Dimensional analysis

As we have already mentioned, length, mass, and time are three *fundamentally different* quantities which are measured in three completely independent units. It, therefore, makes no sense for a prospective law of physics to express an equality between (say) a length and a mass. In other words, the example law

$$m = l, \tag{3}$$

where m is a mass and l is a length, cannot possibly be correct. One easy way of seeing that Eq. (1.3) is invalid (as a law of physics), is to note that this equation is dependent on the adopted system of units: *i.e.*, if $m = l$ in mks units, then $m \neq l$ in fps units, because the conversion factors which must be applied to the left- and right-hand sides differ. Physicists hold very strongly to the assumption that the laws of physics possess *objective reality*: in other words, the laws of physics are the same for all observers. One immediate consequence of this assumption is that a law of physics must take the same form in all possible systems of units that a prospective observer might choose to employ. The only way in which this can be the case is if all laws of physics are *dimensionally consistent*: *i.e.*, the quantities on the left- and right-hand sides of the equality sign in any given law of physics must have the same dimensions (*i.e.*, the same combinations of length, mass, and time). A dimensionally consistent equation naturally takes the same form in all possible systems of units, since the same conversion factors are applied to both sides of the equation when transforming from one system to another.

As an example, let us consider what is probably the most famous equation in physics:

$$E = m\,c^2. \tag{4}$$

Here, E is the energy of a body, m is its mass, and c is the velocity of light in vacuum. The dimensions of energy are $[M][L^2]/[T^2]$, and the dimensions of velocity are $[L]/[T]$. Hence, the dimensions of the left-hand side are $[M][L^2]/[T^2]$, whereas the dimensions of the right-hand side are $[M]([L]/[T])^2 = [M][L^2]/[T^2]$. It follows that Eq. (1.4) is indeed dimensionally consistent. Thus, $E = m\,c^2$ holds good in mks units, in cgs units, in fps units, and in any other sensible set of units. Had Einstein proposed $E = m\,c$, or $E = m\,c^3$, then his error would have been immediately apparent to other physicists, since these prospective laws are not dimensionally consistent. In fact, $E = m\,c^2$ represents the *only* simple, dimensionally consistent way of combining an energy, a mass, and the velocity of light in a law of physics.

The last comment leads naturally to the subject of *dimensional analysis*: *i.e.*, the use of the idea of dimensional consistency to *guess* the forms of simple laws of physics. It should be noted that dimensional analysis is of fairly limited applicability, and is a poor substitute for analysis employing the actual laws of physics; nevertheless, it is occasionally useful. Suppose that a special effects studio wants to film a scene in which the Leaning Tower of Pisa topples to the ground. In order to achieve this, the studio might make a scale model of the tower, which is (say) 1m tall, and then film the model falling over. The only problem is that the resulting footage would look completely unrealistic, because the model tower would fall over too quickly. The studio could easily fix this problem by slowing the film down. The question is by what factor should the film be slowed down in order to make it look realistic?

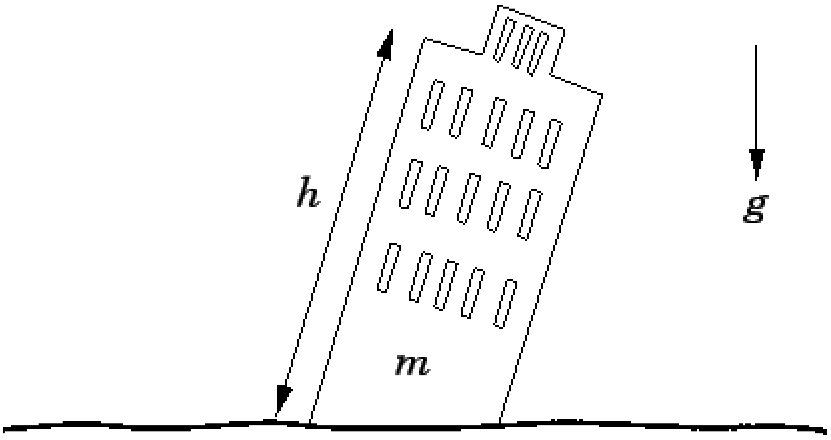

Figure 1: The Leaning Tower of Pisa

Although, at this stage, we do not know how to apply the laws of physics to the problem of a tower falling over, we can, at least, make some educated guesses as to what factors the time t_f required for this process to occur depends on. In fact, it seems reasonable to suppose that t_f depends principally on the mass of the tower, m, the height of the tower, h, and the acceleration due to gravity, g. See Fig. 1. In other words,

$$t_f = C\, m^x\, h^y\, g^z, \tag{5}$$

where C is a dimensionless constant, and x, y, and z are unknown exponents. The exponents x, y, and z can be determined by the requirement that the above equation be dimensionally consistent. Incidentally, the dimensions of an acceleration are $[L]/[T^2]$. Hence, equating the dimensions of both sides of Eq. (1.5), we obtain

$$[T] = [M]^x\, [L]^y\, \left(\frac{[L]}{[T^2]}\right)^z. \tag{6}$$

We can now compare the exponents of $[L]$, $[M]$, and $[T]$ on either side of the above expression: these exponents must all match in order for Eq. (1.5) to be dimensionally consistent. Thus,

$$0 = y + z, \tag{7}$$

$$0 = x, \tag{8}$$

$$1 = -2\, z. \tag{9}$$

It immediately follows that $x = 0$, $y = 1/2$, and $z = -1/2$. Hence,

$$t_f = C\, \sqrt{\frac{h}{g}}. \tag{10}$$

Now, the actual tower of Pisa is approximately 100m tall. It follows that since $t_f \propto \sqrt{h}$ (g is the same for both the real and the model tower) then the 1m high model tower falls over a factor of $\sqrt{100/1} = 10$ times faster than the real tower. Thus, the film must be slowed down by a factor 10 in order to make it look realistic.

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Next: Worked example 1.1: Conversion **Up:** Introduction **Previous:** Precision and significant figures

Richard Fitzpatrick 2002-04-26

1 Scale Models and Similarity

INTRODUCTION

One of the chief objectives of endogenic geology is the study of the past and present motion of matter in the Earth. Such motion is of different types, some of a chemical nature, others of a mechanical. In this book we shall deal with mechanical motion—including strain—of rock and magma bodies, especially the control which gravity exercises over such processes. The atomistic and crystal-physical aspects of the phenomenon of rock flowage are outside the scope of the present work, in which rocks will be treated as continua exhibiting macroscopic mechanical properties such as yield point and ultimate strength, effective viscosity (partly due to recrystallization and transient strain recovery), rigidity, etc. These properties are both isotropic and anisotropic, anistropy being the rule rather than the exception with crystalline rocks.

By treating rocks as continua in a mechanical sense, the theories of fluid dynamics and strength of material may be applied in the study of the evolution of the deformation structure of rock complexes.

Unfortunately, the geometric pattern—architecture—of rock complexes often defies theoretical analysis even if the various rocks in the structure happen to exhibit simple rheological properties. Rocks, however, are far from simple in their mechanical behaviour, a fact which renders a rigorous theoretical dynamic treatment hopelessly complicated for any but the most simple structures.

Greatly intensified experimentation with dynamic scale models and increased application of numerical methods are therefore called for, if a better understanding of the evolution of the structure of the Earth is to be achieved.

SCALE MODELS

The significance of scale-model work in tectonic studies lie in the fact that a correctly constructed dynamic scale model passes through an evolution which simulates exactly that of the original (the prototype), though on a more convenient geometric scale (usually smaller) and with a conveniently changed rate (usually faster).

In the study of geologic structures the situation is generally such that we are in a position to inspect and observe by various means a given final structure without having had a chance to study the initial situation or to witness the actual evolution. This is not exactly a favourable basis on which to build a sound dynamic theory of the evolution of the structure in question. The situation is not improved by the fact that even the final picture is only fragmentary: significant evidence has been removed by erosion or is inaccessible owing to depth of burial. The final picture presented is, therefore, apt to be strongly coloured by the particular experience of the field geologist, and not least by his imagination.

Though it may sometimes be possible to make a reasonable guess as to the pattern of earlier evolutionary stages of a given structure based upon general geological knowledge (we are, for example, reasonably confident that sedimentary strata were deposited as more or less horizontal sheets, the younger above the older), it is certainly no surprise in the light of the above situation that the hypotheses offered as "explanations" of geologic structures are legion. Witness the several contrasted, and often mutually contradictory, hypotheses of orogenesis; of evolution of certain types of folds and of some domal structures; and even of such a universally occurring phenomenon as schistosity.

It seems inevitable that model experiments coupled with theoretical analysis of the dynamics of tectonic processes will contribute greatly to a sound, coherent theory of structural geology and tectonics. By running scale models of tectonic events, one may ultimately hope to separate the physically possible from the physically impossible hypotheses, and the former may be studied in detail to illustrate tectonic processes to an extent not otherwise possible.

SIMILARITY

A model is said to be *geometrically similar* to an original structure if it is a reduced (or enlarged) geometric replica of the original. One may speak of corresponding points in model and original, of corresponding lines or curves, of corresponding surfaces and corresponding volumes. The ratio between the distances between any two corresponding points in model and in original is constant for a given model–original pair. This ratio is called the model ratio of length,

$$\frac{l_m}{l_o} = l_r \tag{1.1}$$

(The model ratios employed in the following are chiefly from the classic work of Hubbert, 1937. See also the paper by Koeniqsberger and Morath, 1913).

The model ratios of area and volume, respectively, are consequently

$$\frac{A_m}{A_o} = l_r^2 \tag{1.2}$$

and

$$\frac{V_m}{V_o} = l_r^3 \tag{1.3}$$

A model that imitates movements in an original structures is said to be *kinematically similar* to the original if corresponding particles are found at

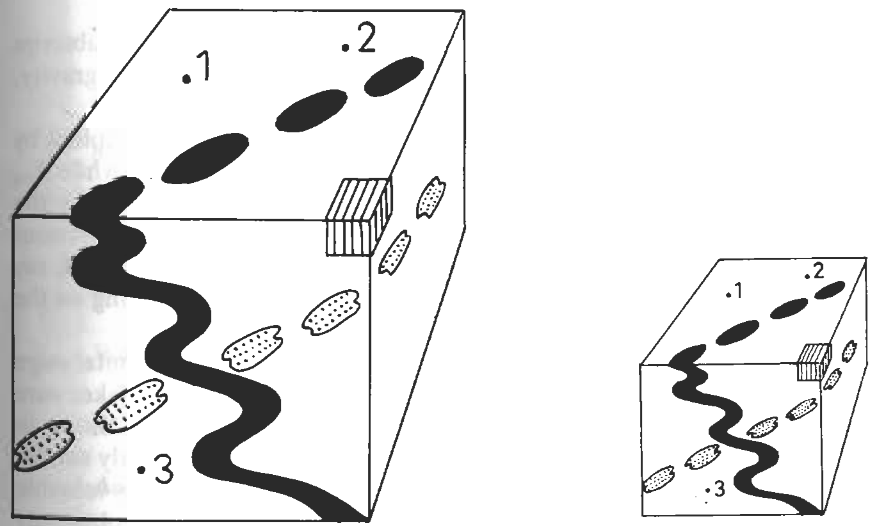

Fig. 1.1. Two geometrically similar structures.

corresponding points at corresponding times (Langhaar, 1962); that is, the model remains geometrically similar to the original during the evolution of the two structures, provided the evolutionary stages are compared at corresponding times. The meaning of the term "corresponding times" will become clear when we introduce the model ratio of time, t_r, which is the ratio between the lengths of time needed for the completion of corresponding movements in model and original. Thus

$$\frac{t_m}{t_o} = t_r \tag{1.4}$$

where t_o is the time needed for a given movement in the original and t_m the time needed for the corresponding motion in the model.

The model ratio of time is constant for motions anywhere throughout the two kinematically similar structures, otherwise the model becomes distorted relative to the original and the two structures cease to be geometrically similar.

Geometrically and kinematically similar structures are *dynamically similar* if the ratio is constant between the various kinds of mechanical forces, compared kind for kind, that act on any two corresponding particles in original and model. Assigning the symbol F_r to the model ratio of force, the above condition is written

$$F_r = \frac{F_{mg}}{F_{og}} = \frac{F_{mi}}{F_{oi}} = \frac{F_{mv}}{F_{ov}} = \frac{F_{mc}}{F_{oc}} = \frac{F_{mf}}{F_{of}} \tag{1.5}$$

where F is strength of force on corresponding particles (volumes). Subscript o and m refer to original and model, while g, i, v, e and f refer to gravity, inertia, viscous, elastic and frictional force, respectively.

F_{og}, for example, equals the acceleration due to gravity multiplied by the mass contained in the volume l_o^3 of the original structure, while F_{mg} equals the acceleration to which the model is exposed multiplied by the mass contained in the volume l_m^3. F_{ov} is the force caused by the viscous stress acting on the surface area of a defined region within the original, say the area l_o^2, and F_{mv} is the force caused by the viscous stress acting on the area l_m^2 of the corresponding region in the model.

If one constructs a model which is geometrically similar to the inital stage of a spontaneously evolving natural structure, and moreover makes sure that the mechanical forces in model and original are related as shown in expression (1.5), then the model will develop in a manner exactly similar to that of the original. A study of the evolution of such models is as valuable as the study of the evolution of the original structure. Quite obviously then, model testing is an ideal tool for the study of the slow geotectonic processes.

The above statement is made under the assumption that non-mechanical processes such as chemical reactions, diffusion, heat flow and nuclear processes do not occur. Hence the chemical processes which often are extremely significant during rock deformation (e.g. pressure-solution schistosity and the continuous filling of expanding fissures by growing minerals) are unfortunately not imitated in the scale-model experiments discussed in this book.

From the condition of constant model ratio of force in a dynamically similar model–original pair follows the condition of constant model ratio of mass per corresponding volume throughout the two structures. It is clear

that the model ratio of the body force of inertia or of gravity or other body-force field is not constant throughout the two structures unless the ratio of mass is constant. (The ratio of mass refers to mass in corresponding volumes, just as the ratio of force refers to force acting on corresponding volumes of matter in the structures.)

