Exercises FYS5555

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1 Introduction to CompHep

1.1 Warm-up

Work through Appendix B in High p_T Physics at Hadron Colliders [1]. Note that there have been some updates to compHep since the book was written so things may have changed slightly.

1.2 Helicity supression in pion decays

Find the decays rate of $\pi^- \to e^- \bar{\nu}_e$ and $\pi^- \to \mu^- \bar{\nu}_\mu$. What is the ratio of the two decay rates? How does your result agree with the measured/theoretical values? (see Chapter 11.6 in Thomson [2]). Can you explain the result?

1.3 Lepton Universality

Given the following two formulas, use compHep to check lepton universality of the weak charged-current coupling.

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \equiv \frac{1}{\tau_\tau} = \frac{G_F^{(e)} G_F^{(\mu)} m_\mu^5}{192\pi^3} \tag{1}$$

$$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \equiv \frac{1}{\tau_\tau} = \frac{G_F^{(e)} G_F^{(\tau)} m_\tau^5}{192\pi^3} \tag{2}$$

The G_Fs are the weak charged-current coupling strength for the various leptons.

1.4 Violation of Unitarity I

Consider the process

$$e^+ e^- \to W^+ W^- \tag{3}$$

a) Which SM Feynman graphs contribute to this process?

b) Calculate the cross section as a function of the centre of mass energy ($\sqrt{s}$) for each of the processes. Plot them together in the same plot.

c) Calculate the cross section excluding the Z-diagrams. Discuss.

d) Calculate the cross section including all diagrams. How do they compare with Figure 1.5 in High p_T Physics at Hadron Colliders [1]?

1.5 Violation of Unitarity II

Consider the scattering process

$$W^+W^- \to W^+W^- \tag{4}$$

a) Which diagrams contribute to this process?

b) Calculate the cross section as a function of $\sqrt{s}$, exluding the diagrams with Higgs. What do you observe?

c) Look at the cross section as a function of the cosine of the angle between the initial and scattered $W^\pm$. What do you observe?

d) Calculate the cross section as a function of $\sqrt{s}$ including all diagrams. Compare with the results from b).

2 Collider Physics

2.1 Invariant Mass

Using the following relations

$$E = E_T \cosh y \tag{5}$$

$$p_x = p_T \cos\phi \tag{6}$$

$$p_y = p_T \sin\phi \tag{7}$$

$$\beta_T = \frac{p_T}{E_T} \tag{8}$$

show that the invariant mass,

$$M_{12}^2 = (p_1 + p_2)^\mu (p_1 + p_2)_\mu \tag{9}$$

can be written as

$$M_{12}^2 = m_1^2 + m_2^2 + 2E_{T_1}E_{T_2}\left(\cosh\Delta y - \beta_{T_1}\beta_{T_2}\cos\Delta\phi\right) \tag{10}$$

2.2 Transverse Mass

a) Using the result of the previous exercise find an expression for the invariant mass of a leptonic W decay.

b) Plot the distribution of the invariant mass as a function of ϕ between 0 and π

c) Use the fact that the W-mass is known (80.4 GeV) and assume the W decays at rest to find an expression for the difference in rapidity, $\Delta\eta$, between the electron and the neutrino.

Hint 1: In a) consider only the transverse plane since you can not measure the p_z component of the neutrino (i.e. the same as setting $\eta = 0$)
Hint 2: Assume masses of leptons and neutrinos can be neglected

2.3 Di-jet production at the LHC

It was shown during lectures (slide 28-31) and in Chapter 3.3 and 3.4 in [1] that the cross section of di-jet production at the LHC through gluon-gluon scattering/annihilation can be expressed as

$$\left(\frac{d\sigma}{dMdy}\right)_{y=0} = \frac{2C\left[xg(x)\right]d\hat{\sigma}(1+2\to 3+4)}{M^3} \tag{11}$$

where C is a color factor, $xg(x)$ is the gluon distribution function and M is the mass of the 2-parton system. By integrating over M from M_0 to ∞ and y from $-y_{max}$ to y_{max} in 11 we can find an expression of the total cross section

$$\sigma = \frac{\Delta y C\left[xg(x)\right]\pi\alpha_s^2\left|A\right|^2}{M_0^2}$$

a) insert values of the parameters (see [1] or Collider Physics II slides from lectures) and find the expected total cross section.

b) look at the $gg \to gg$ process in compHep and calculate the total cross section. Compare it with what you got above.

c) check the $1/M^3$ dependence on the cross section predicted from 11

d) try different lower cut-values on the p_T of the outgoing gluons and see how the total cross section changes.

Hint 1: Multiply by a factor of $\hbar^2 = 0.4\, mb\, GeV^2$ to get the result in milibarns (mb)
Hint 2: Use `p1` as the initial composite particle in the scattering process and select the $GG->GG$ sub-process before you start calculations.

References

[1] Dan Green. *High p_T Physics at Hadron Colliders.* Cambridge University Press, 2004.

[2] Mark Thomson. *Modern Particle Physics.* Cambridge University Press, 2013.