Result Mid Term Exam Fys5120

Spring semester 2013

Problem 1)

$r = p + k$

Electron self energy

$$-i\Sigma(p) = e_0^2\, \hat{\Sigma}(p)$$

$$= e_0^2 \left\{ \left[\hat{\Sigma}(p)\right]_{\eta=0} + \eta \left[\Delta\hat{\Sigma}(p)\right] \right\}$$

where η is the gauge parameter in the photon propagator $D^{\mu\nu}(k)$ in eq. (1) in Mid-Term problems. Thus $\left[\hat{\Sigma}(p)\right]_{\eta=0}$ correspond to the $(-g^{\mu\nu})$ term of $D^{\mu\nu}(k)$.

Using standard methods in Peskin & Schroeder we find (in $d=4-\varepsilon$ dimensions):

$$\left[\hat{\Sigma}(p)\right]_{\eta=0} = 2(d-1)\, B\!\left(\tfrac{d}{2},\tfrac{d}{2}\right) \gamma\cdot p\, (-p^2)^{\left(\frac{d}{2}-2\right)} \frac{i}{(4\pi)^{d/2}} \cdot \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)}$$

where $B(\alpha,\beta)$ is defined in eq. (2) of MT-problems.

Note that we have used

$$(2-d)\, B\!\left(\tfrac{d}{2}-1,\tfrac{d}{2}\right) = -2(d-1)\, B\!\left(\tfrac{d}{2},\tfrac{d}{2}\right)$$

(Remember $m_0 = 0$)

As usual, we have used the standard integrals

$$I_n \equiv \int \frac{\not{d}l}{[l^2-\Delta+i\varepsilon]^n} \quad ; \quad \Delta = -x(1-x)p^2$$

$$\not{d}l \equiv \frac{d^d l}{(2\pi)^d} \quad : \qquad (n=1,2,3,\cdots)$$

Note that $\Delta \circ I_3 = \left(\frac{d}{4}-1\right)\cdot I_2$,

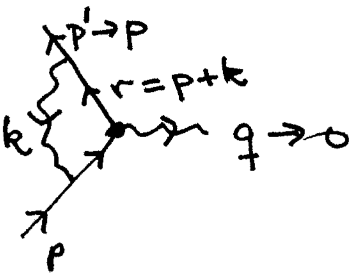
and $B\left(\frac{d}{2}+1, \frac{d}{2}-1\right) = \frac{d}{d-2}\, B\left(\frac{d}{2}, \frac{d}{2}\right)$.

Using such relations, we obtain

$$[\Delta \hat{\Sigma}(p)] = -[\hat{\Sigma}(p)]_{\eta=0} \quad \text{, such that}$$

$$\Sigma'(p) = 0 \quad \text{for } \eta = 1 \text{ (Landau gauge)}$$

Problem 2)



Contribution from Vertex correction diagram:

$$\Delta\Gamma^\mu(p'=p;p) = e_0^3\left[\hat{\Gamma}^\mu_{\eta=0} + \eta\cdot\delta\hat{\Gamma}^\mu\right]$$

where $\hat{\Gamma}^\mu_{\eta=0}$ is due to the $(-g^{\mu\nu})$ part of $D^{\mu\nu}(k)$.

We use the following Dirac algebra in d dimensions:

$$\gamma^\alpha \gamma^\mu \gamma_\alpha = (2-d)\gamma^\mu \;;\; \gamma^\alpha\gamma^\mu\gamma^\nu\gamma_\alpha = 4g^{\mu\nu} - (4-d)\gamma^\mu\gamma^\nu$$

$$\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\sigma\gamma_\alpha = -2\gamma^\sigma\gamma^\nu\gamma^\mu + (4-d)\gamma^\mu\gamma^\nu\gamma^\sigma$$

Using also relations for $B(\alpha,\beta)$ functions above, we obtain

$$\boxed{\hat{\Gamma}^\mu_{\eta=0} = -2(d-1)B(\tfrac{d}{2},\tfrac{d}{2})\frac{i}{(4\pi)^{d/2}}\frac{\Gamma(2-d/2)}{\Gamma(2)}\left\{(-p^2)^{(\frac{d}{2}-2)}\gamma^\mu + (4-d)(-p^2)^{\frac{d}{2}-3}\gamma p\, p^\mu\right\}}$$

