Diffusion excersise, FYS-PGP 4300

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These analytical excersises are intended to make you familiar with some solutions to the diffusion equation and to obtain valuable information from typical distributions.

DIFFUSION FROM A POINT SOURCE

Verify that

$$c(x,t) = \frac{A}{\sqrt{t}} e^{-x^2/4Dt} \quad (1)$$

is a solution to

$$\frac{\partial c}{\partial t} - D\frac{\partial^2 c}{\partial x^2} \quad (2)$$

DIFFUSION IN LIQUIDS

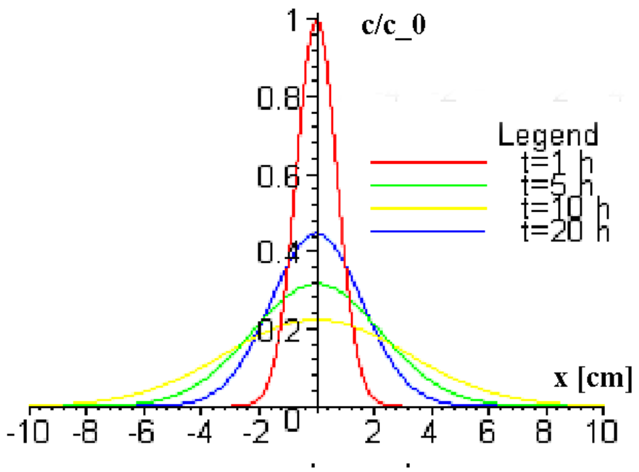

FIG. 1: Concentration curves $c(x,t)$ for diffusion from a point source at the origin at time $t = 0$.

Figure 1 shows typical concentration curves, $c(x,t)$, for diffusion from a point source at the origin $x = 0$ at time $t = 0$. Measure the width of the curves to determine the diffusion constant.

DISTRIBUTED SOURCE, THE ERROR FUNCTION

When the concentration distribution at time 0 is a step function: $c(x \leq 0, t = 0) = c_0$, $c(x > 0, t = 0) = 0$ the solution to the diffusion equation is the integrated effect over point sources between $x = 0$ and $x = -\infty$:

$$c(x,t) = \int_x^{\infty} \frac{c_0}{2\sqrt{\pi D t}} e^{-\xi^2/4Dt} d\xi \quad (3)$$

Use the transformation

$$\eta = \frac{\xi}{2\sqrt{Dt}} \quad (4)$$

to express $c(x,t)$ in terms of the error function:

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-\eta^2} d\eta \quad (5)$$

Use Matlab to plot the curves $c(x,t)$ and $c(\eta)/c_0$ at 1, 5, 10 and 20 hours for the diffusion coefficient you calculated in the first excersise. (The error function in Matlab is erf().)

DIFFUSION DATING OF TECTONIC EVENTS

In the adjoined Nature article a new model for Eclogitization of the crust is proposed. The main basis of their claim are the diffusion profiles shown in figure 2. What boundary conditions have they used? Are these the only appropriate BC? What do you think the authors mean by "maximum" and "mean" diffusion distances?

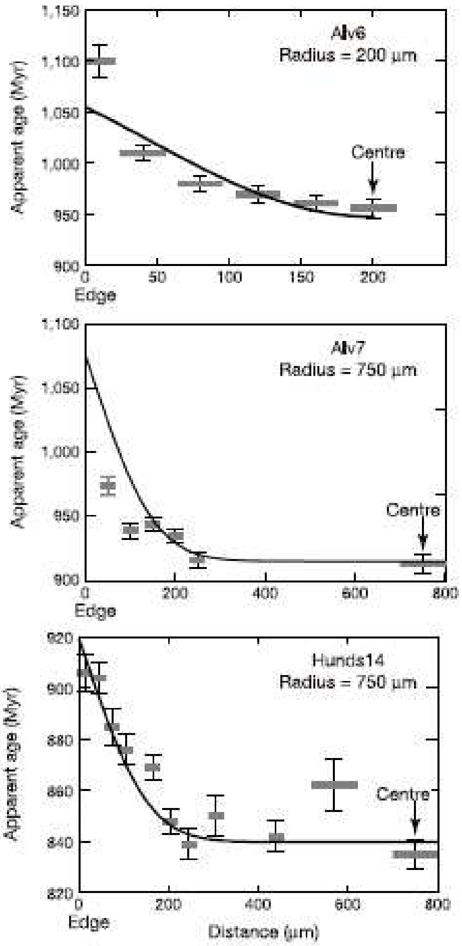

FIG. 2: Apparent 40Ar-39Ar age versus distance profiles across phlogopite from Alv6, Alv7 and Hunds14. On the horizontal axis, the length of the symbol represents the width of the trench analysed; the error on the age is $\pm 2\sigma$. Superimposed on these profiles are the theoretically derived apparent-age curves for volume diffusion of 40ArE into different sized phlogopites from the grain boundary due to a thermal spike of 526 8C lasting 18 kyr. Observed maximum diffusion distances of 200 mm are entirely consistent with the mean (integrated) diffusion distances of 79 mm discussed in the text. 40ArE concentrations in the grain-boundary network are: Hunds14, $1.00 \cdot 10^{-8}$ mol/g; Alv6, $1.37 \cdot 10^{-8}$ mol/g; Alv7, $1.25 \cdot 10^{-8}$ mol/g. These concentrations are remarkably similar to each other, suggesting that the fluids served as a common argon reservoir. Argon diffusion parameters for phlogopite were used (activation energy E=242.3 kJ/mol and pre-exponential coefficient $D = 7.5 \cdot 10^{-5}\mathrm{m}^2/\mathrm{s}$), together with a cylindrical diffusion geometry. Note that some of the ages within the profile are anomalously high relative to their position in the grain, and probably reflect the introduction of 40ArE along structural defects.