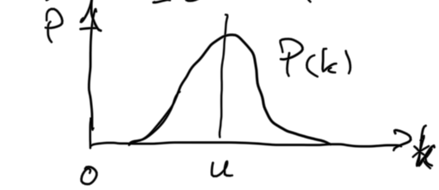
Diffusion and friction in fluids

Diffusion ~ · random motion
- independent of details

4.1.3 1D random walk



steplengths kL

$u = \langle k_j \rangle = \sum_k k P_k$ - drift

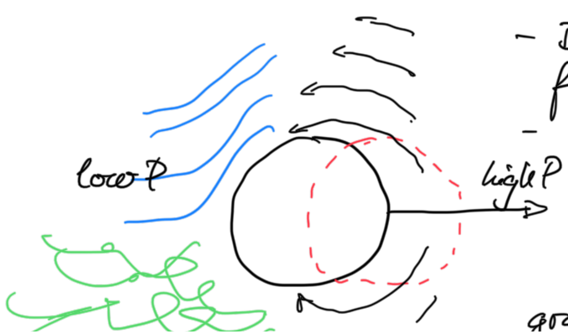
$\langle x_N \rangle = NuL$

$\sigma_N^2 = \langle (x_N - \langle x_N \rangle)^2 \rangle = 2Dt$

This is general. Calculating σ_N^2 in detail is model dependent.

Dimensional analysis → drag

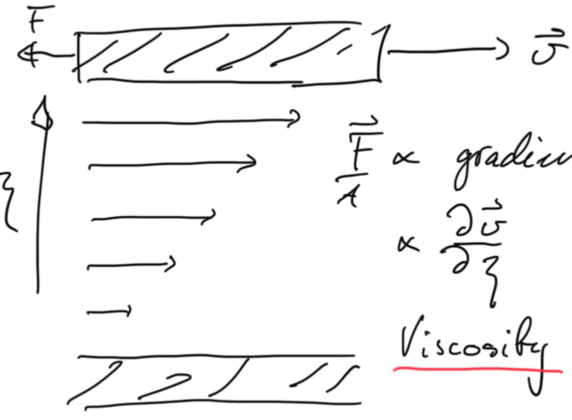
		SI
Length	L	m
time	T	s
mass	M	kg
area		
volume		
speed	LT^{-1}	
acceler.	LT^{-2}	
force	MLT^{-2}	N
work	ML^2T^{-2}	J
stress / pressure	$MT^{-2}L^{-1}$	Pa



- Displacement of fluid
- gradient in velocity

$v \Rightarrow$ viscous drag $\quad v$

Viscosity



$\frac{\vec{F}}{A} \propto$ gradient of v

$\propto \frac{\partial \vec{v}}{\partial z}$

Viscosity $\quad \frac{F}{A} = \eta \frac{\partial \vec{v}}{\partial z}$

$\left[\frac{F}{A}\right] = ML^{-1}T^{-2}$

$\left[\frac{\partial v}{\partial z}\right] = T^{-1}$ $\quad \Rightarrow [\eta] = ML^{-1}T^{-1} = Pa\cdot s$

Important quantities

- Velocity $\quad \vec{v} \quad LT^{-1}$
- Size $\quad \vec{r} \quad L$
- Density $\quad \rho \quad ML^{-3}$
- Viscosity $\quad \eta \quad ML^{-1}T^{-1}$
- Drag $\quad F_d \quad MLT^{-2}$

Buckingham's Π-theorem: The number of independ. dimensionless parameters necessary to describe the system is no. of physical quantities $-$ no. of indep. dimensions

$$5 - 3 = 2$$

Dimless param: $\quad Re = \frac{\rho v r}{\eta} \qquad \frac{\rho v \text{ - inertial}}{\frac{\eta}{r} \text{ - viscous}}$

