physics and engineering at Western Michigan University (WMU).

Apparatus

The main components of the experimental arrangement are shown in figure 1. The latex spheres in dilute aqueous suspension are projected with a microscope and television camera onto a monitor. With the eyepiece removed, a 20× objective provides adequate magnification. The TV camera lens is also removed and replaced with a 10–15 cm tube which helps to align the camera and to exclude some of the stray relections. A heat filter between the illuminator and the microscope mirror is used to maintain a constant suspension temperature.

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The cover glass for the depression slides is sealed with nail polish. Latex spheres of 0.93, 1.10 and 1.35 μm diameter were tested with good results. When not in use the slides were rotated slowly in a special rotator as suggested by Webb. With a 60 cm distance between the bottom of the TV camera and the microscope stage, the image diameters of the 1.1 μm spheres on the monitor screen were on the order of 1 cm.

Experimental procedure

As is common in the USA, students work in pairs (occasionally in groups of three). With a plastic transparency taped to the monitor screen, one of the partners gives the time signal. The other partner marks with a fine felt tip pen the centre of the sphere's positions and labels them 1, 2, 3... If the sphere drifts off the screen, another is selected and followed. When a total of 25 *displacements* are obtained, the transparency is removed. Then the second partner repeats the procedure. The temperature is measured by placing the bulb of a thermometer between the top of the slide cover and the microscope objective. Each transparency is labelled with the student's name, date, sphere diameter and temperature. The displacement calibration is determined by replacing the slide with a transmission grating and measuring the spacing between its lines on the monitor screen.

The transparency with the recorded sphere positions is taped onto a sheet of graph paper and, using an arbitrary origin, the x and y position coordinates read off and recorded. Alternatively, if a copying machine is available, the transparency record can be transferred onto graph paper and copies made for the partners and the lab instructor. This was done at WMU. The two transparencies yield 50 displacements in each of the two directions. Thus, 100 data points are available for analysis. Time is usually adequate in a three-hour laboratory period to repeat the observations with a different time interval.

A hand calculator with a basic statistics program was made available in the laboratory. Each student computed the mean, variance and standard deviation for the displacements before leaving the laboratory.

It is possible for a single person to perform the experiment provided an audible, automatic time signal is available. The circuit for such a timer was designed by R Hiltbrand and is given in the Appendix. It was used by all the students at WMU.

Data analysis and results

Seven WMU students performed the experiment at 15 and 30 s intervals. Three students used somewhat different times because of an accidental misalignment of the timer control knob. A 'random walk' of a sphere at 15 s intervals plotted by one of the observers is shown in figure 2. A histogram of the data by this student and his partner (team no 1) is displayed in figure 3, which also shows a Gaussian curve fitted to the midpoints of the groups (Snedecor and Cochran 1980). A plot on probability graph paper was used to test the goodness of fit. The graph (figure 4) shows that with only 100 data points the distribution of displacements is a good approximation to a normal or Gaussian distribution.

The probability graph serves another useful purpose: checking the calculations for the mean and the standard deviation of the displacements. The mean can be read off at the 50th percentile: two standard deviations are included between the 16th and 84th percentiles.

The mean and standard deviation for the 100 monitor screen displacements at 15 s intervals for

Figure 2 A random walk of a 1.10 μm latex sphere at 15 s time intervals

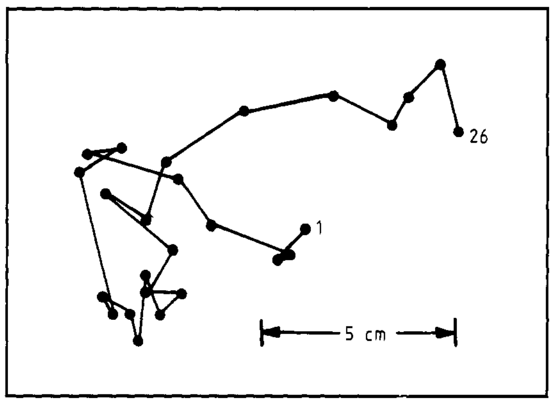
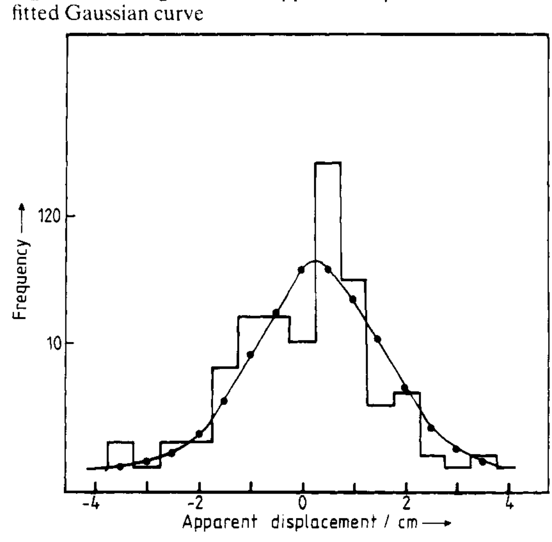


