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http://www.uio.no/studier/emner/matnat/fys/FYS5120/v24/

Lecture spring 2024:

Quantum field theory II

Problem sheet 1

Problem 1

As a warm-up, to re-fresh our knowledge of canonical quantization, let us consider the Lagrangian for a free complex scalar field ϕ,

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2 \,.$$

Compared to the real case, this Lagrangian exhibits an additional symmetry, $\phi \to \exp[i\alpha]\phi$, where α is an arbitrary phase.

a) Show that this is indeed a symmetry, and user Noether's theorem to demonstrate that the corresponding conserved charge is given by $Q \propto \int d^3x\, \Im(\dot{\phi}^* \phi)$.

b) Derive the Hamiltonian, and diagonalize it by introducing creation and annihilation operators.

c) Rewrite the conserved charge in terms of these annihilation and creation operators. How does the charge between the two types of particles differ?

Problem 2

Consider a hermitian $N \times N$ matrix B and calculate the N-dimensional Gaussian integral

$$\int d^N p \, e^{-\frac{1}{2} p_m^* B_{mn} p_n + \frac{1}{2} J_n^* p_n + \frac{1}{2} J_n p_n^*} \,,$$

where m, n run from 1 to N. How does the result generalize to the infinite-dimensional case? Finally, calculate these integrals also over Grassmann variables (i.e. use $p_i \to \theta_i$ with $\theta^i \theta^j = -\theta^j \theta^i$; and the same for J_i).

[Hint: Consider first the 1D integral, which you can solve by completing the square, and then using $\left[\int dp \exp(-\frac{1}{2}p^2)\right]^2 = 2\pi$*.]*

Problem 3

The generating functional for a free, real scalar field is proportional to

$$Z_0[J] \exp\left[-\int d^4x \int d^4y \frac{1}{2} J(x) D_F(x-y) J(y) \right] ,$$

where D_F is the Fermi propagator.

a) Show this claim explicitly, starting from the definition

$$Z_0[J] \equiv \int \mathcal{D}\phi \exp\left[i \int d^4x \left(-\frac{1}{2}\phi(\Box + m^2 - i\epsilon)\phi + J\phi \right) \right] ,$$

and using the defining properties of Green's functions.

b) Using the functional formalism, confirm explicitly that the two-point correlation function $\langle 0|T\{\hat{\phi}(x)\hat{\phi}(y)\}|0\rangle$ equals $D_F(x-y)$.

c) Repeat the same exercise as in b), but this time in Fourier space.

Problem 4

Using the newly introduced path integral formalism, start from the QED Lagrangian and derive all Feynman rules from the corresponding generating functional!