$Re \gg 1 \quad$ turbulence

$Re \ll 1 \quad$ only viscous damping

F_d not used - make dim. less

$[F_d] = MLT^{-2} \qquad M$ in η and ρ

$[\ \ ] \quad [F_d] \quad MLT^{-2} \quad L^4T^{-2} \quad v^2 \quad L^2T^{-2}$

$\left[\frac{\eta}{\rho}\right] = \frac{\ldots}{ML^{-3}} = L \ldots \qquad r^2 \quad L^2$

$$\Rightarrow \quad C_R = \frac{F_d}{\rho v^2 r^2}$$

II

$$\frac{F_d}{\eta} = \frac{MLT^{-2}}{ML^{-1}T^{-1}} = L^2 T^{-1} \qquad /v \quad /r$$

$$C_s = \frac{F_d}{\eta v r}$$

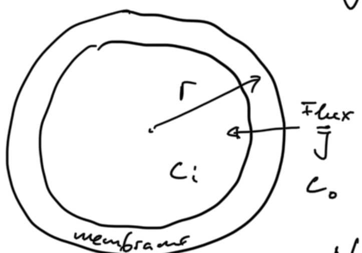
Stokes drag $\quad F_d = C_s \eta v r \qquad$ allow Re $\qquad$ Sphere $C_s = 6\pi$

Viscous friction $\quad \zeta = \frac{F_d}{v}$

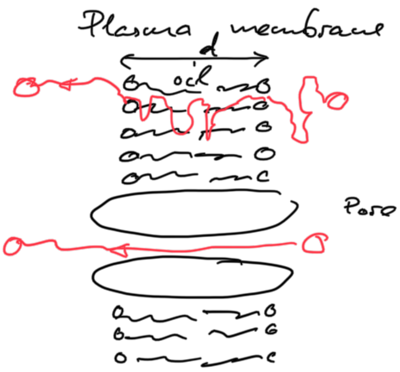
$$\underline{\zeta = 6\pi \eta r}$$

Exercise: Yours from 4C, p120 $\Rightarrow$ $\boxed{\zeta D = k_B T \quad \text{Einstein relation}}$

Permeation through a membrane



$$c = \frac{N}{V}$$



Conc. difference $\Delta c = c_o - c_i$

Flux $\quad J = \frac{1}{A}\frac{\Delta N}{\Delta t} \qquad$ flow rate per unit area

How does c_i change with time? (c_o - const)

$$\frac{\partial c_i}{\partial t} = -\frac{\partial \Delta c}{\partial t} = \frac{1}{V}\frac{\partial N}{\partial t} = \frac{3}{r} \cdot J \qquad A = 4\pi r^2, \quad V = \frac{4}{3}\pi r^3$$

Permeability P: $\quad J = P \Delta c \qquad \frac{V}{A} = \frac{r}{3}$

$$\Rightarrow \quad -\frac{\partial \Delta c}{\partial t} = \frac{3}{r} P \Delta c$$

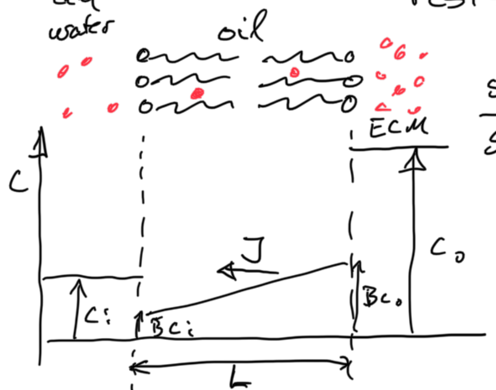
$$\int_{\Delta c_0}^{\Delta c} \frac{\partial \Delta c}{\Delta c} = -\frac{3P}{r} \int_0^t \partial t$$

$$\ln \Delta c - \ln \Delta c_0 = -\frac{3P}{r} t$$

$$\frac{\Delta c}{\Delta c_0} = e^{-t/\tau} \quad , \; \tau = \frac{r}{3P}$$

Permeability is + diffusion through membrane

+ resistance to entry / exit



$$\frac{\text{solubility in oil}}{\text{solubility in water}} = B \quad \text{partition coefficient}$$

Diffusion in membrane $\quad J = -D_m B \frac{\Delta c}{L} = P \Delta c$

$$\Rightarrow |P| = \frac{D_m B}{L}$$

B is small $\Rightarrow$ passive permeation is small

$\Rightarrow$ "active" permeation biologically more important

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For next lecture : Life at low Reynolds number

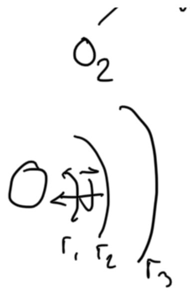
p 3-4 Intro, swimming
p 5-6 Scallop + other swimmers
p 7-8 Corkscrew → p9 Datum
p 9-10 Swimming & diffusion

Ivar : simple implementation of Vicsek model ?

