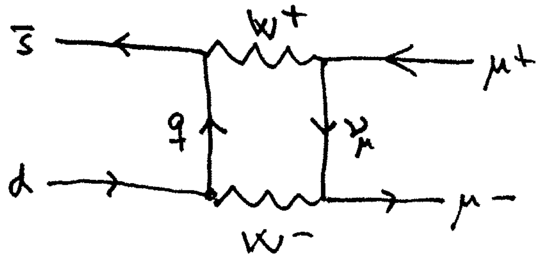
Problems for Fys 5120 – April 2013

* First; – try (– as mentioned in lectures) Problem 10.3, – and also 10.4 (– at least to order λ^2) in P&S (page 345)

* Calculate $\bar{s}d \to \mu^+\mu^-$ (i.e. $K^0 \to \mu^+\mu^-$) from diagram (a), p. 725 for "arbitrary" quark mass m_q (actually $q = u, c, t$)



– for zero external momenta

This is a (relatively) good approximation for $K^0 \to \mu^+\mu^-$

Note that for zero external momenta, Feynman parametrization is an unnecessary complication.

(Instead "splitting of fractions" like

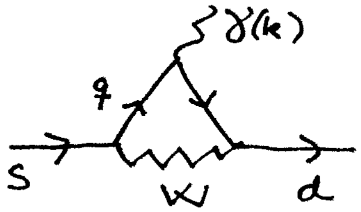
$$\frac{1}{(x+a)(x+b)} = \frac{-1}{a-b}\left(\frac{1}{x+a} - \frac{1}{x+b}\right)$$ might be useful....)

Show that the leading term for $\bar{s}d \to \mu^+\mu^-$

PA-2

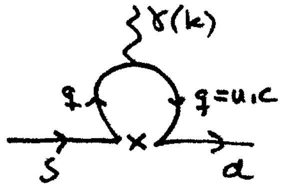
cancels by the "GIM-mechanism",
cfr. quark mixing ((20.148) p.724)
(– generalized to 3 families by Kobayashi & Maskawa)
– and that terms $\sim \frac{g_W^{4}}{M_W^4} m_q^2$ $(q = u, c, t)$
survives

* Consider the amplitude for $s \to d\gamma$



Show that the leading divergent term again cancels by GIM (quark mixing)

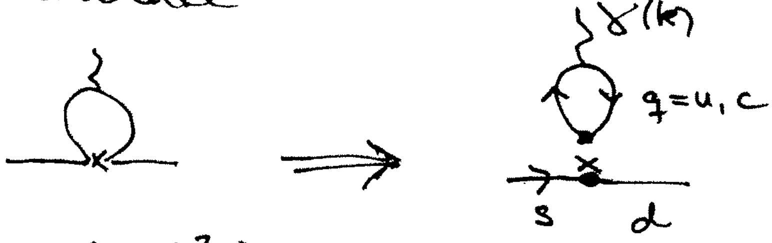
For $q = u$ and $q = c$, $m_q^2 \ll M_W^2$ is a good approximation, and we may try to shrink W to a point:



If we also use a Fierz-transformation

$$(\bar\psi_d \gamma^\mu L \psi_q)(\bar\psi_q \gamma_\mu L \psi_s) = (\bar\psi_d \gamma^\mu L \psi_s)(\bar\psi_q \gamma_\mu L \psi_q),$$

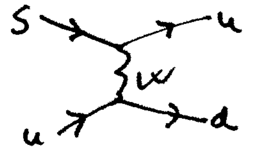
we may use the result from Vacuum polarization in QED to calculate the loop like:



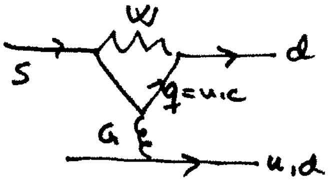
Show this !

(– to order k^2)

Also, replace the photon by a gluon, and show that a "new" non-leptonic interaction (In addition to



) from

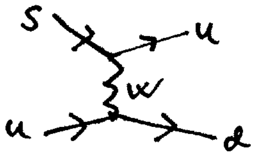


is generated

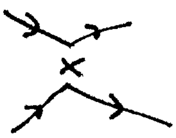
Find (scetch) the mathematical structure of this contribution (the "Penguin diagram")

* Try to go through the Yang-Mills (for int QCD) versions of basic loops at pages 522 to 531 in detail.

* Calculate the QCD correction diagram(s) to the basic non-leptonic "current-current interaction

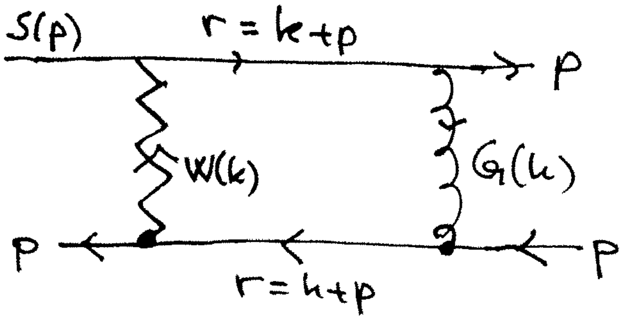


$\xrightarrow{M_W^2 \gg |P_{ext}|^2}$



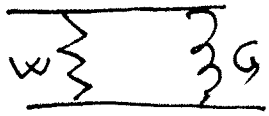
$\sim G_F$

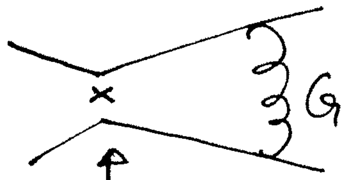
i.e. diagrams like – {with same ext momentum p for all 4 ext. legs} – for $p^2 = -\mu^2$



(– or use $p \to 0$ and μ as lower cut-off)

cfr. sect. 18.2 in P&S

Compare 

with the diagram (divergent) 

$\sim G_F$

with cut-off regularization. What is actually the cut-off?

* Technical hints:

→ Use $\gamma^\mu \gamma^\nu \gamma^\sigma = g^{\mu\nu}\gamma^\sigma - g^{\mu\sigma}\gamma^\nu + g^{\nu\sigma}\gamma^\mu + i\,\varepsilon^{\mu\nu\sigma\lambda}\gamma_\lambda\gamma_5$,

→ $\varepsilon^{\mu\nu\sigma\lambda}\varepsilon^{\alpha\beta\delta}{}_\lambda = a\, g^{\mu\alpha}g^{\nu\beta}g^{\sigma\delta} + b(ggg) + \ldots$

(indices permuted)

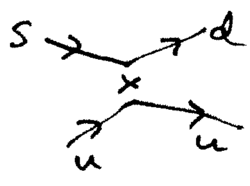
→ $2(t^a)_{in}(t^a)_{lj} = \delta_{ij}\delta_{ln} - \frac{1}{N_c}\delta_{in}\delta_{lj}$

($i,n = 1,2,..N_c$: For $SU(3)_c$, $N_c = 3$)

→ and Fierz-transf

* Explain that 

generates new interaction



$\sim \alpha_s G_F$