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Lecture spring 2024:

Quantum field theory II

Problem sheet 2

Problem 5

To further train our skills in 'reading off' vertex rules, what are the vertex rules corresponding to the following interaction terms? (and what is the dimension of the respective operator ?)

a) $\mathcal{L} \supset \lambda_a \, (\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi)$

b) $\mathcal{L} \supset \lambda_b \, (\bar{\psi}\gamma^5\gamma^\mu\psi)\partial_\mu\phi$

c) $\mathcal{L} \supset \lambda_c \, F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi$, where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

d) $\mathcal{L} \supset \lambda_d \, \phi^2\partial_\mu(\bar{\psi}\gamma^\mu\psi)$

Problem 6

Let us consider the phenomenology of a **complex scalar field** Φ, charged under an Abelian gauge group $U(1)'$ (the prime simply denoting that this is not the standard model $U(1)$).

a) Write down the Lagrangian for this theory, including the most general form for the potential V of Φ, and state explicitly the gauge transformations that leave this Lagrangian invariant!

b) Now add a Dirac fermion Ψ that has a *right*-handed component charged under $U(1)'$, and a *left*-handed component that is neutral! Why must both the gauge boson and the fermion be massless, but not the scalar?

c) From now on, assume that Ψ and Φ have identical charges under the $U(1)'$. Which additional term in the Lagrangian does this allow? Not taking into account Φ self-interactions stemming from V, there are now 4 interaction vertices in this theory. State all of them, as well as the associated Feynman rules!
[Hint: Be careful with arrow directions!]

Problem 7

In the lecture, we will derive the Yang-Mills Lagrangian for fermion fields. The same procedure works also for (multiplets of) scalar fields, which leads to

$$\mathcal{L} = -\frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + (D_\mu\phi)^\dagger(D^\mu\phi) - m^2\phi^\dagger\phi\,.$$

Here ϕ is a complex scalar field in some representation r of G (i.e. it is a multiplet with the same number of components as the dimension of r). What are the Feynman rules for the two vertices that describe interactions between the scalar field and the gauge field?

Problem 8

In this context, we will also derive the following transformation laws for fermions and gauge fields under infinitesimal gauge transformations:

$$\psi \to (1 + i\alpha^a t^a)\,\psi$$

$$A^a_\mu \to A^a_\mu + \frac{1}{g}(\partial_\mu\alpha^a) + f^{abc}A^b_\mu\alpha^c\,,$$

where $\alpha^a(x)$ are infinitesimal real functions, t^a are the generators of the symmetry group and $f^{abc}t^c \equiv -i[t^a, t^b]$ gives the structure constants that define their algebra. How do these expressions generalize for arbitrary (i.e. not necessarily infinitesimal) gauge transformations $\alpha^a(x)$?

[NB: The transformation law for the vector fields can in general only be written in an implicit way!]