The requirement of constant model ratio of mechanical forces when compared kind for kind in model and in prototype leads to the condition that the ratio between any two dissimilar forces in the model must be equal to the ratio between the same two forces in the prototype. Thus, for example, in models of fluid flow in which both inertia and viscosity are

Table 1.1 Model ratios of some mechanical quantities

Quantity	Dimensional formula	Model ratio
Length	L	$\frac{l_m}{l_o} = l_r$
Mass	M	$\frac{m_m}{m_o} = m_r$
Time	T	$\frac{t_m}{t_o} = t_r$
Area	L^2	$\frac{l_m^2}{l_o^2} = l_r^2$
Volume	L^3	$\frac{l_m^3}{l_o^3} = l_r^3$
Velocity	LT^{-1}	$\frac{l_m t_m^{-1}}{l_o t_o^{-1}} = l_r t_r^{-1}$
Acceleration	LT^{-2}	$\frac{l_m t_m^{-2}}{l_o t_o^{-2}} = l_r t_r^{-2}$
Density	ML^{-3}	$\frac{m_m l_m^{-3}}{m_o l_o^{-3}} = m_r l_r^{-3}$
Force	MLT^{-2}	$\frac{m_m l_m t_m^{-2}}{m_o l_o t_o^{-2}} = m_r l_r t_r^{-2}$
Stress	$ML^{-1}T^{-2}$	$\frac{m_m l_m^{-1} t_m^{-2}}{m_o l_o^{-1} t_o^{-2}} = m_r l_r^{-1} t_r^{-2}$
Strain	L^o	$\frac{\Delta l_m l_m^{-1}}{\Delta l_o l_o^{-1}} = 1$
Viscosity	$ML^{-1}T^{-1}$	$\frac{m_m l_m^{-1} t_m^{-1}}{m_o l_o^{-1} t_o^{-1}} = m_r l_r^{-1} t_r^{-1}$

Subscript m refers to model, o to original and r to model ratio.

important, the Reynolds number must be the same in both model and prototype if dynamic similarity is sought. The Reynolds number is defined as the ratio between the inertial and the viscous forces (Table 6.1, p. 41).

Length, time and mass are basic concepts in the conventional theory of mechanics. All other mechanical concepts, such as velocity, acceleration, strength, stress, viscosity, etc., may be defined in terms of length, time and mass. Similarly, the model ratios of all mechanical concepts are determined for a model–original pair by the model ratios of the three basic parameters, length, time and mass. Table 1.1 shows some pertinent relationships. For a comprehensive study of model theory, the works by Hubbert (1937), Porter (1958), Langhaar (1962) and Kline (1965) should be consulted. The symposium on model works in geophysics edited by Long (1953) is also important.

For a study of tectonic problems by means of model experiments, it is of course desirable that the models be dynamically and kinematically as similar to the original as possible, to ensure that the tectonic evolution in question is really imitated by the model. Only then will an analysis of the mechanics of the model throw light on the tectonic event under study.

For several reasons, which we shall discuss subsequently, we are generally obliged to work with models that are but partially similar to the geologic structures. Fortunately, it is often possible to select model materials and conditions such that the lack of strict dynamic similarity does not seriously affect the particular processes one wishes to study.

Before discussing the practical limitations of model investigation of tectonic phenomena, let us consider a theoretical model of tectonic evolution.

2 Theoretical Model of a Rock Complex in Tectonic Evolution

FORCES ACTIVE IN TECTONIC PROCESSES

A dynamic treatment of tectonic evolution is conveniently introduced by considering the mechanical forces that act on the various parts of an arbitrarily limited three-dimensional region of the Earth that contains the structure in question. Such a region will be called a tectonic system—or simply a system—in the following discussion.

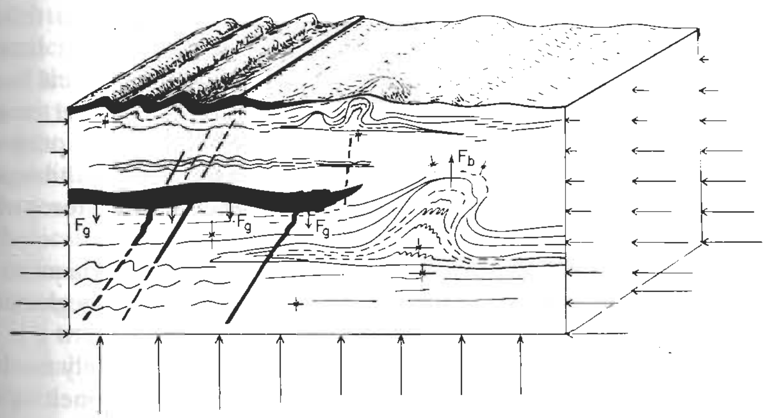

Fig. 2.1. A tectonic system as defined in the text. Arrows at boundary indicate a stress system not corresponding to static stability in the field of gravity. (Possible shear stress at boundary not indicated.) F_g is gravitational force acting on a basic sill; F_b is buoyant force acting on a granite dome. Local stresses are indicated at selected points.

A tectonic system is generally affected by the following types of mechanical forces that control its dynamic evolution (Fig. 2.1).

(1) The stresses that act at the boundary of the system. These stresses are transmitted through the interior of the system in a manner controlled by the geometric pattern of the system, by the mechanical

6 Difficulties Encountered in Experiments with Tectonic Models

For a number of reasons it is not possible in practice to construct experimental dynamic models which in every respect are true-to-scale replicas of tectonic phenomena. Models that only partly satisfy the requirements of the theory of dynamic similarity have to suffice.

The difficulties encountered in experimental model investigation of tectonic processes are chiefly of the following kinds.

(1) Incomplete knowledge of rheological data of the rocks in the prototype. Very little is known about the strength, effective viscosity, yield point, etc. of rocks under the conditions at which they yield in active tectonic systems. Data of this nature are unfortunately produced only slowly in contemporary experimental research.

(2) Difficulties in finding model materials with rheological properties suited to simulating rocks and other terrestrial materials in ordinary dynamic scale models. Particularly scarce are materials that mimic reasonably well the characteristic visco-elastic behaviour of rocks, i.e. their ability to yield coherently or plastically under gentle long-lasting stresses while being almost perfectly elastic at sudden impacts. (Stitching wax and silicone putty are among the very few solid substances exhibiting such behaviour suited for model work.)

In models intended to imitate global-scale processes (e.g. convection currents in the mantle) impossible demands are made on the compressibility of the model materials. From top to bottom of the mantle the density changes, presumably owing to compression of a homogeneous (?) solid substance, from 3·3 to 5·6 g cm^{-3}. No known solid would be correspondingly compressed by its own weight through a model mantle of manageable thickness, say a few decimetres at most.

(3) When materials with acceptable imitation properties are found, they are often soft and mechanically weak to a degree that makes model construction and model study—e.g. by sectioning and microscopic

inspection—quite impracticable or at best a most messy operation.

However, the softness and weakness required of model materials are limited to models of tectonic phenomena over which gravity exercises a significant control. In tectonic systems in which the gravitational pull is negligible relative to the stresses at the boundary, the strength and viscosity of the model materials are limited only by the maximum stresses which the experimenter is able to transmit to the system. In the study of buckle folds, for example, of a kind not affected by gravity (e.g. folds of rather short wavelength) the hardest of materials may well be used as model material if one has access to a press strong enough to produce buckling. The evolution of huge undulations, such as geosynclines and geanticlines, on the other hand, is closely governed by the force of gravity; models of such structures do not reproduce the phenomena realistically unless the materials are soft and weak enough to sag, and even fracture, under their own weight. This is even more true of models of such features as the rise and evolution of salt domes, batholiths and the like; structures of this kind do not form at all in models unless the materials are soft and weak.

(4) In models whose outer frame is at rest (or in non-accelerated motion) in the Earth's field of gravity, the model ratios of time and length are interrelated, $l_r t_r^{-2} = 1$. As model and tectonic prototype are both exposed to the same gravitational field, acceleration in free fall is the same in the two systems. The model ratio of acceleration, a_r, is therefore unity, which for strictly correct scale models must hold for acceleration in general throughout the structures. Consideration of dimensions, however, leads to $a_r = l_r t_r^{-2}$ and consequently $l_r t_r^{-2} = 1$ (see Table 1.1, p. 5).

The requirement $l_r t_r^{-2} = 1$ would make model study of tectonics quite impossible by virtue of the great contrast in time and geometric dimensions of tectonic systems and manageable models. For example, for a tectonic study a model ratio of length of, say, $l_r = 10^{-5}$ is a common figure (10 km in nature corresponds to 10 cm in model). A time span of, say, 10^6 years may be a reasonable figure for the evolution of the structure in question.

Now, from the model ratio of length follows the ratio of time

$$t_r = \sqrt{10^{-5}} \approx 3{\cdot}16 \times 10^{-3}, \qquad (6.1)$$

which means that a strictly dynamically similar model requires $t_m = 3{\cdot}16 \times 10^{-3} \times 10^6 = 3160$ years to undergo the evolution we want to study—not exactly a practical time span for the experimenter.

However, the acceleration—in terms of rate of change of velocity, but certainly not in terms of force per unit mass in a body–force field—of most tectonic processes is negligible except, for example, in earthquakes and the flow of magmas. This is equivalent to saying that the inertial terms are negligible in the fluid-dynamic equations when applied to tectonic processes. For this reason no significant error is introduced by disregarding the condition $l_r t_r^{-2} = 1$ and treating l_r and t_r as independent variable in most tectonic problems.

The reasoning implicit in the above comments is not limited to negligible inertial forces but is generally applicable in model work. Although strict dynamic similarity does require that the model ratio of force be constant throughout for a model–prototype pair, if some types of force can be shown to be negligible for the phenomenon under study, then no important error is introduced by relaxing the prototype–model ratio requirement for just these types of force. Such relaxations of the strict theoretical requirements with respect to insignificant types of force in experimental models not only simplify the construction and handling of models but often open the only avenue to experimental approach. The relaxation of the restriction $l_r t_r^{-2} = 1$ due to negligible inertial forces is a case in point.

Other examples are provided by tectonic structures not significantly controlled by the force of gravity. For models designed for the study of such structures (e.g. practically all small-scale folds, boudins, and pinch-and-swell and similar structures in rocks), the ratio of the gravitational force acting on corresponding masses in model and natural system need not equal the ratios of the forces applied on the boundaries of model and natural system. This we can show by model theory.

The model ratio of the forces due to gravity equals the model ratio of mass times the model ratio of acceleration due to gravity, which is unity in ordinary model work. But the ratio of mass is identical with the ratio of density times the ratio of volume, hence

$$\frac{F_{mg}}{F_{og}} = F_r = m_r a_r = \rho_r l_r^3 a_r = \rho_r l_r^3 \tag{6.2}$$

since $a_r = 1$. The ratio of the stress distribution caused by gravity in the two structures is correspondingly

$$\sigma_r = F_r l_r^{-2} = \rho_r l_r \tag{6.3}$$

showing that at constant ρ_r, the stress due to gravity decreases with decreasing linear dimension of the model of a given natural structure. In order to reproduce flowage and collapse correctly, the strength of the model material must correspondingly decrease with diminishing model size, hence the

requirement of weak and soft material in gravity-controlled tectonic models. See Hubbert (1937, 1945) and Ramberg (1968, 1970).

However, if in nature the force of gravity is negligible relative to the viscous or elastic forces and therefore also relative to the boundary forces needed to produce the desired deformation, then the condition $\sigma_r = \rho_r l_r$ needs not be satisfied, and one may apply without restriction in our models any stresses necessary to produce the wanted deformation quite irrespective of the model ratio of force to which these stresses correspond.

The buckling and boudinage experiments performed by Ramberg (1955, 1963b), Huddleston and Stephansson (1973), Strömgård (1973), Ramberg and Strömgård (1971) and Ghosh and Ramberg (1968) are examples of models in which gravity exercises no appreciable control over the structural pattern. The relative significance of the compressive surface force and the body force of gravity in layered systems are discussed in detail in Chapter 8.

In fluid dynamics some standard dimensionless expressions for the ratios between various forces are in use, some of which are given in Table 6.1,

Table 6.1 Force ratios useful in model work

$F_v \hat{=} \mu v l$	$F_p \hat{=} \Delta p l^2$	$F_g \hat{=} \rho l^3 g$	
$\frac{F_i}{F_v} = \frac{v l \rho}{\mu}$ Reynolds number	$\frac{F_p}{F_i} = \frac{\Delta p}{\frac{1}{2}\rho v^2}$ Pressure coefficient	$\frac{F_i}{F_g} = \frac{v^2}{gl}$ Froude number	$F_i \hat{=} \rho v^2 l^2$
	$\frac{F_p}{F_v} = \frac{\Delta pl}{\mu v}$ Stokes number	$\frac{F_g}{F_v} = \frac{\rho l^2 g}{\mu v}$ Nameless ratio	F_v
		$\frac{F_g}{F_p} = \frac{\rho l g}{\Delta p}$ Suggested: Smoluchowski number	F_p

F_i = inertia force; ρ = density;
F_v = viscous force; l = length;
F_p = pressure force; v = velocity;
F_g = gravity force; μ = viscosity;
p = pressure or stress;
$\hat{=}$. "dimensionally equal to"; g = acceleration of gravity.

reproduced from Kline (1965). Six of these ratios are so often used that they have acquired specific names, Reynolds number probably being the best-known.

The values which these dimensionless numbers achieve in a system show the relative importance of the forces involved. For example, a small Reynolds number means small inertial force relative to the viscous force, a high Froude number signifies strong inertial force as compared with the force of gravity, etc. For our purposes the Stokes number and especially the ratios F_g/F_v and F_g/F_p, which so far have not been honoured with a person's name (probably because the ratios between gravity and viscosity and gravity and pressure force are of little significance in ordinary fluid dynamic problems), are the most significant force ratios.

In general, the situation in tectonic systems is such that the value of both the Reynolds number and the Froude number is less than the limit above which inertial forces are influential. (Note, however, that this limit depends upon the geometry of the system; see also Chapters 7 and 8.)

At strict dynamic similarity, which requires that the ratio between forces in model and in prototype be constant if compared kind for kind, the ratio between two dissimilar forces, such as those given in Table 6.1, in the model must assume the same value as the ratio between the same forces in the prototype.

From

$$\frac{F_{mg}}{F_{og}} = \frac{F_{mi}}{F_{oi}} = \frac{F_{mv}}{F_{ov}} \tag{6.4}$$

follows that

$$\frac{F_{mg}}{F_{mi}} = \frac{F_{og}}{F_{oi}}; \frac{F_{mv}}{F_{mi}} = \frac{F_{ov}}{F_{oi}}; \frac{F_{mg}}{F_{mv}} = \frac{F_{og}}{F_{ov}} \tag{6.5}$$

As we have seen, it is not necessary to meet the last requirement if one (or both) of the two forces involved in the ratio is insignificant in both model and prototype. There is generally no need, therefore, to attempt to obtain the same Reynolds number or the same Froude number for a model as those which existed in the tectonic prototype during its evolution. One has only to ensure that the inertial forces are negligible also in the models.

On the other hand, the ratio F_g/F_v in models of most large-scale tectonic structures must approximate very closely the ratio in the natural structures because both gravitational and viscous forces are of paramount significance in tectonic evolution. This applies also to the ratio F_g/F_p and the Stokes number.

Tectonic deformations have been mentioned in which neither gravity nor inertia was a controlling force—e.g. the evolution of small buckle folds

and boudins. In models of such structures, however, it is necessary that the Stokes number, which gives the ratio between pressure, or stress, force and viscous force in a system, be the same in model and in prototype.