Figure 3 Histogram of 100 apparent displacements with a fitted Gaussian curve



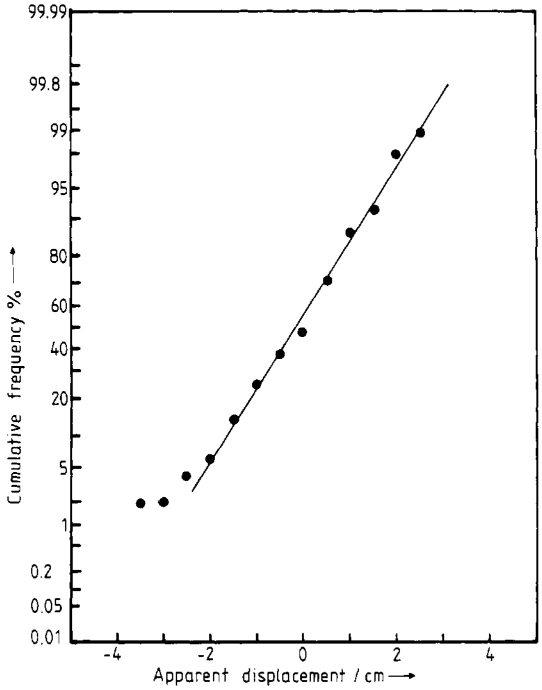


Figure 4 A plot of the data in figure 3 on probability graph paper

Figure 5 A computer-generated histogram and Gaussian of 350 displacements at 15 s intervals observed by seven students

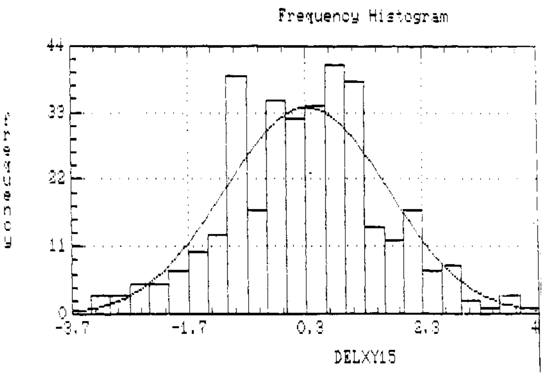


Figure 6 A computer readout of a normal probability plot for the data of figure 5

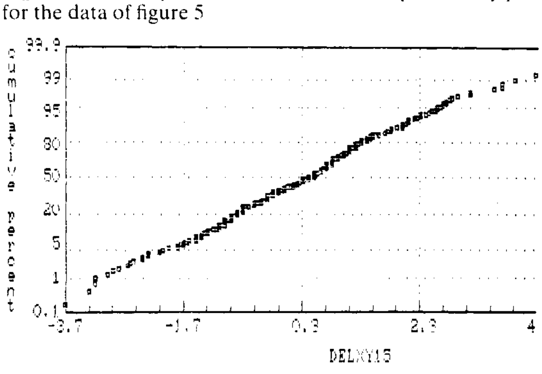


Table 1 Summary of an experiment on Brownian motion, Western Michigan University, autumn 1986

Observers, no	*Time intervals*, s	*Data points*	$N\times10^{26}$ kg mol^{-1}
Students 10	15–18.6	500	6.3
10	30–35.4	250	7.1
Faculty 4	14.6	150	6.0

the two students of team no 1 were 1.23 cm and 0.065 cm respectively. The screen calibration was 2.77 μm cm^{-1}. With T=297 K and the sphere diameter of 1.10 μm, the calculated value of Avogadro's number was $N=5.5\times10^{26}$ kg mol^{-1}.

