UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: Relativistic quantum field theory (FYS5120)

Day of exam: March 24, 2023

Exam hours: 3 hours

This examination paper consists of 4 pages. (including the title page)

Appendices: none

Permitted materials: 3 A4 pages (two-sided) with own notes.

Make sure that your copy of this examination paper is complete before answering.

Midterm exam

Lecture autumn 2023: Advanced quantum field theory (FYS5120)

⇝ **Carefully read** *all questions before you start to answer them! Note that you don't have to answer the questions in the order presented here, so start with those that you feel most sure about. Problems where more slightly involved calculations are needed are marked with an asterisk (∗). Keep your descriptions self-contained, but as short and concise as possible! Answers given in English are preferred; however, feel free to write in Norwegian/Swedish/Danish if you struggle with formulations!*
Maximal number of available points: **40**.

Good luck!

Problem 1

In the **path integral formalism**, the n-point correlation function for any type of bosonic field ϕ can be written as

$$\langle\Omega|\phi(x_1)...\phi(x_n)|\Omega\rangle = \frac{\int \mathcal{D}\phi\, \phi(x_1)...\phi(x_n) \exp\left[i\int d^4x\, \mathcal{L}\right]}{\int \mathcal{D}\phi\, \exp\left[i\int d^4x\, \mathcal{L}\right]}\,.$$

a) As an application of this formalism, we learned that one can simply 'read off' the vertex rules from a given interaction term in the Lagrangian. Draw the Feynman diagram and state the corresponding Feynman rule for each of the following interaction terms:

   1. $\mathcal{L} \supset -\lambda\,(\bar\psi\gamma_\mu\psi)(\bar\psi\gamma^\mu\psi)$
   2. $\mathcal{L} \supset -\lambda\,|\phi|^2(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*)A^\mu$

   Which mass dimension does the coupling λ have in each of these cases?
   (*5 points*)

b) What is the most important conceptual difference between quantum and classical fields, and how do you think this would manifest itself in the path integral formalism? As a (new) application, consider Yukawa theory, with $\mathcal{L}_{\rm int} = -\lambda\phi\bar\psi\psi$, and argue that the interaction vertex for fermions with an *external* (classical) scalar field $\phi_{\rm cl}$ is given by $i\lambda\phi_{\rm cl}$. Can you think of an application of this 'vertex' rule in the standard model? (*3 points*)

Problem 2

Consider the following **toy model** Lagrangian:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + i\overline{\psi}\not{D}P_L\psi + |D_\mu\phi|^2 - V(|\phi|),$$

where $F_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$, and ϕ is a complex scalar field with potential V. The scalar ϕ and the fermion ψ have different charges, such that the covariant derivative is given by $D_\mu = \partial_\mu + igQ_{\psi,\phi}B_\mu$.

a) State the *field* transformations under which $\mathcal{L}$ is symmetric! Which of those symmetries forbids a mass term for the 'photon' B_μ, and which for the fermion ψ? *(4 points)*

b) Now assume that $V(|\phi|)$ is of such a form that the scalar field acquires a non-vanishing vacuum expectation value $v \equiv \langle\phi\rangle \neq 0$. How does the field content of the theory change, compared to a situation with $V(|\phi|) = m^2|\phi|^2$, and what is this mechanism referred to? Why is there no Goldstone boson? Show explicitly which mass the 'photon' acquires in this way! *(4 points)*

c) For the case described in b), consider massive gauge boson production, $\psi\bar{\psi} \to BB$, and write down the amplitude $i\mathcal{M}$ that follows from using Feynman rules at leading order in perturbation theory. Argue that in the limit of high CMS energies, only gauge bosons with *transverse* polarizations can be produced this way. *(4 points)*

d)* Next, consider instead the annihilation into massless real scalar particles π, i.e. $\psi\bar{\psi} \to \pi\pi$, assuming that the couplings of π are obtained by replacing $B_\mu \to (\partial_\mu\pi)/m_B$ in the Lagrangian from problem b). Compute the (tree-level) unpolarized differential cross section $d\sigma/dt$ for this case! *(5 points)*
[<u>*Hint*</u>*: You will encounter a seemingly daunting trace of 8 gamma matrices; this can be calculated by hand, almost magically, by realizing that* $\not{p}\not{k}\not{p}$ *becomes a very simply expression if* $p^2 = 0$ *– which one? The final result is even simpler.*]

e) **Bonus question**. Discuss the connection between the results of c) and d). From the computation in d), you will have noticed that $m_\psi = 0$ was crucial in arriving at the conclusion. Is there any way of realizing a non-vanishing fermion mass, $m_\psi \neq 0$, in our model (i.e. without adding new fields or changing the symmetries, but possibly by adding further terms to $\mathcal{L}$)? If yes, how would the production rate of longitudinal gauge bosons scale with v and the coupling contant(s)? *(Up to 5 bonus points)*

<u>Problem 3</u>

We now turn to aspects of the **standard model** of particle physics (SM).

a) State the full gauge group of the SM, and explain how the number of SM *gauge bosons* follows from the underlying *unbroken* symmetry. What is the total number of bosonic (including scalar) degrees of freedom in the SM? Then repeat the counting of bosonic degrees of freedom after spontaneous symmetry breaking, i.e. for the actually observed bosonic field content of the SM. What is the gauge group now? (*5 points*)

b) Why do some of the SM *fermion* fields appear in doublets and triplets (and with respect to what)? State all SM fermion fields that are part of a doublet, as well as all SM fields that are part of a triplet. Can you think of a fermion that is a singlet under *all* SM gauge interactions? (*3 points*)

c)* Consider the SM process $u\bar{u} \to d\bar{d}$ and draw all contributing tree-level diagram(s) that only involve the strong coupling α_s and the electromagnetic coupling α_em, respectively. Write down the corresponding amplitudes for these cases. Calculate the *ratio* of the unpolarized cross sections for gluon and photon exchange! (*7 points*)
[*Hint: you don't have to fully calculate the individual $|\mathcal{M}|^2$ to obtain the result.*]

Formulas that you might find useful:

- Properties of Lie algebra generators

$$(t^a)^\dagger = t^a; \qquad [f^a, f^b] = if^{abc}t^c; \qquad (t^a_G)_{bc} = -if^{abc} \tag{1}$$

- Casimir operators

$$t^a_r t^a_r \equiv \mathbf{1} \times C_2(r) = \mathbf{1} \times \begin{cases} \frac{N^2-1}{N} & \text{for fundamental rep. of SU(N)} \\ N & \text{for adjoint rep. of SU(N)} \end{cases} \tag{2}$$

$$\mathrm{Tr}[t^a_r t^b_r] \equiv \delta^{ab} \times C(r) = \delta^{ab} \times \begin{cases} \frac{1}{2} & \text{for fundamental rep. of SU(N)} \\ N & \text{for adjoint rep. of SU(N)} \end{cases} \tag{3}$$

- Helicity projections

$$P_R \equiv \frac{1+\gamma^5}{2}, \quad P_L \equiv \frac{1-\gamma^5}{2}; \qquad [\gamma^5, \gamma^\mu] = 0, \quad (\gamma^5)^2 = 1 \tag{4}$$

- Dirac algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{5}$$