The effect of surface tension forces on structural pattern in tectonics can be ignored, since the viscous and gravitational forces are so much stronger. In small models, however, whose fold pattern, for example, may be less than, say, 1 cm across, surface tension may play an important role relative to viscosity and gravity. In such cases the ratios between the surface tension force and the viscous and gravity forces should be watched lest the models deviate noticeably from dynamic similarity.

GRAVITY, DEFORMATION AND THE EARTH'S CRUST

Second Edition

In theory, experiments and geological application

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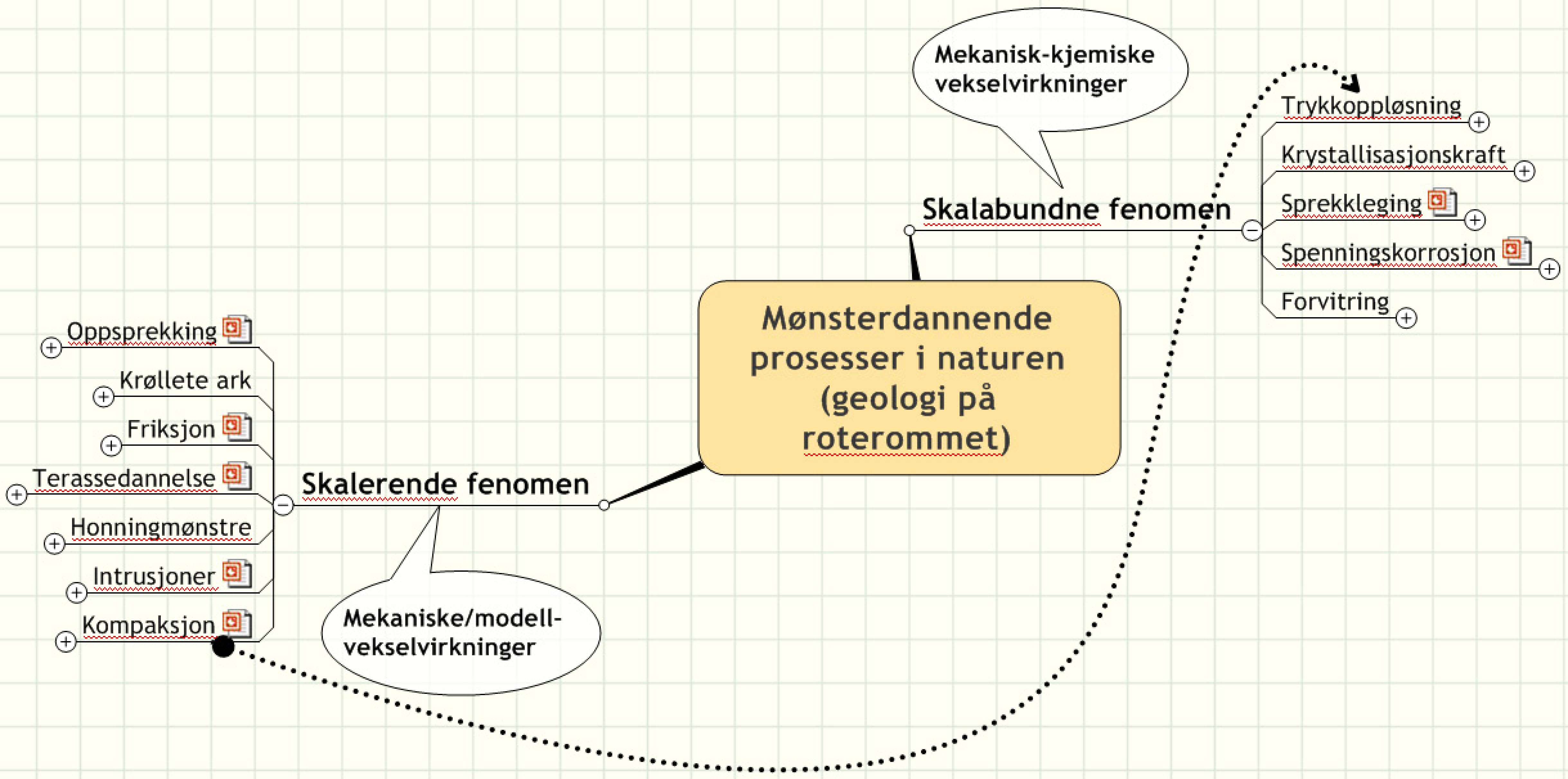
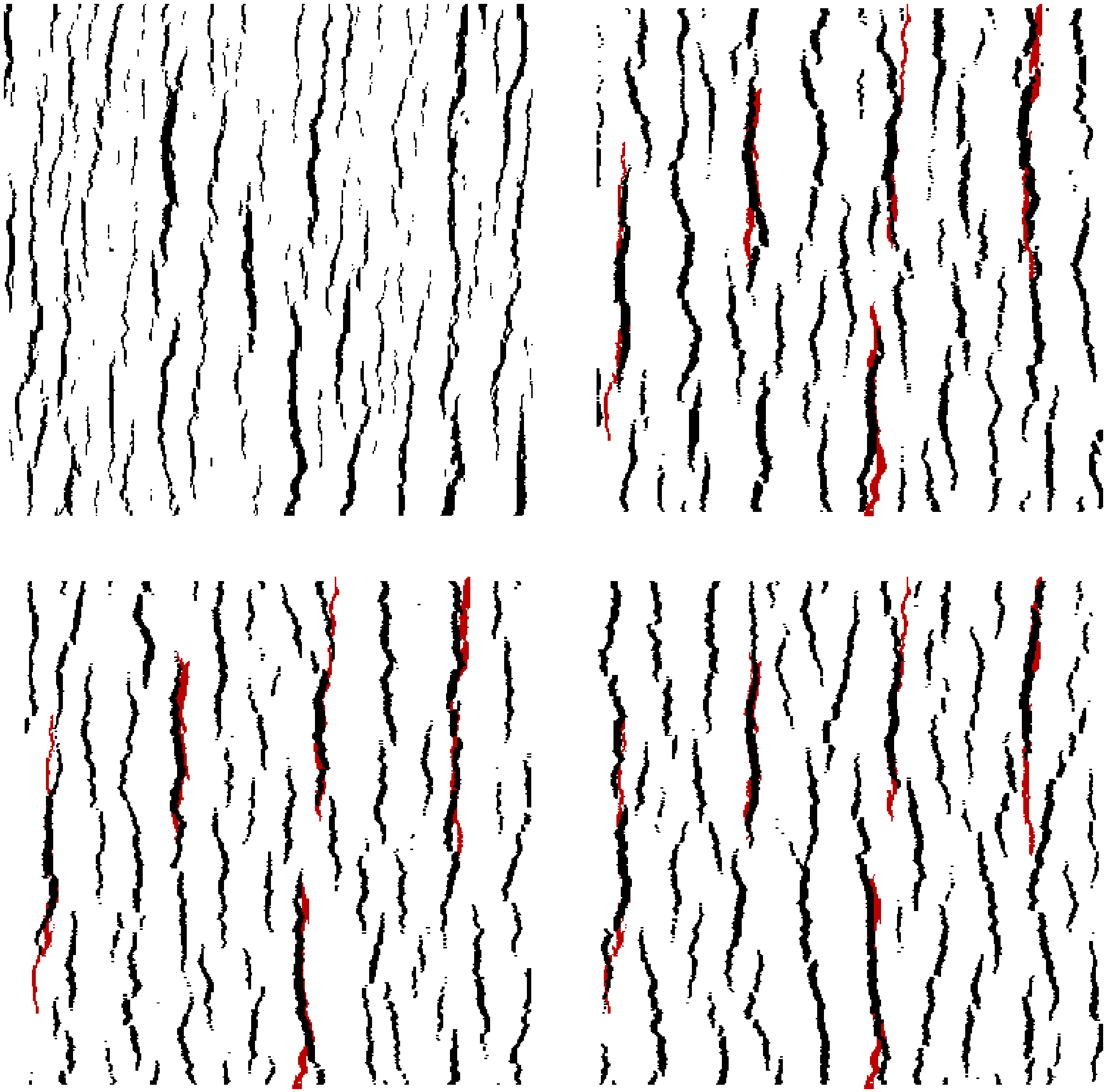
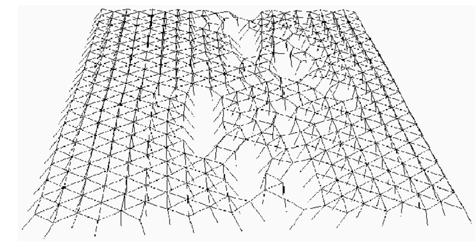
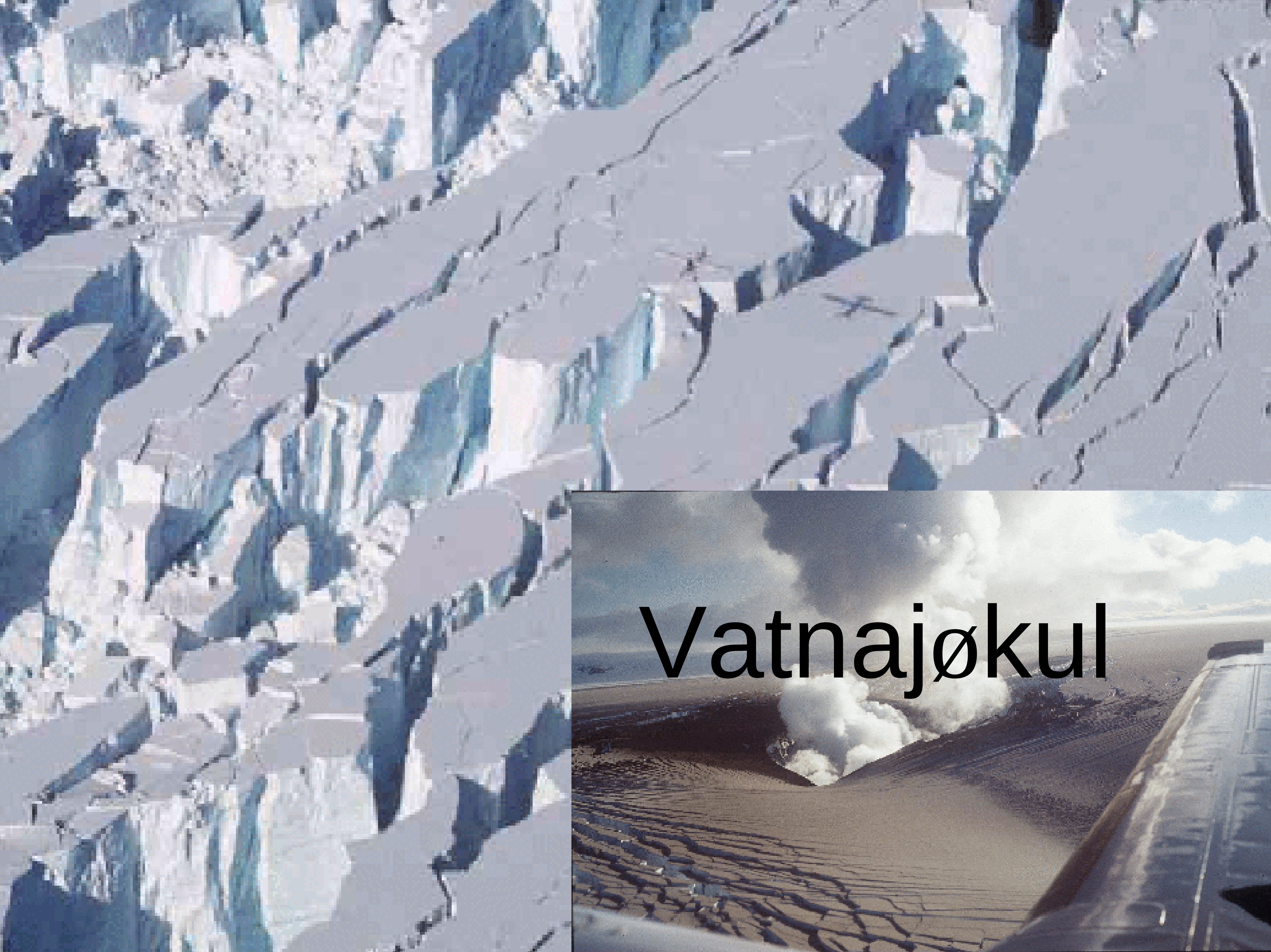

Mekanisk-kjemiske vekselvirkninger
Skalabundne fenomen
Trykkoppløsning
Krystallisasjonskraft
Sprekkleging
Spenningskorrosjon
Forvitring
Mønsterdannende prosesser i naturen (geologi på roterommet)
Oppsprekking
Krøllete ark
Friksjon
Terassedannelse
Honningmønstre
Intrusjoner
Kompaksjon
Skalerende fenomen
Mekaniske/modell-vekselvirkninger

