Problem sheet 3

FYS5120-Advanced Quantum Field Theory

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$\rightsquigarrow$ These problems are scheduled for discussion on **Wednesday, 9 February 2022**. If you spot any typos and/or mistakes please send an email to ***lasselb@fys.uio.no*** or ***jonaeid@math.uio.no***.

Problem 7: Perturbation theory

Perturbation theory is handled in an especially nice way using path integrals. To explore this we will consider φ^4 theory.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4$$

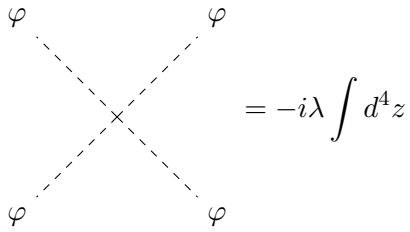
The path integral is given by

$$\int \mathcal{D}\varphi \exp(iS[\varphi]) = \int \mathcal{D}\varphi \exp\left(i\int d^4x \left[\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4\right]\right).$$

From here we simply split the interaction term into its own exponential and series expand that exponential function:

$$\int \mathcal{D}\varphi \exp(iS[\varphi]) = \int \mathcal{D}\varphi \left(1 - i\frac{\lambda}{4!}\int d^4z\varphi(z)^4 + \left(-i\frac{\lambda}{4!}\right)^2 \int d^4z \int d^4w \varphi(z)^4\varphi(w)^4 + ...\right) \exp(iS_0[\varphi])$$

With this, define the generating functional $\mathcal{Z}[J]$ for this theory and use it to show that the vertex rule for this theory is given by:



$$= -i\lambda \int d^4z$$

Problem 8: Scalar QED

Consider the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) - m^2\varphi^\dagger\varphi$$

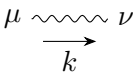
where

$$D_\mu = \partial_\mu + ieA_\mu$$

is the usual gauge-covariant derivative for a $U(1)$ symmetry. By defining an appropriate generating functional for this theory do the following exercises:

a) Calculate the vertices:

b) Calculate the propagator:



Remark: This is done in detail in chapter 9.4 of Peskin and Schroeder's book, we encourage you to follow their steps.

c) Convince yourself that you can get the same results you got in a) and b) by "simply" reading the Lagrangian.

d) To order e^2 in perturbation theory draw all the Feynman diagrams that contribute to the Photon propagator and write down their corresponding matrix elements. *Do not attempt to compute the matrix elements!*