*Torsten Bringmann* (torsten.bringmann@fys.uio.no)
http://www.uio.no/studier/emner/matnat/fys/FYS5120/v24/

Lecture spring 2024:

# Quantum field theory II

## Problem sheet 6

**Problem 18**

The electron self-energy at $\mathcal{O}(\alpha)$, i.e. at lowest loop-order, is given by

$$\Sigma_2(p) = 2e^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{x\not{p} - 2m_0}{[k^2 - \Delta + i\varepsilon]^2}\,,$$

where $\Delta \equiv (1-x)m^2 - p^2 x)$ and $m_0$ is the (bare) electron mass parameter in the Lagrangian.

a) Derive the above expressions from the QED Feynman rules, using Feynman parameters to bring the loop propagators into the stated form.

b) Regularize the above integral using Pauli-Villard, and compute both infinite and finite parts.

c) Repeat the same for dimensional regularization.

**Problem 19**

Use the results from the previous problem to show that, in dimensional regularization, the on-shell renormalization conditions for the electron self-energy $\Sigma$ imply that the counterterms to lowest order are given by

$$\delta_2 = \frac{\alpha}{\pi}\left\{-\frac{1}{2\epsilon} - \frac{1}{4}\log\frac{\tilde{\mu}^2}{m_P^2} - 1 - \frac{1}{2}\log\frac{m_\gamma^2}{m_p^2}\right\}$$
$$\delta_m = \frac{\alpha}{\pi}\left\{-\frac{3}{2\epsilon} - \frac{3}{4}\log\frac{\tilde{\mu}^2}{m_P^2} - 1\right\}$$

Confirm explicitly that $\Sigma$ (to this order) is independent of $\tilde{\mu}$ ! Why is this expected? What happens to the photon mass $m_\gamma$ ?

**Problem 20**

Show that the *photon* self-energy, to leading order in dim. reg., is given by

$$\Pi_2^{\mu\nu} = (p^2 g^{\mu\nu} - p^\mu p^\nu)\Pi_2(p^2)\,,$$

with

$$\Pi_2 = -2\frac{\alpha}{\pi}\int_0^1 dx\, x(1-x)\left[\frac{1}{\epsilon} + \log\frac{\tilde{\mu}^2}{m_0^2 - p^2 x(1-x)}\right] + \mathcal{O}(\epsilon)\,.$$