Experiment & Algortihmic Model

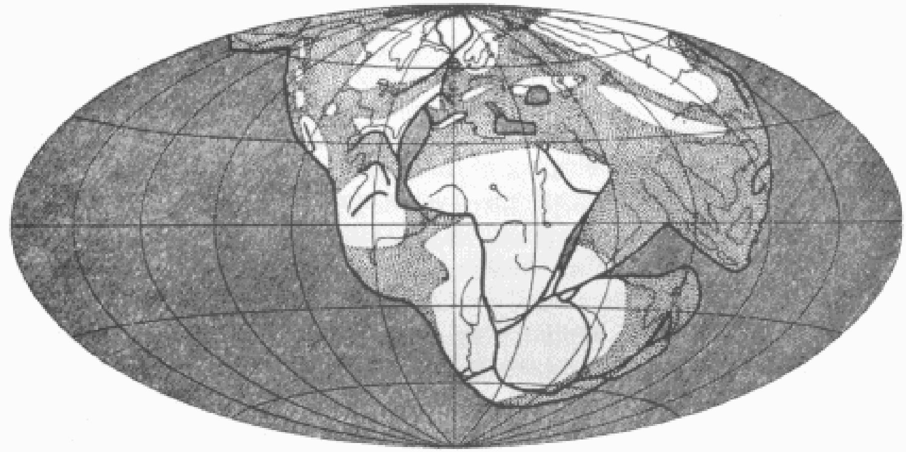


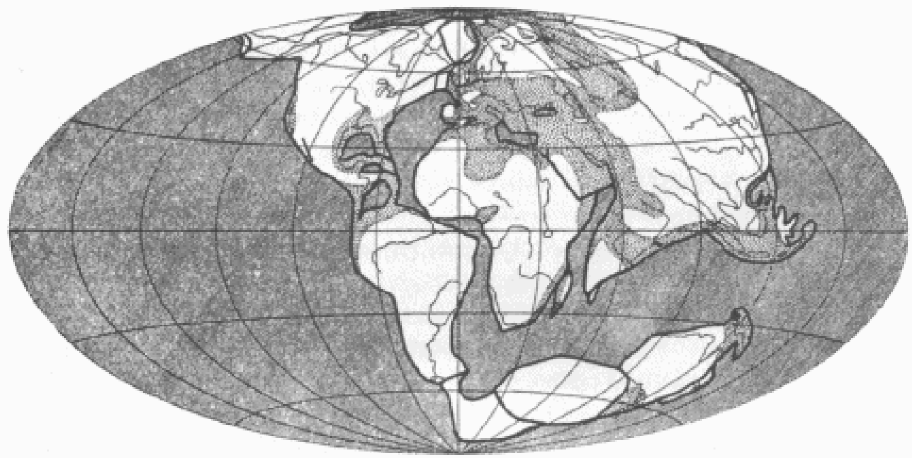


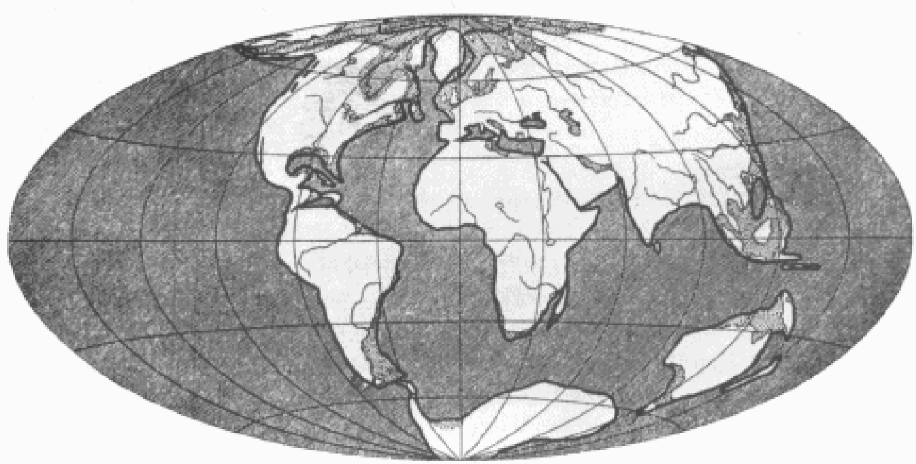

Vatnajøkul

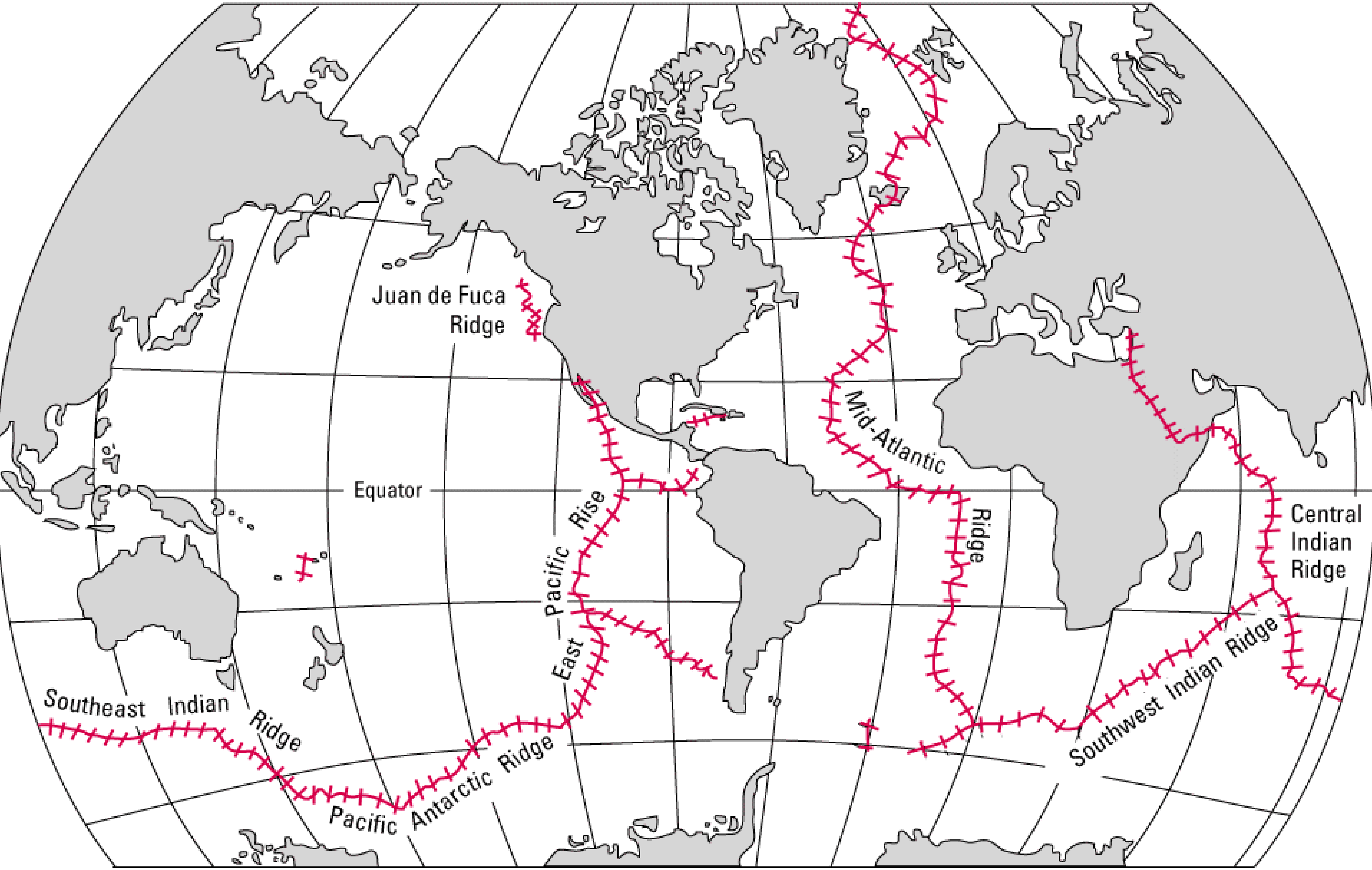
Fractures Hisarøy

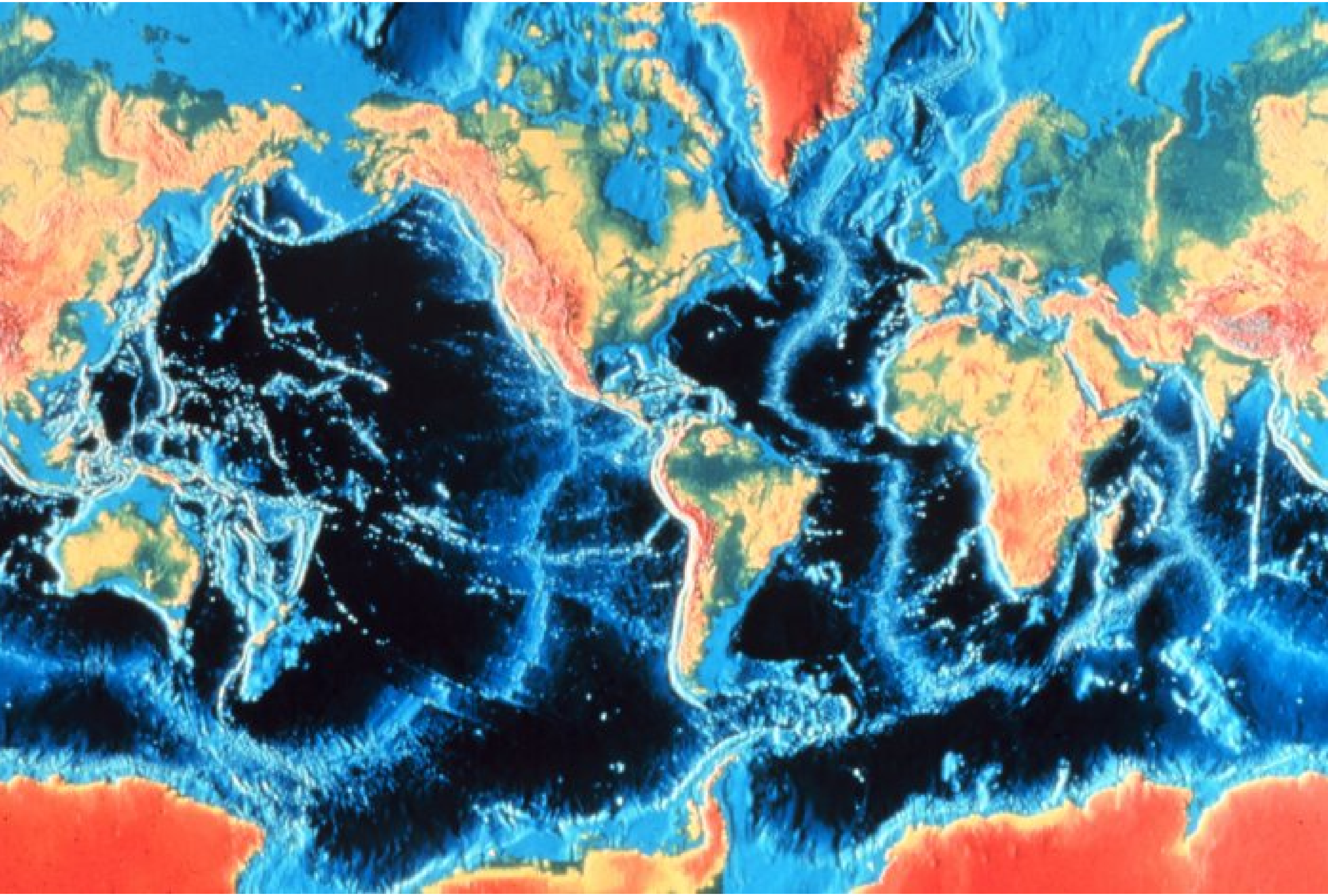
Fractures: Experiments

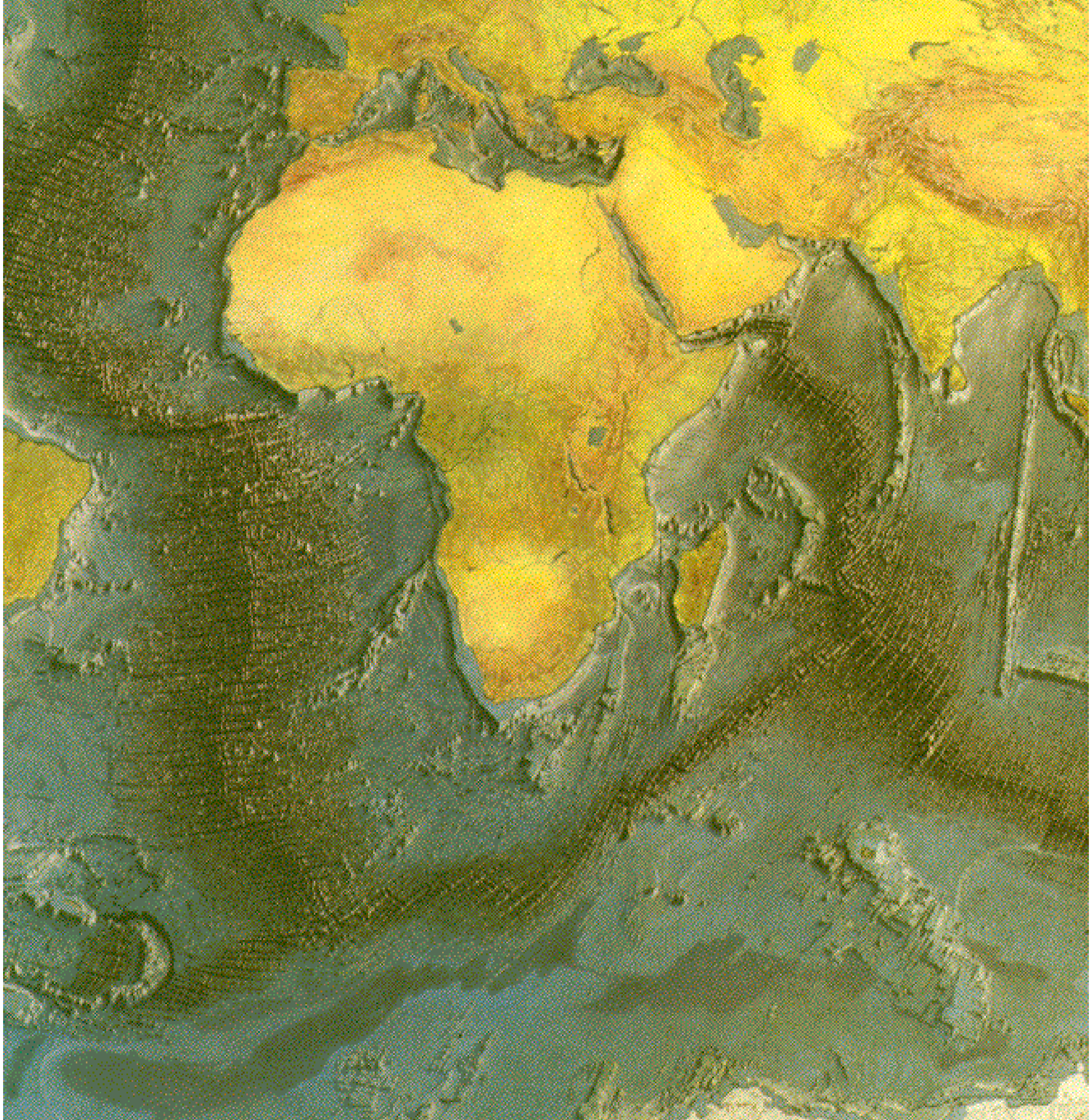






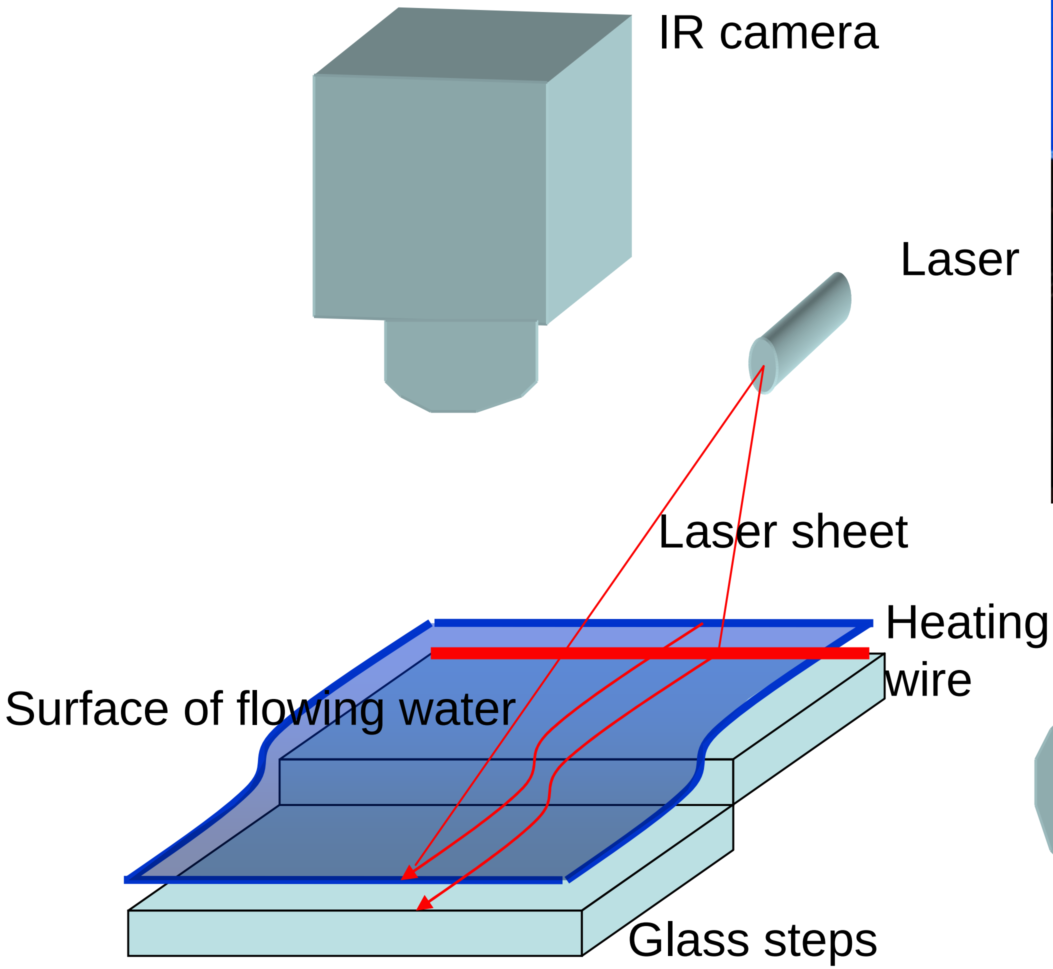

Juan de Fuca Ridge
Equator
East Pacific Rise
Mid-Atlantic Ridge
Central Indian Ridge
Southeast Indian Ridge
Pacific Antarctic Ridge
Southwest Indian Ridge