The two partners and another team recorded 100 displacements on two transparencies at 30 s intervals. The resulting standard deviation was 1.68 cm and $N=6.3\times10^{26}$ kg mol^{-1}. The ratio of the standard deviations for the 30 and 15 s displacements, $S_{30}/S_{15}=1.68/1.23=1.37$. This experimental ratio is in good agreement with the predicted 1.41 value from the Einstein–Smoluchowski law.

One of the students with a lot of computer experience used a Zenith microcomputer and STATGRAPHICS software† to make the calculations and plot the graphs of this data shown in figure 5 and 6. The value of N was 5.4×10^{26} kg mol^{-1}. Then seven of the students also recorded 150 displacements at 30 s, obtaining a value of $N=6.9\times10^{26}$ kg mol^{-1}.

Two faculty members and the laboratory supervisor each plotted a random walk under the same conditions as the students. Their values of N for the 15 s interval were 5.7, 6.5 and 5.7×10^{26} kg mol^{-1}, for an average of 6.0×10^{26} kg mol^{-1}. Was this a matter of greater observational experience or luck? Probably both! The results of all the observations are summarised in table 1.

Error analysis

The largest uncertainty is associated with the targeting of the sphere on the monitor screen: ±1 mm at best or approximately 0.3 μm in the slide suspension. Thus the uncertainty in the mean square displacement for the 15 s time interval could be as high as 15% and for the 30 s interval, 10%. The diameter of the sphere used in our experiment was given by the manufacturer to 1%. The temperature has the least uncertaintly, perhaps 0.3%. Thus, the overall uncertainty in the Avogadro number is on the order

† Statistical Graphics Corporation, STATIGRAPHICS, Software Publishing Corporation, 2115 East Jefferson St, Rockville, MD 20852, USA.

of $\pm 15\%$ for the 30 s intervals and $\pm 20\%$ for the 15 s measurements. The experimental values of N fall within these limits.

Summary and conclusions

The availability of latex microspheres, compact television cameras and electronic calculators make it possible to perform an experiment on Brownian movement in one laboratory period. A more accurate value of N can be determined by other methods. However, the experiment described above has several valuable pedagogical outcomes. Undergraduate students get experience with several experimental techniques: (i) recording a 'random walk' of a microsphere; (ii) plotting a histogram of displacements; (iii) fitting a Gaussian curve to the histogram; (iv) checking the goodness of fit analytically or with probability graph paper; (v) calibrating screen displacements with a diffraction grating; (vi) calculating Avogadro's number from the experimental data; (vii) verifying data validity with the Einstein–Smoluchowski Law.

The experiment also provides valuable practice in unit conversion and error analysis. Another instructive feature: the experiment makes the students aware of Einstein's work other than relativity. The students' reactions to the experiment were positive: 'interesting', 'challenging', 'fun'.

Acknowledgments

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Appendix

A compact, variable timer–beeper

In some laboratory exercises a timer with an audible signal is useful. For example, in the experiment on Brownian motion described above, a single student can pinpoint the moving sphere at the beep of the timer preset for the desired time interval. The timer is also useful in such experiments as the timing of radioactive decay and charge–discharge of a capacitor.

The circuit diagram shown in figure 1 was designed by R Hiltbrand (Western Michigan University, USA). The timer is calibrated with a stop watch and the time intervals marked around the knob-pointer of the potentiometer. The 'dimensions' of the timer are 10×5×5 cm.

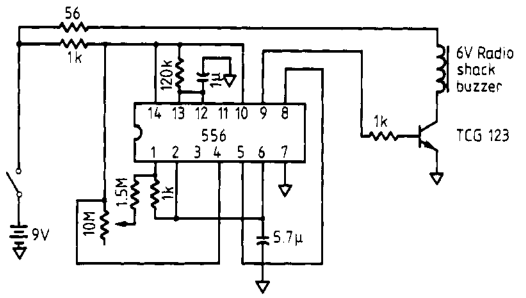

Figure A1 Schematic circuit for the timer–beeper

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Snedecor G and Cochran W 1980 *Statistical methods* 7th edn (Ames, 10: Iowa State University Press) (The normal curve fitting is found in most elementary statistics texts).

Webb J le P 1980 'Einstein and Brownian motion' *Phys. Educ.* **15** 116

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