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Lecture spring 2024:

Quantum field theory II

Compulsory problem set

⇝ Deadline for handing in answers is **Thursday, 16 Mai 2024**. You must pass this assignment in order to proceed to the final (oral) exam.

It's only the questions marked with '$*$' that require more detailed calculations. For ease of correcting, please try to follow Schwarz' conventions for counter terms, self-energies etc. You don't have to re-derive results quoted in the lecture or previous exercises; also for intermediate results you can simply refer to the corresponding equation in Schwarz – so, in order to save time, make sure to not re-invent the wheel!

Problem I

Consider a theory with a Dirac fermion ψ that couples to a real scalar ϕ with a Yukawa coupling y:

$$\mathcal{L}_{\phi,\psi} = i\bar{\psi}\partial\!\!\!/\psi - \frac{1}{2}\phi\Box\phi + y\phi\bar{\psi}\psi$$

*a) Which are the 3 counter terms of this theory? Calculate them in the $\overline{\text{MS}}$ scheme, to leading order in $\alpha \equiv y^2/(4\pi)$! *(You'll anyway need the full expressions for the self-energies later, so don't take 'shortcuts' to just extract the divergent parts for those. For the vertex correction, however, this is highly advisable.)*

b) You will have seen that there are no mass terms generated radiatively, i.e. both fields stay massless even when taking into account radiative corrections. What is the symmetry that prevents a mass term for the fermion ψ? Explain why this would in any case make it 'technically natural' to have even extremely light fermions in this theory!

c) What about the scalar mass – can you identify a corresponding argument for why no mass term is generated?

Problem II

On closer inspection, it turns out that the theory stated in problem I is not complete.

a) Which *additional* dim $\leq$ 4 operator does the theory generate at loop level (beyond those stated in $\mathcal{L}_{\phi,\psi}$)? Is the coefficient of this operator calculable? If yes: explain how you can do so, leading to a finite result that can be compared to measurements; if not: explain how to treat this operator instead. (Also state the symmetry that forbids the remaining Lorentz-invariant dim ≤ 4 operators that one can in principle construct out of a fermion and a scalar !)

b) Based on your insight from a), revisit the reasoning about fermion and scalar masses from the previous problem. (Re-)calculate the scalar self-energy! Why does the self-energy receive an imaginary part from the fermion loop, but not from the other contribution(s)?

*c) To summarize the discussion so far, write down the *actual* Lagrangian of the theory introduced in problem I. State the full counterterms of all operators in the MS scheme (to leading order)! What about dim > 4 operators in this theory – are they always calculable (and why)?

*d) Calculate the counter terms for the scalar mass and field operators in the on-shell scheme. Use the result to express, to leading order, the $\overline{\text{MS}}$ mass of ϕ in terms of its pole mass! From this, calculate the 'running' of the $\overline{\text{MS}}$ mass – i.e. express the $\overline{\text{MS}}$ mass exclusively as a function of *i)* the renormalization scale μ and *ii)* the $\overline{\text{MS}}$ mass 'measured' at some other renormalization scale μ_0.

Problem III

Let us now slightly modify our initial starting point from problem I, and instead consider the Lagrangian

$$\mathcal{L}_{\phi,\psi} = i\overline{\psi}\partial\!\!\!/\psi - \frac{1}{2}\phi\Box\phi + iy\phi\overline{\psi}\gamma^5\psi$$

a) Compared to the theory considered above, which discrete symmetry changed ? (related: why did we have to add an initial 'i' in the interaction Lagrangian, assuming that y is still real ?) Which symmetry remained unaffected? Which of the results from problem I & II changes due to this modified interaction term, and how?

b) Now imagine adding an explicit Dirac fermion mass term, $m_\psi\overline{\psi}\psi$, as starting point of the discussion. How does this affect *i)* the theory discussed in problem I & II, and *ii)* the theory with the modified Yukawa interaction? (Just point out the qualitative differences in the calculations, and explain their origin !)

*c) Let us now instead add an explicit term $\frac{1}{3!}\mu_\phi\phi^3$ to the Lagrangian from problem I. Show that this generates a fermion mass term, and calculate the (leading-order) infinite part that must be compensated for by a new counter term δ_m. Which other new dim ≤ 4 term does this new interaction generate? Explain the physical significance of this term, and how it relates to the fermion mass term.