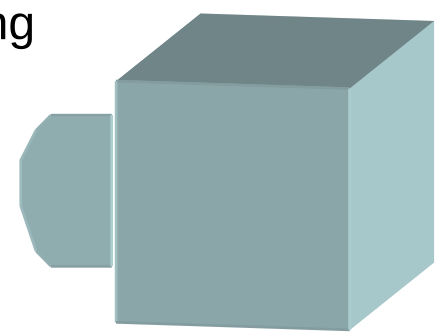




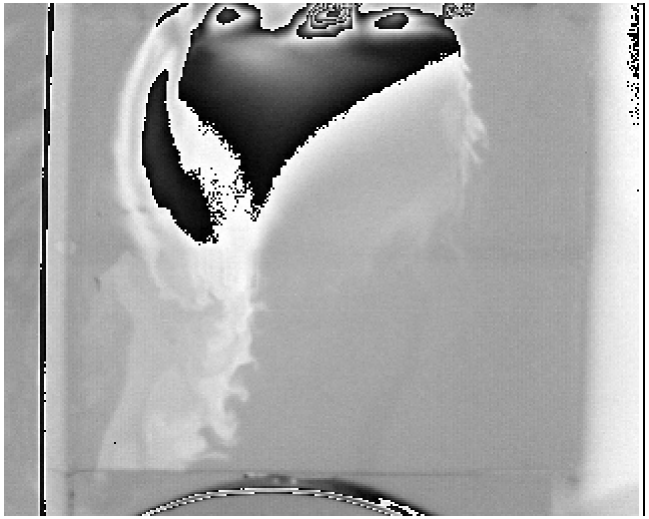
Terassedannelse

fluid flow at steps

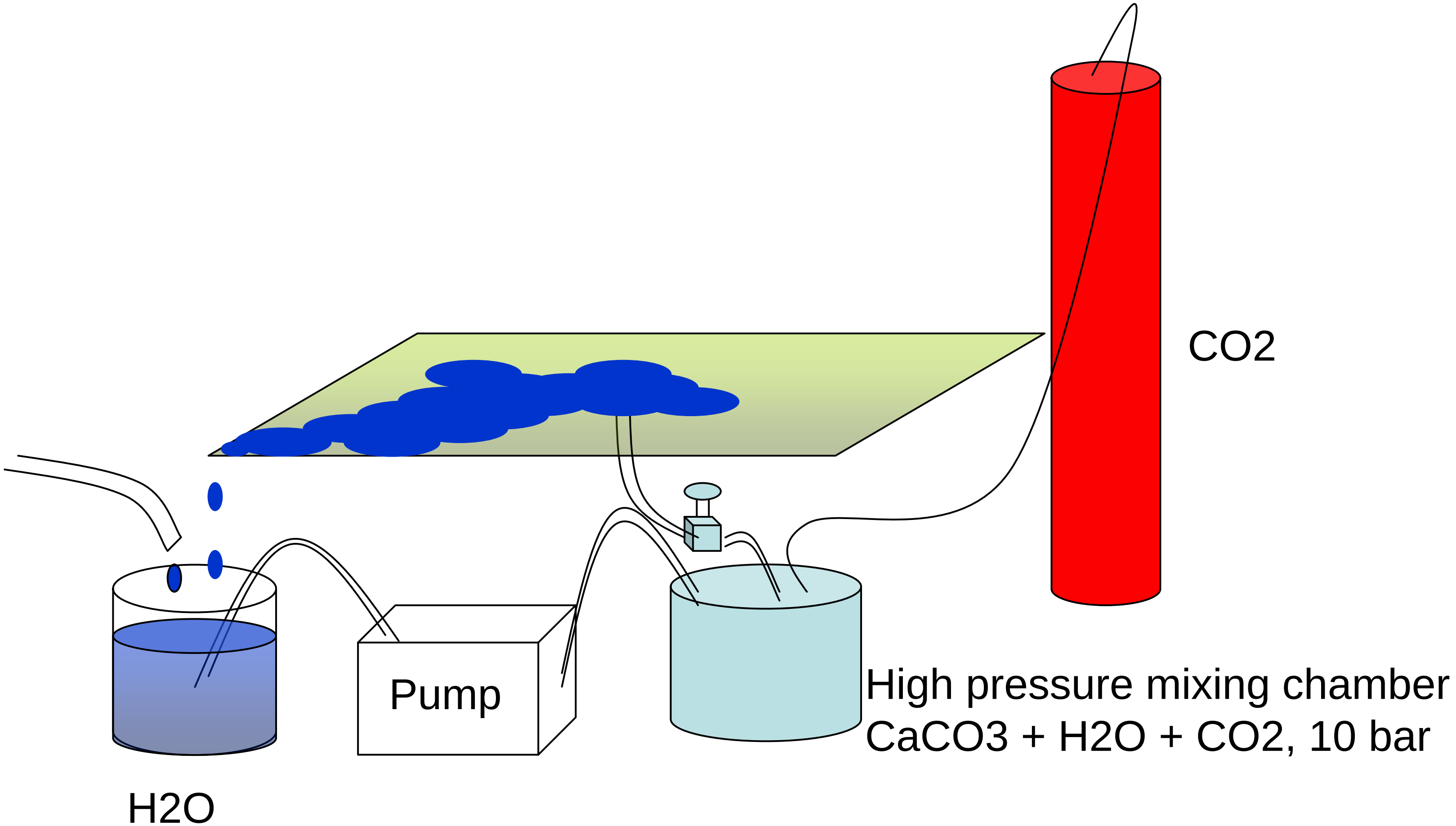




heat diffusion flow at steps

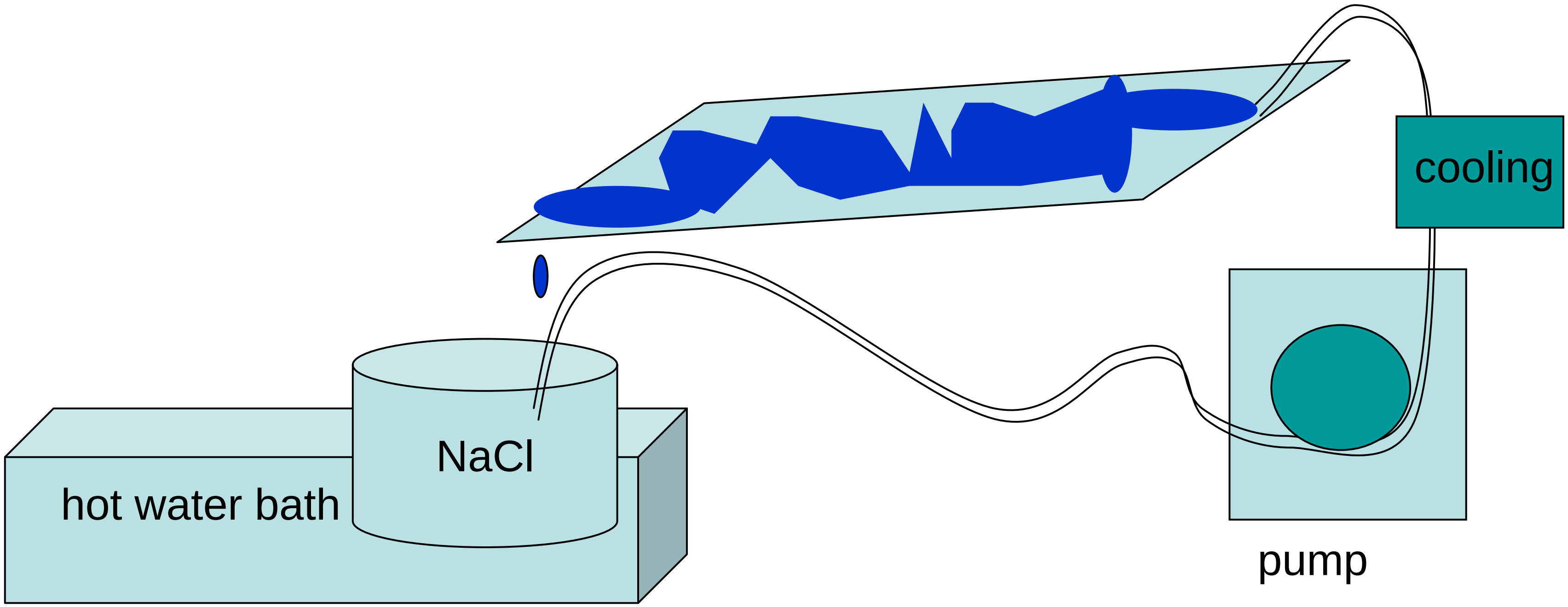


hydrothermal system in lab



- pH at start and end of "pools" similar to Troll springs
- 300 mg/l Ca at start of pools (260 mg/l is equilibrium)
- 500 mg/l is equilibrium conc. at 10 bar, 20 C
- improve with: powder, filter, higher pumping capacity, cooling, tilting + rough base

