Exercises FYS-KJM4480, Fall semester 2009

Exercises week 45, November 2-6 2009

Exercise 16

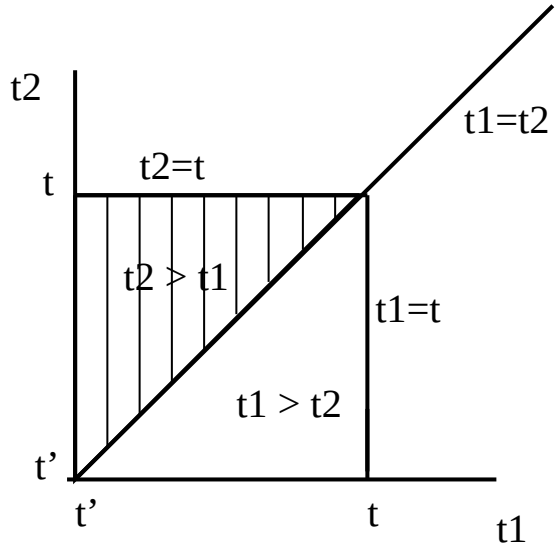
Show that

$$\int_{t'}^{t} dt_1 \int_{t'}^{t_1} dt_2 H_1(t_1)H_1(t_2) = \frac{1}{2}\int_{t'}^{t} dt_1 \int_{t'}^{t} dt_2 T\left[H_1(t_1)H_1(t_2)\right]$$

Hint: Use the definition of T in order to distinguish between $t_1 > t_2$ and $t_1 < t_2$;

$$\int_{t'}^{t} dt_1 \int_{t'}^{t} dt_2 T\left[H_1(t_1)H_1(t_2)\right] = \int_{t'}^{t} dt_1 \left\{ \int_{t'}^{t_1} dt_2 H_1(t_1)H_1(t_2) + \int_{t_1}^{t} dt_2 H_1(t_2)H_1(t_1) \right\}$$

Show that the last term on the right-hand side equals the first term (change the order of the integrations and thereafter integration variables). The area of integration for the first term is shown in the figure below.



Exercise 17

In project 2 you found an expression for the interaction part of the Hamiltonian for the electron gas given by

$$H_I = \frac{e^2}{2V}\sum_{\sigma_1\sigma_2}\sum_{\vec{q}\neq 0,\vec{k},\vec{p}} \frac{4\pi}{q^2} a^{\dagger}_{\vec{k}+\vec{q},\sigma_1} a^{\dagger}_{\vec{p}-\vec{q},\sigma_2} a_{\vec{p}\sigma_2} a_{\vec{k}\sigma_1}$$

a) Find all diagrams to second order in perturbation theory. Set up the corresponding expressions and discuss their behavior.
b) What happens in case you keep the convergence factor μ finite?

This exercise will also be discussed next week as I don't expect that you will finish it during this week. It requires a non-negligible amount of work but illustrates the difference between an interaction with infinite range and a finite-ranged one.