Dag : Interest of swarming of E coli, active matter

4.6.2 p 138 Bacterial metabolism : O_2 limit

O_2 - consumed by bacteria



- transp. from outside
  - × Ocean : turbulence
  - × ECM, mud, pond : diffusion

Assume instant uptake at bacteria surface

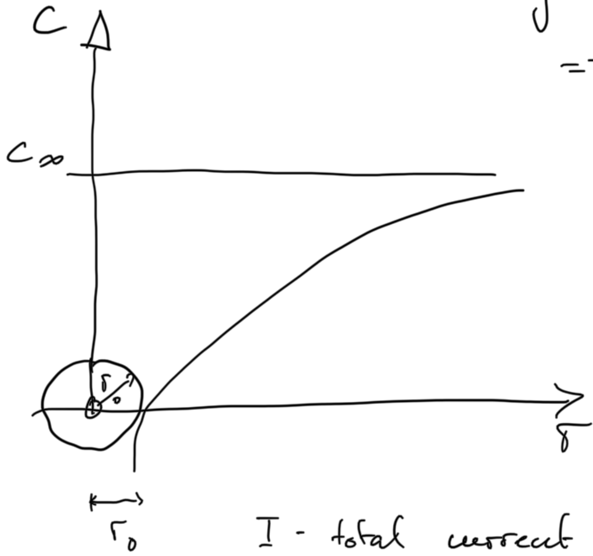
Flux $j(r) = \frac{I}{4\pi r^2} = -D\frac{\partial c}{\partial r}$

$$\Rightarrow \frac{-I}{4\pi D}\frac{\partial r}{r^2} = \partial c$$

$$\frac{-I}{4\pi D}\,\frac{1}{r}\Big|_{r_0}^{\infty} = c\Big|_{0}^{c_\infty}$$

$$\frac{-I}{4\pi D}\left[-\frac{1}{r_0}\right] = c_\infty$$

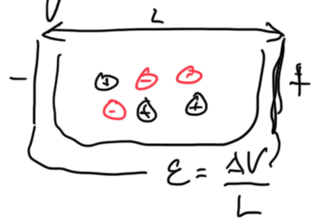
$$\Rightarrow \underline{I = 4\pi D c_\infty r_0}$$



I - total current of O_2

Ions at membranes - a first look

Electrolyte in electric field

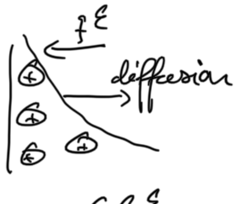


Electric force on ions: $F = q\mathcal{E}$

Drift velocity $v = \frac{F}{\zeta}$

$\Rightarrow$ Ion flux $j = c \cdot v = \frac{cq\mathcal{E}}{\zeta}$

Charge buildup at walls



Stationary $j = \frac{cq\mathcal{E}}{\zeta} - D\frac{\partial c}{\partial x} = 0$ $\quad\Big|\quad j = D\left(-\frac{\partial c}{\partial x} + \frac{q\mathcal{E}}{k_BT}\right)$

Nernst Planck

Einstein relation : $\zeta D = k_B T$

$$\Rightarrow \quad cq\mathcal{E} - k_BT\frac{\partial c}{\partial x} = 0$$

$$\frac{q\mathcal{E}}{k_BT}\,\partial x = \frac{\partial c}{c}$$

$$\frac{qE}{k_B T} L = \ln \frac{c_{right}}{c_{left}}$$

Membrane :