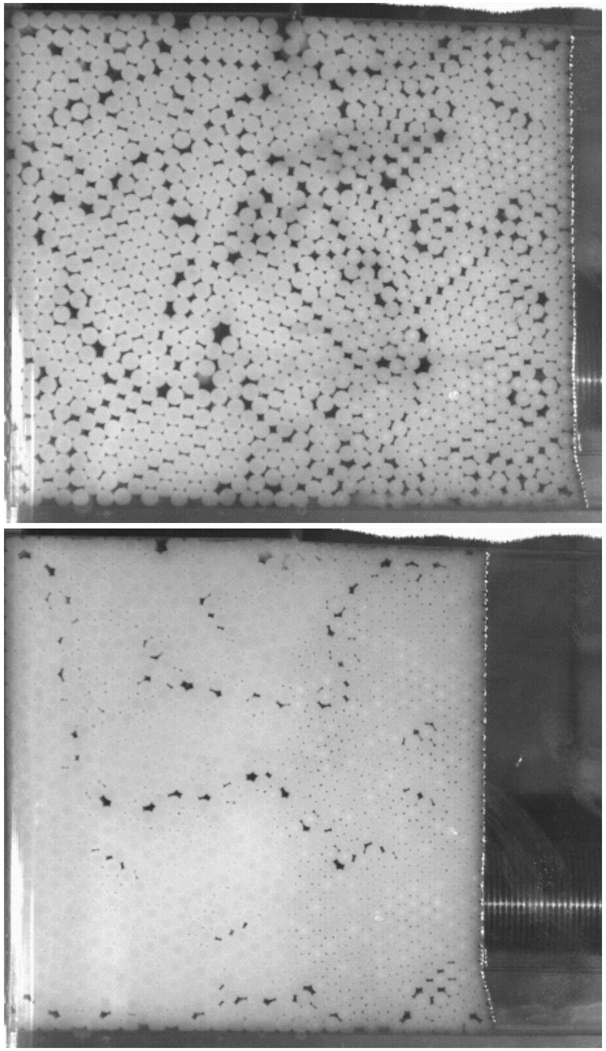
sodium chloride pools

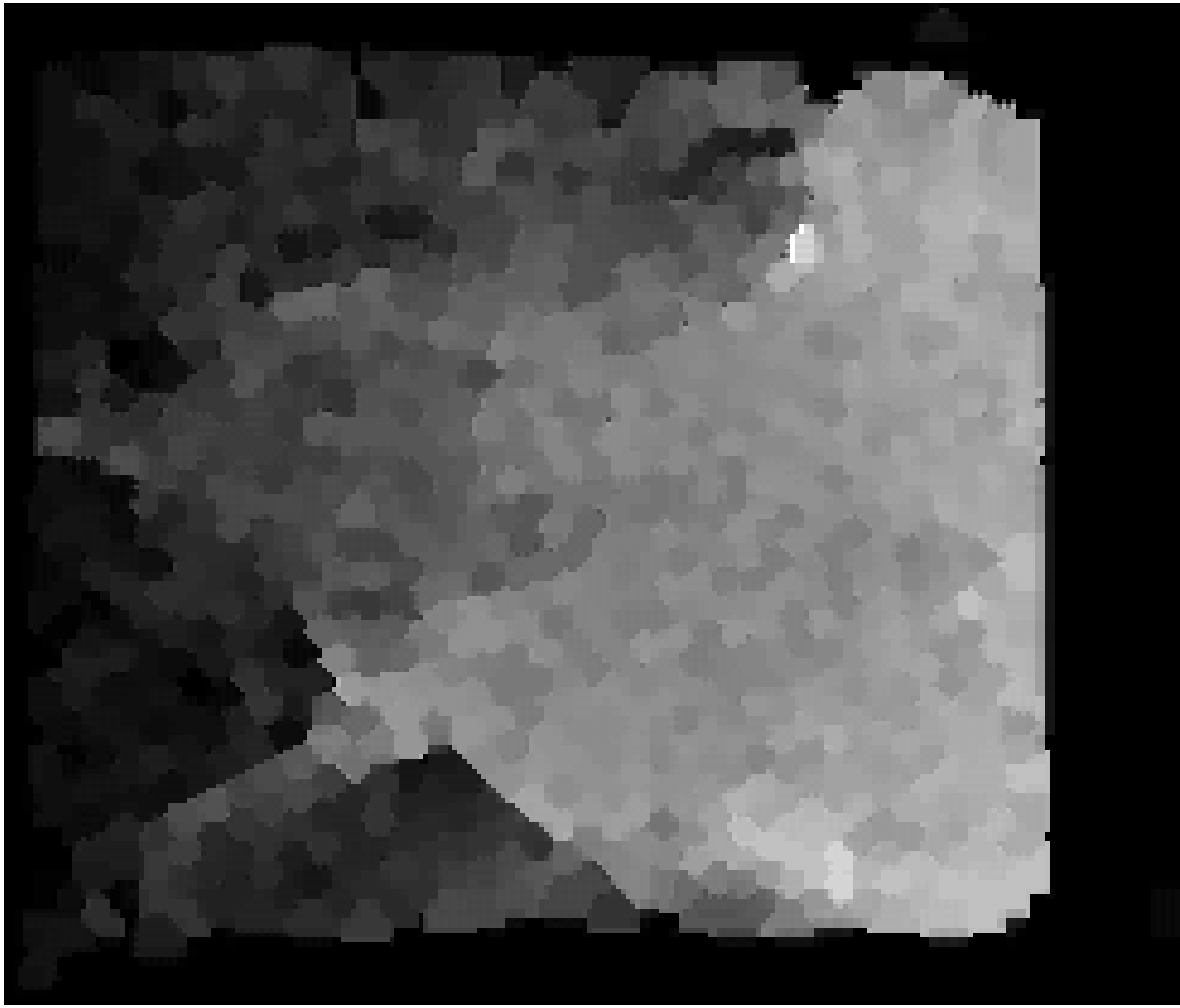


sodium chloride pools

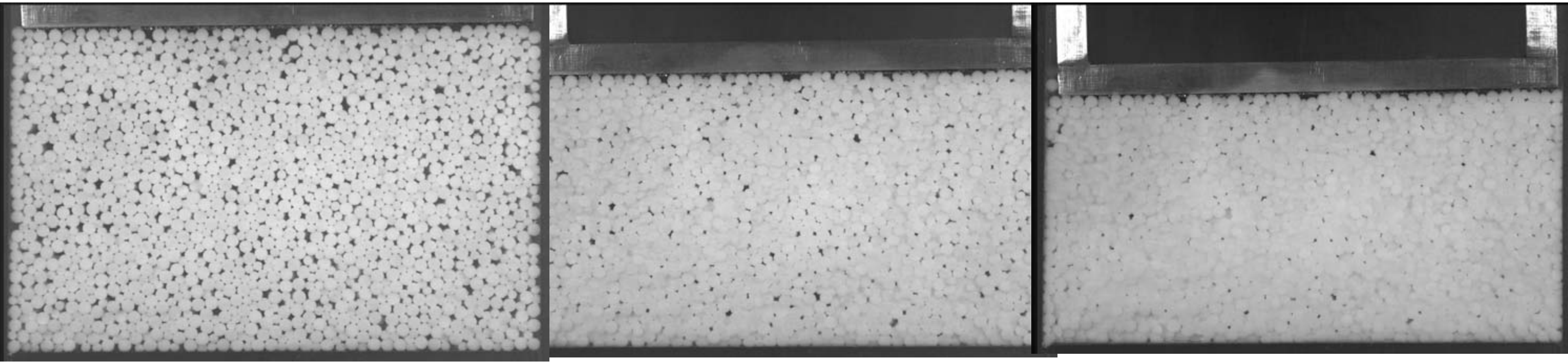
Ice, Oslo

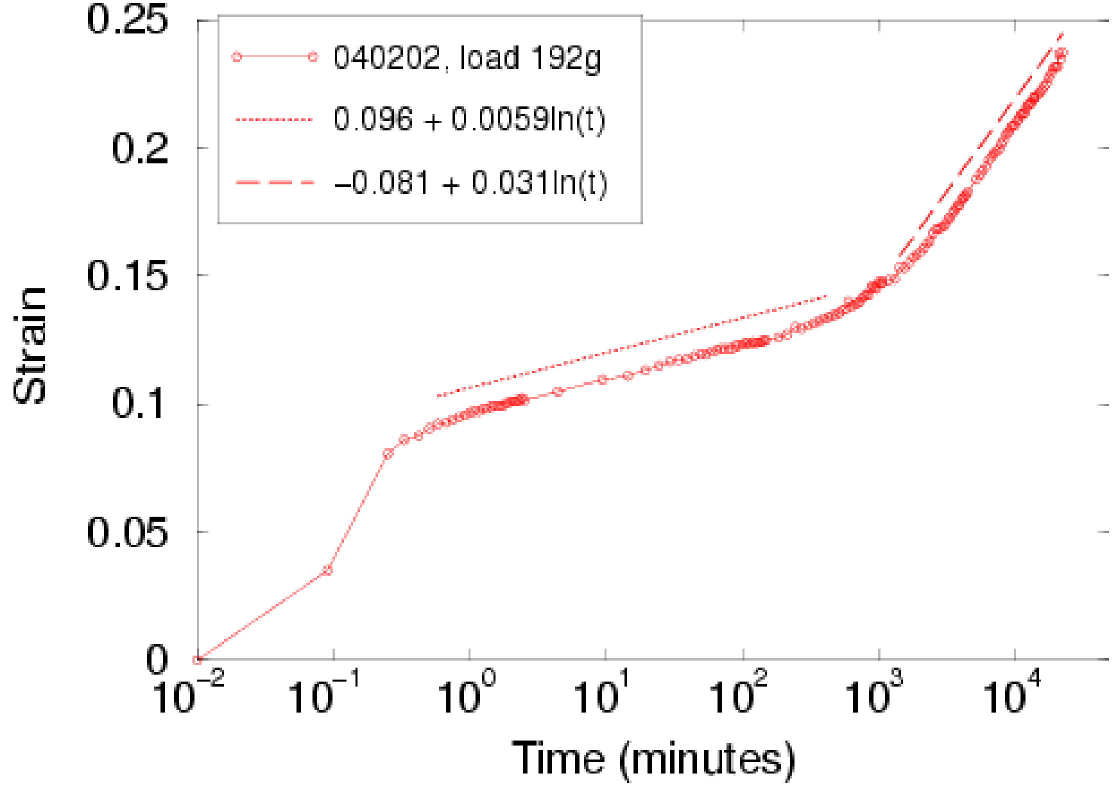
Compaction of plastically deforming grains



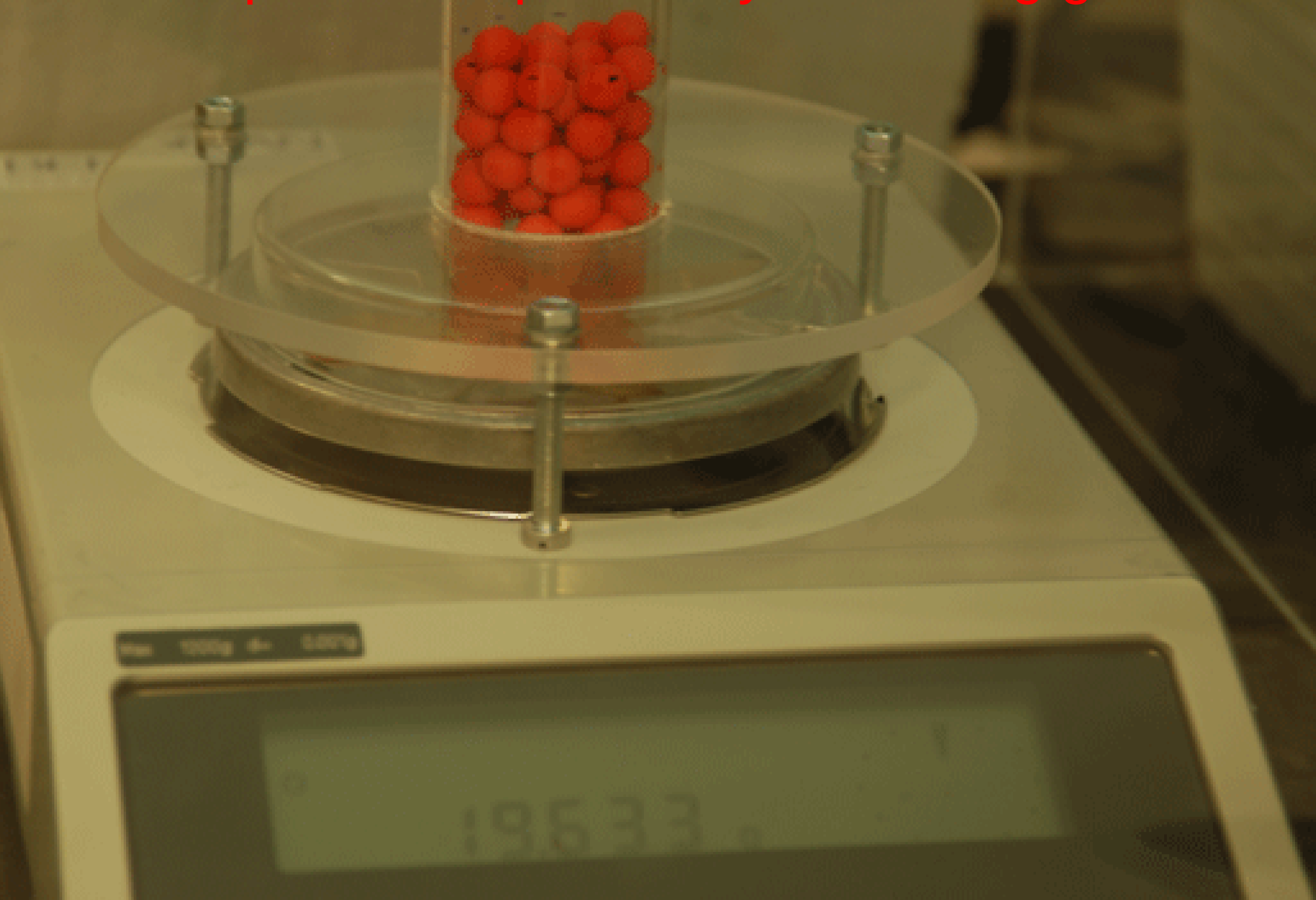


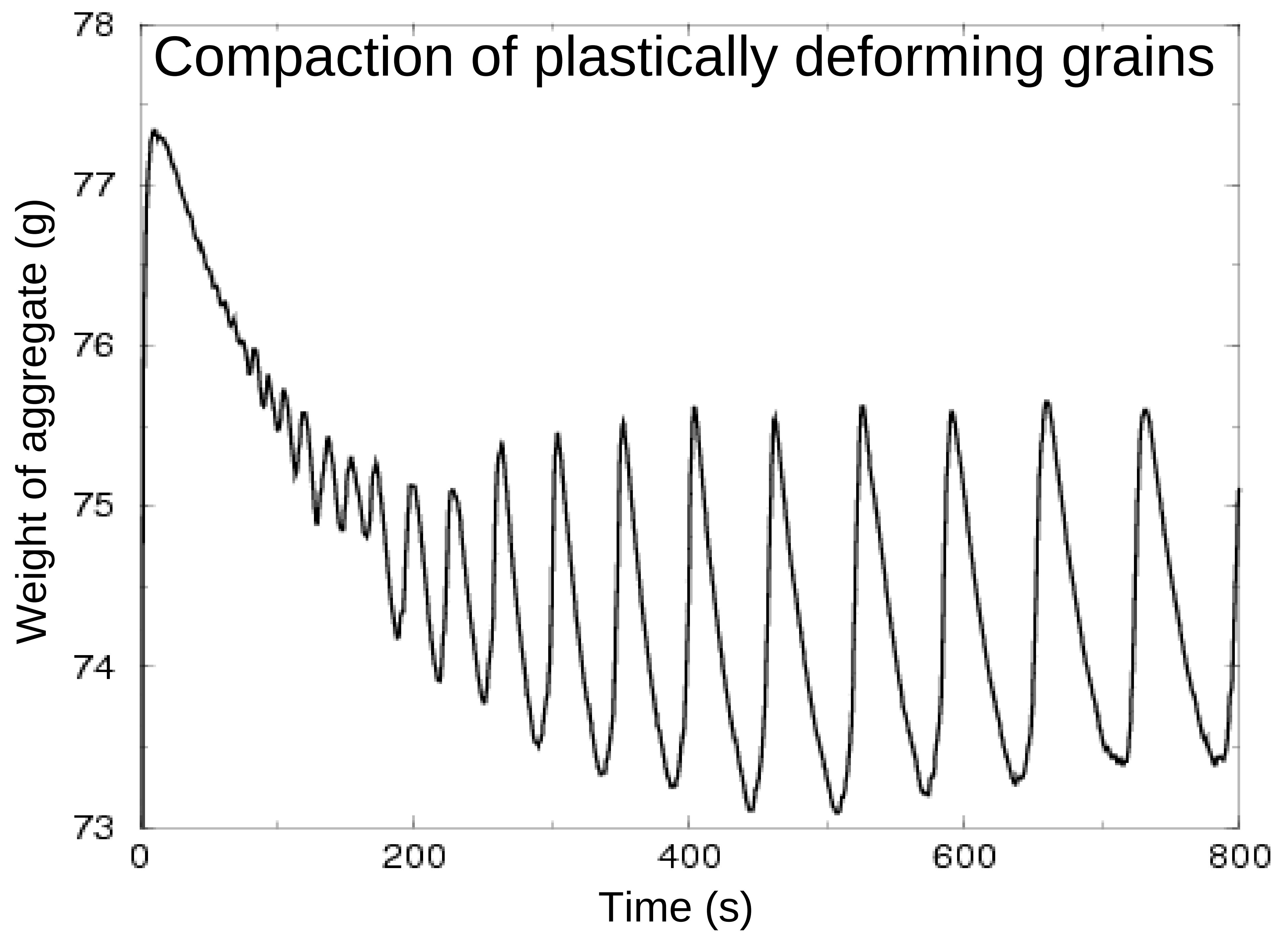
Compaction of plastically deforming grains





