Passive and active walkers

Dag Kristian Dysthe
Department of Physics, University of Oslo, PObox 1048 Blindern, 0316 Oslo

This project will connect theory of diffusion and flow at low Reynolds numbers with simulations of random walkers and experiments on Brownian motion and the motion of E coli.

I. REPORT ON PROJECT

The objective of this project is to get an insight on transport and motion in fluids at the micron scale. This project contains several experiments, simulations and theory. I have given some tasks to give some guideline to what to include from some of the subprojects. You should try to put together a report that treats the different topics and subprojects as a whole.

II. DIFFUSION

This is treated in the textbook and elsewhere. I will only repeat the diffusion equations and the simplest solution. The so-called Fick's diffusion equation in one dimension is written:

$$J = -D\frac{\partial \rho}{\partial x}, \tag{1}$$

where J is the mass flux, ρ the mass density distribution, x the space coordinate and D the diffusion coefficient. The continuity equation (here in 1D) states that the change of mass inside a volume equals the difference between what flows in to and out of the volume:

$$\frac{\partial \rho}{\partial t} + \frac{\partial J}{\partial x} = 0, \tag{2}$$

where t is the time. When we combine the two equations we get the partial differential equation for diffusion:

$$\frac{\partial \rho}{\partial t} + D\frac{\partial^2 \rho}{\partial x^2} = 0 \tag{3}$$

Starting with particles in $x = 0$ at time $t = 0$: $\rho(t = 0, x) = \delta(x)$, where δ is the Kroeneker delta function the diffusion equation has solution (you may easily verify this):

$$\rho(t, x) = \frac{1}{\sqrt{4\pi Dt}}\exp(-\frac{x^2}{4Dt}) \tag{4}$$

This is a Gaussian distribution with mean $x = 0$ and standard deviation $\sqrt{4Dt}$. Thus the width of the distribution is proportional to the square root of the diffusion coefficient. When data is sparse and noisy one always obtains a more precise estimate of the width by integrating over all the data. The second moment of the distribution is:

$$\int_{-\infty}^{\infty} x^2 \rho(t, x)dt = 2Dt \tag{5}$$

where I have used that

$$\int_{-\infty}^{\infty} y^2 e^{-x^2/a}dt = \frac{\sqrt{\pi}}{2}a^{3/2}. \tag{6}$$

The second moment can be seen as the mean square deviation (from the starting point $x = 0$) of x.

III. LIFE AT LOW REYNOLDS NUMBER

This paper by Purcell [1] is a classic.

IV. RANDOM WALKS

The subject of random walks (RW) has been lectured, it is treated in the textbook and in a supplementary document [2]. A simple Matlab program, `rw1d_vector` illustrates a 1D RW simulation of wlakers that are started at $x = 0$ and properties of the resulting distributions.

Task 1

Compare the distribution of the 1D RW with the solution to the diffusion equation for molecules started at $x = 0$. Do a similar RW simulation in 2D in Matlab or Python. Finally you may try to do the same in 3D and measure the 2D MSD as we do in the experiments on Brownian motion.

V. BROWNIAN MOTION

This has been treated in the lectures and the textbook. The original paper by Albert Einstein in German [3] and translated to english [4] was a hallmark paper in physics. Langevin's treatment 3 years later of the same phenomenon is simpler to follow [5].

You have performed experiments on Brownian particles of different sizes.

Task 2

Describe the experiments, the results obtained and the image analysis. Compare your results to the theory and to your RW simulations. Compare the particle diameters you obtain from your measurements and compare to the diameters specified by the manufacturer. Discuss this in relation to the statistical uncertainty from your analysis and to what you consider the main sources of error in the experiments.

VI. SWIMMING OF E COLI

This has been treated in the lectures and the textbook. Some original papers by the father of E coli physics, Howard Berg, have been included [6–8] sa background. If you have time and special interest in active matter you may have a look at the newer papers on bacterial swarming [9–11].

You have performed experiments on the Nissle strain of E coli.

Task 3

Describe the experiments, the results obtained and the image analysis. Compare your results to the theory and to your RW simulations. For those who have a special interest in active matter: Try to do experiments on high density suspensions of E coli and use correlation methods to find flow fields instead of tracking individual particles.

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