Using in addition $\Delta \cdot I_4 = (\frac{d}{6}-1)\cdot I_3$, we obtain after some tedious calculation that

$$\delta\hat{\Gamma}^\mu = -(\hat{\Gamma}^\mu_{\eta=0}), \text{ such that}$$

$$\boxed{[\Delta\Gamma^\mu(p',p)]_{p'=p} = 0} \quad \text{for } \eta=1 \text{ (Landau gauge)}$$

Problem 3)

Using

$$\frac{\partial}{\partial p_\mu}\left[\gamma\cdot p\,(-p^2)^{\frac{d}{2}-2}\right] =$$

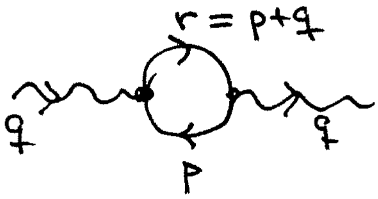
$$\gamma^\mu(-p^2)^{\frac{d}{2}-2} + \gamma\cdot p\cdot\left(\frac{d}{2}-2\right)(-p^2)^{\frac{d}{2}-3}\cdot(-2p^\mu),$$

we easily get

$$e_0\frac{\partial[\Sigma(p)_{\eta=0}]}{\partial p_\mu} = -e_0^3\hat{\Gamma}^\mu_{\eta=0}, \text{ and sim. for}$$

"η-part" : $\quad e_0\dfrac{\partial}{\partial p_\mu}\Delta\Sigma(p) = -e_0^3\delta\Gamma^\mu$

Problem 4 /



We obtain (using standard methods)

$$i\,\Pi^{\mu\nu}(q) = -e_0^2\,\hat{\Pi}^{\mu\nu}(q)\quad,\ \text{where}$$

$$\hat{\Pi}^{\mu\nu} = f(d)\int_0^1 dx\,\Big\{\frac{(2-d)}{d}\,g^{\mu\nu}\left[I_1+\Delta\cdot I_2\right]$$
$$-x(1-x)\,I_2\,(2q^\mu q^\nu - q^2 g^{\mu\nu})\Big\},$$

$$I_n = \frac{i(-1)^n}{(4\pi)^{d/2}}\,\frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)}\,\frac{1}{\Delta^{n-d/2}}\,;\quad \Delta \equiv -x(1-x)q^2$$

$$f(d) \equiv \mathrm{Tr}(\mathbb{1}_{\mathrm{Dirac}}) = 4 + \mathcal{O}((4-d))$$

We use $\quad I_1 + \Delta\cdot I_2 = \frac{d}{d-2}\cdot\Delta\cdot I_2\,,$

giving

$$i\,\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\left(-2e_0^2 f(d)\right)B\!\left(\tfrac{d}{2},\tfrac{d}{2}\right)\frac{i}{(4\pi)^{d/2}}\,\frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)}\,(-q^2)^{\frac{d}{2}-2}$$

$$\boxed{\equiv +ie_0^2\,(q^2 g^{\mu\nu} - q^\mu q^\nu)\,\hat{\Pi}(q^2)}$$

Renormalized charge e_R given by

$$e_R^2 = e_0^2\left[1 + e_0^2\,\hat{\Pi}(q^2=-\mu^2)\right],$$

or

$$\alpha_R = \frac{e_R^2}{4\pi} = \alpha(\mu^2) = \alpha_0\left[1 + \alpha_0\cdot 4\pi\,\hat{\Pi}(q^2=-\mu^2)\right]$$

Using
$$\Gamma\left(2-\frac{d}{2}\right)\frac{\partial}{\partial\mu}\left(\mu^2\right)^{\frac{d}{2}-2} = -\Gamma\left(3-\frac{d}{2}\right)\frac{2\mu}{(\mu^2)^{3-d/2}},$$
$$= \text{finite !}$$

we find

$$\mu\frac{\partial}{\partial\mu}\alpha(\mu^2) = K\cdot[\alpha(\mu^2)]^2 + \mathcal{O}(\alpha^3)$$

(We had a misprint in eq. (6) of problems)

with $K = \frac{2}{3\pi}$

Solution:

$$\frac{1}{\alpha_2} = \frac{1}{\alpha_1} - K\ln\frac{\mu_2}{\mu_1}$$

or: for $\mu_1 = \mu$ and $\mu_2 = Q$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1-\frac{\alpha(\mu^2)}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}$$

I have given max 10 points per problem (–i.e. Max score = 50 points) when correction/reading the written answers

RMT-6

Result (points for the 8 candiates)

Fys 5120 , Spring 2013

Candidate Number	Score (points)
1	41
3	33
4	30
5	37
6	50
7	38
8	49
9	49