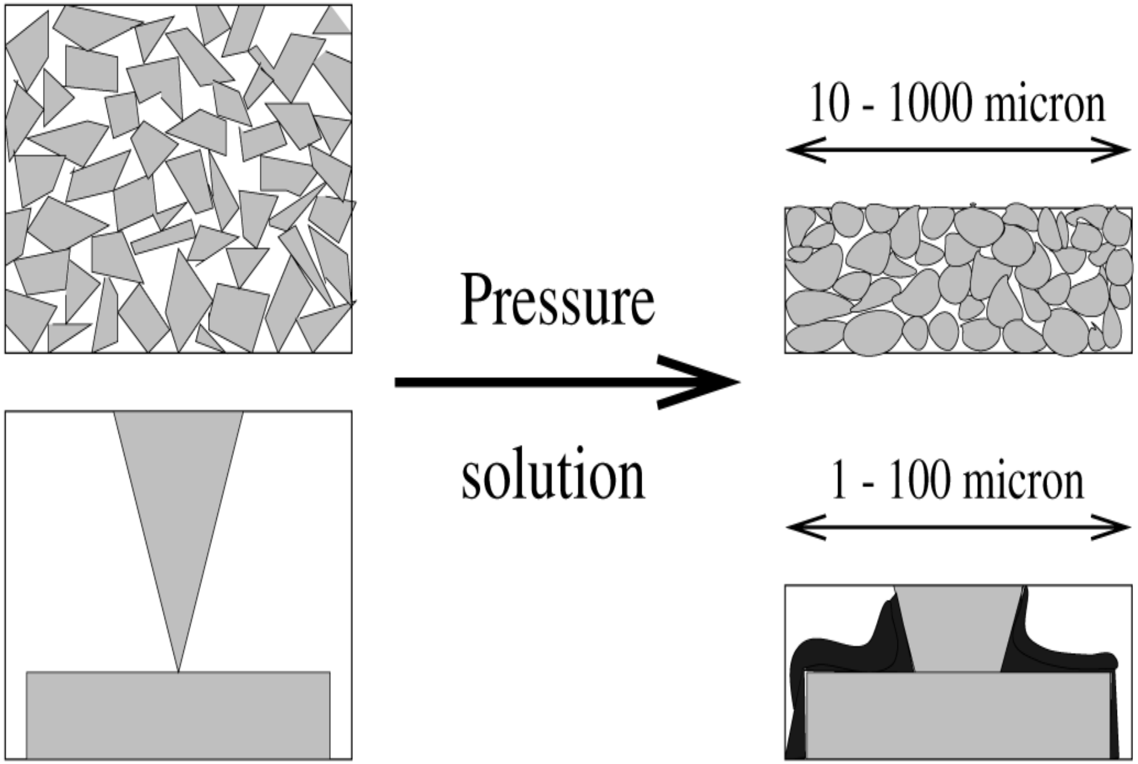
Compaction of plastically deforming grains





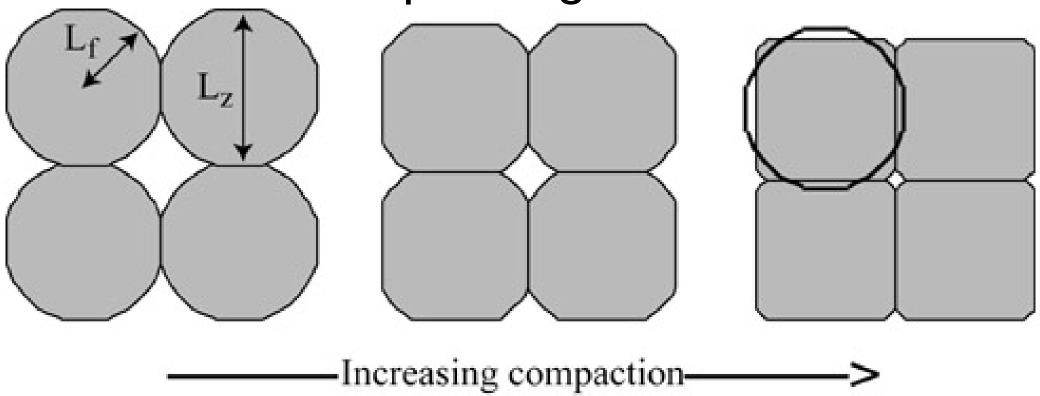
Compaction of plastically deforming grains
Weight of aggregate (g)
Time (s)
73
74
75
76
77
78
0
200
400
600
800

Compaction by pressure solution creep

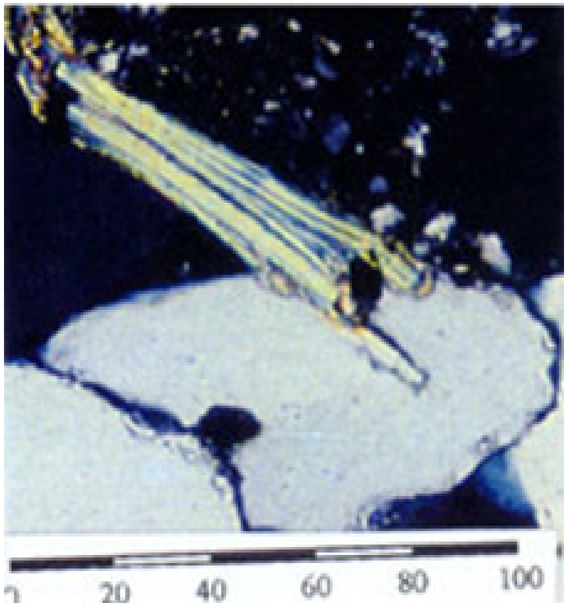


- Dissolution of stressed **surface** of solid
- Diffusion of dissolved mass
- Precipitation on less stressed **surface** of solid

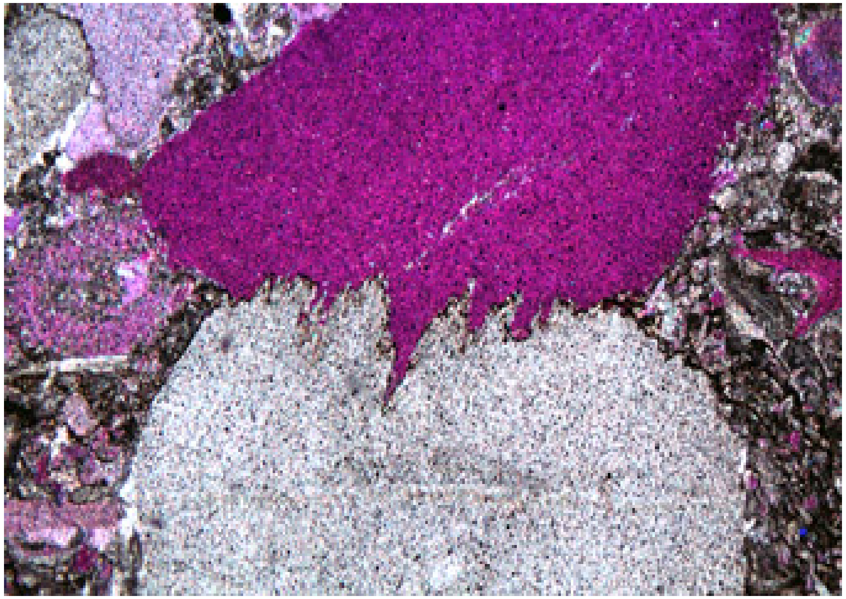
Static packing models:



Pressure solution

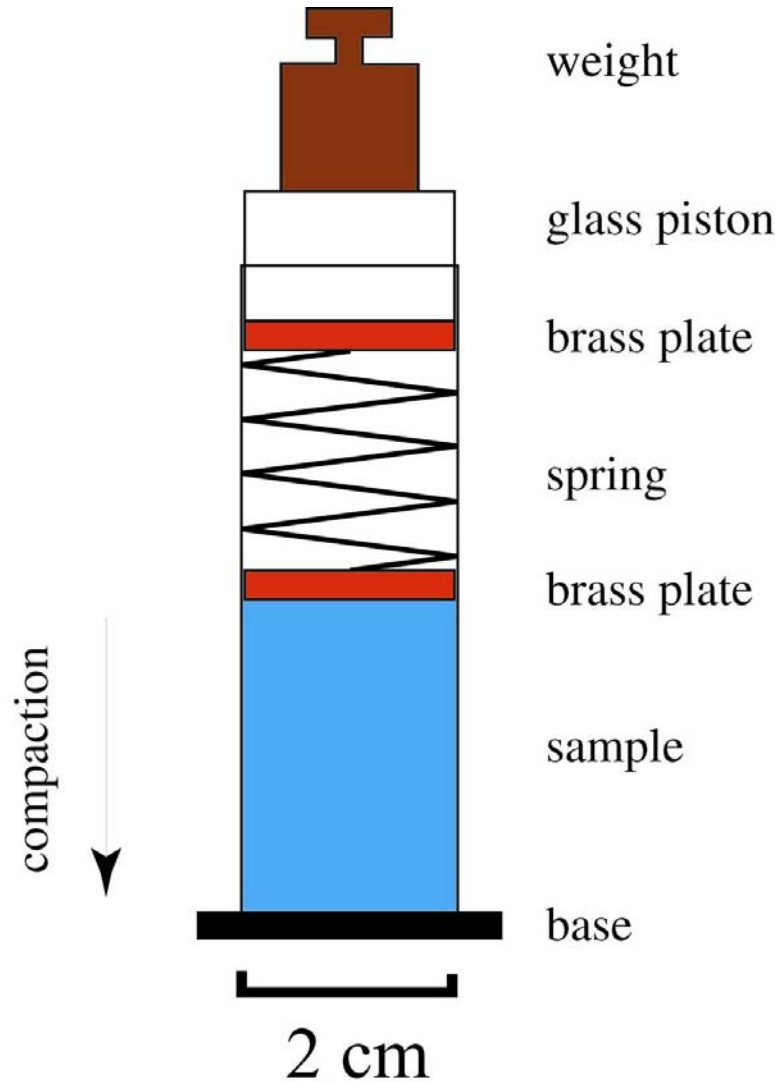


mica indenting a quartz

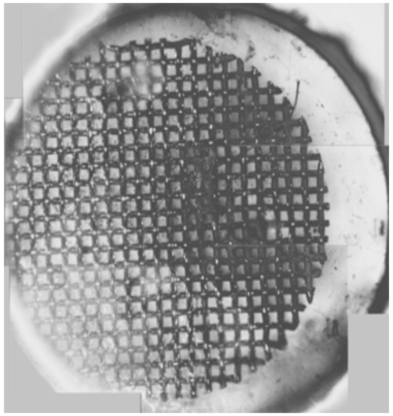


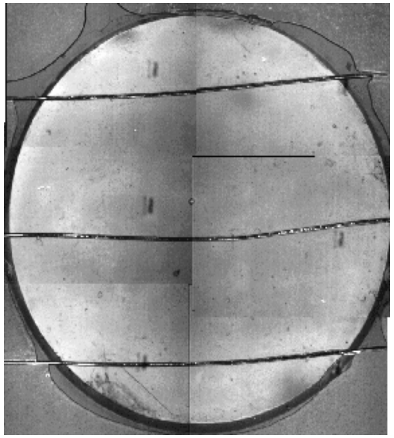
stylolite between two calcite of sea-urchin fossils

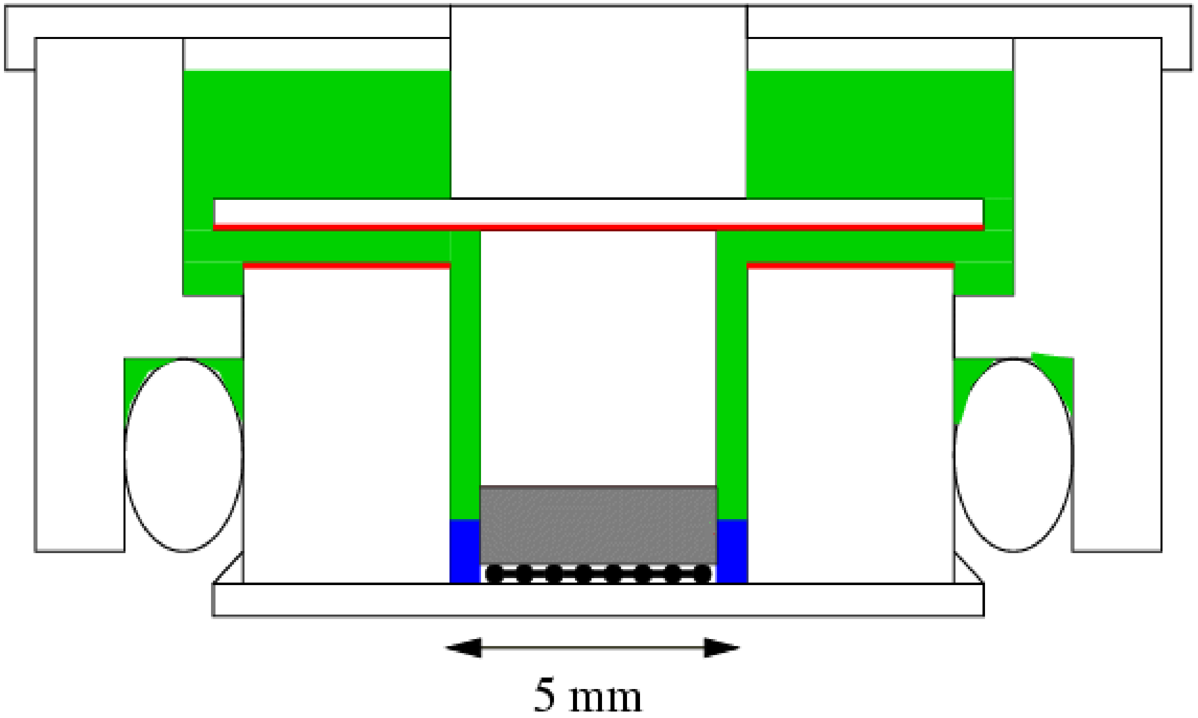
Experimental set-up



Sample holder – single grain experiments

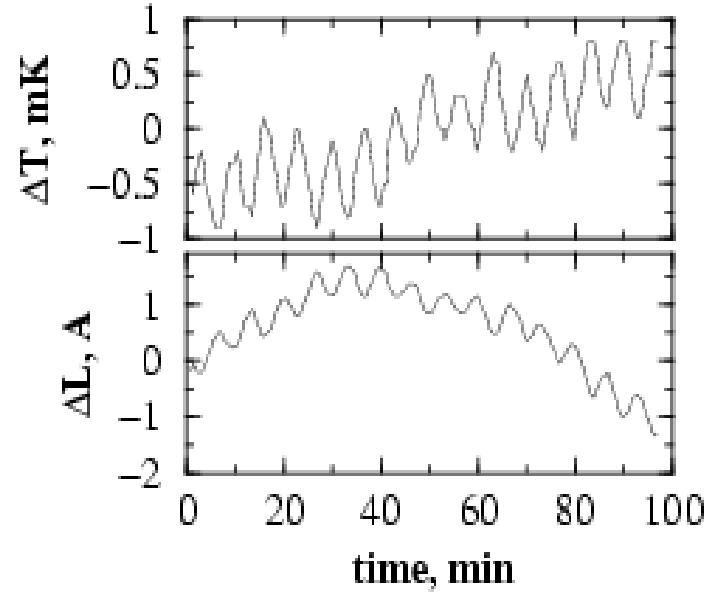






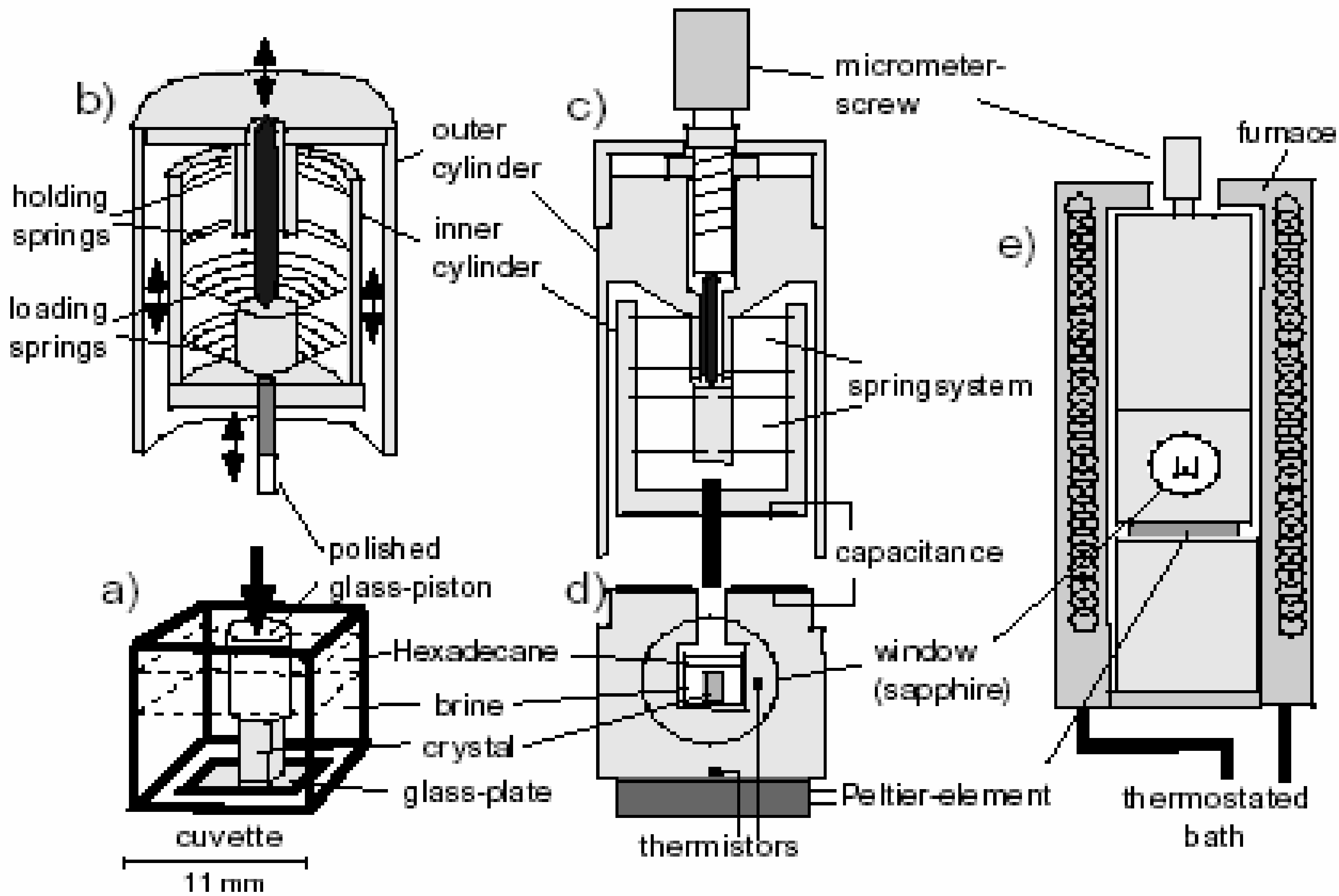
Ångstrøm resolution capacitance dilatometry

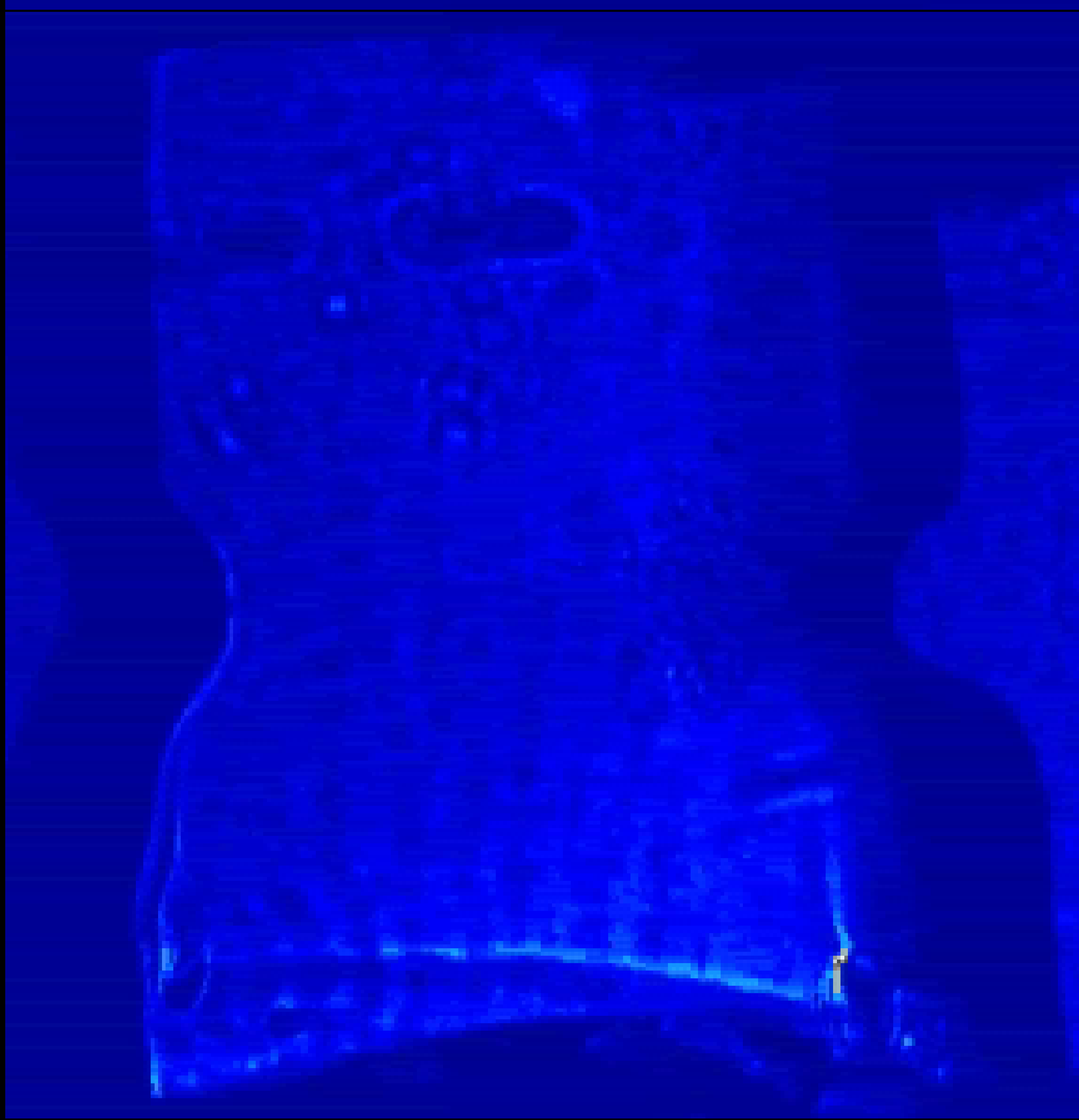
Forced thermal oscillations

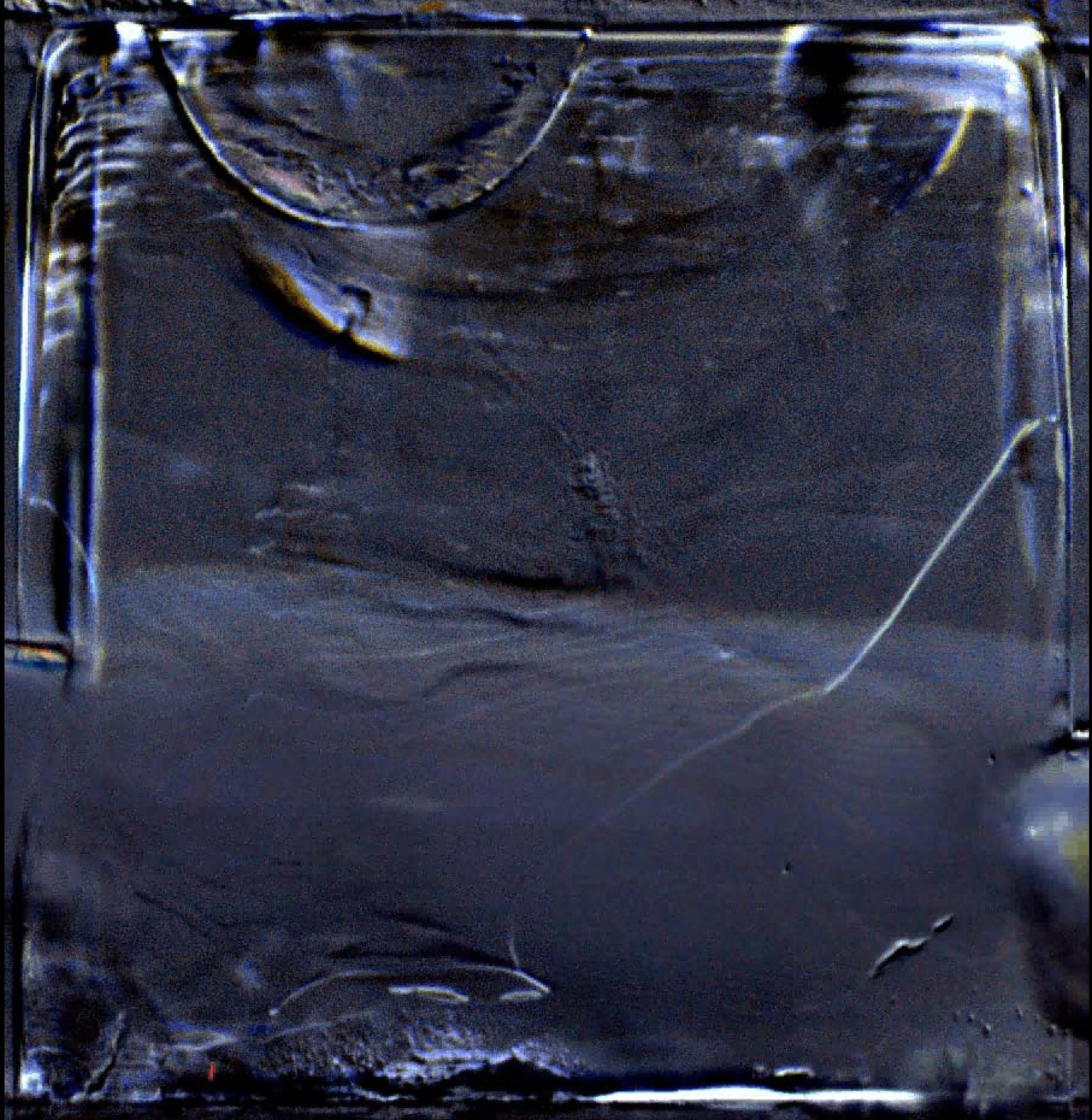


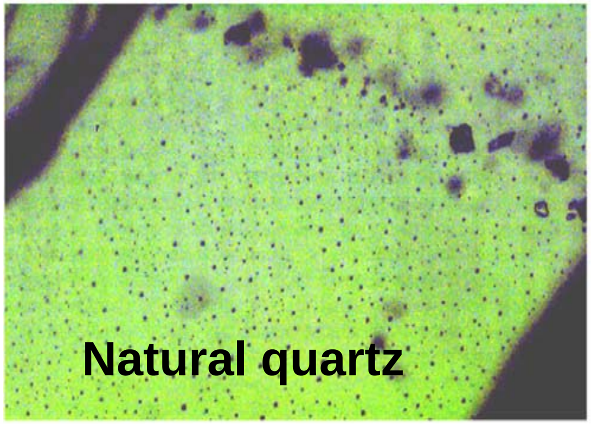
Electrical, thermal and mechanical limits to:

- Resolution, ±0.2 Å
- Stability, ±5 Å in one hour, ±100 Å for > 100 hours


b)
outer cylinder
inner cylinder
holding springs
loading springs
polished glass-piston
a)
Hexadecane
brine
crystal
glass-plate
cuvette
11 mm
c)
micrometer-screw
spring system
capacitance
d)
window (sapphire)
Peltier-element
thermistors
e)
furnace
thermostated bath

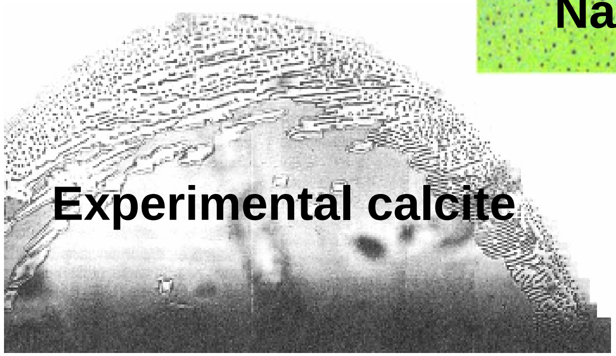






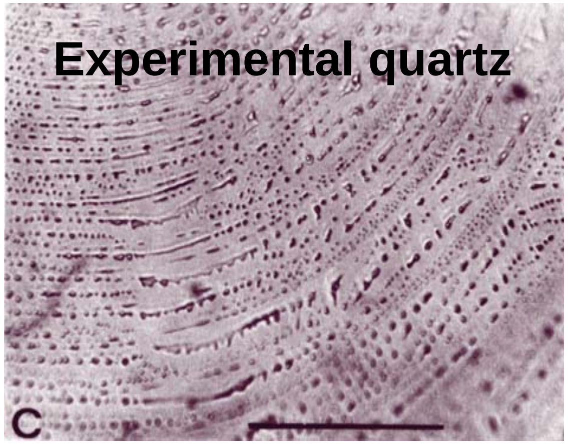
Natural quartz

Gel + olive oil



Experimental calcite

Experimental quartz



Gel + motor oil