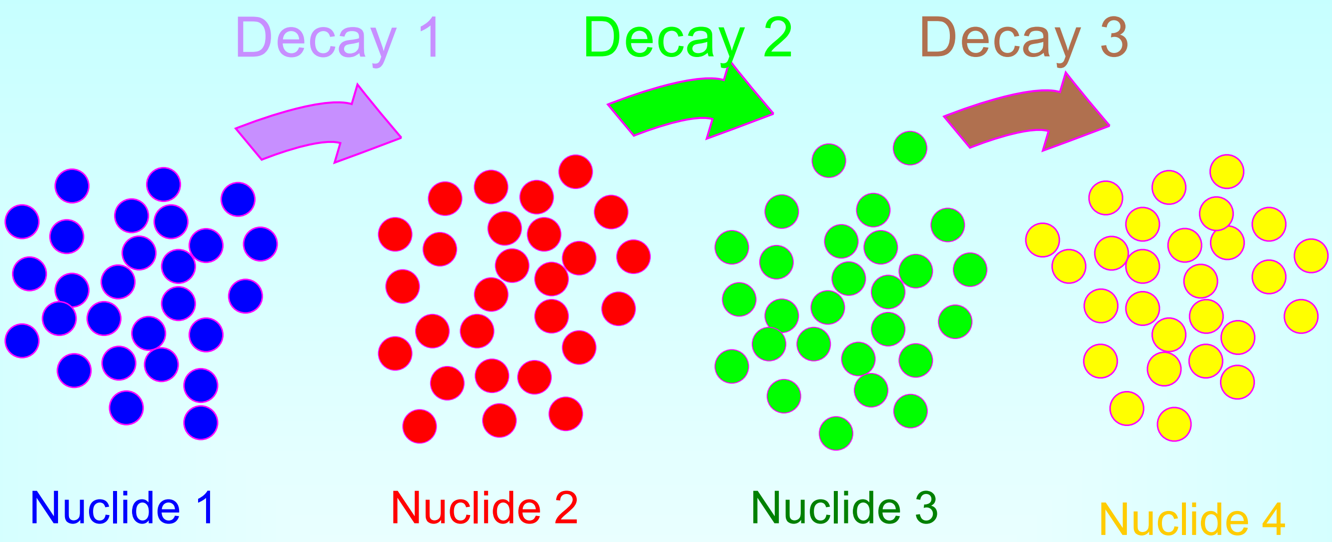
Genetically dependent nuclides

- When a radioactive nuclide disintegrates to a nucløide which in turn also is radioactive, we say that the two are genetically dependent
- There can be many consecutive nuclides in a genetic series, for instance: in the disintegration of ^{238}U, the nucleus ends in ^{206}Pb after 14 disintegrations.

Important example:

$$^{99}Mo\ (66h) \xrightarrow{\beta^-} {}^{99m}Tc\ (6.0h) \xrightarrow{IT} {}^{99}Tc\ (213\,000\ y)$$

Genetic dependence



For genetically dependent nuclides it is important to remember that **the same** atom changees all the time, and goes through different stages before ending up as stable.

Genetically dependent nuclides ctd.

Other important examples:

^{210}Pb (22.3y) $\xrightarrow{\beta^-}$ ^{210}Bi (5.0d) $\xrightarrow{\beta^-}$ ^{210}Po (138 d) $\xrightarrow{\alpha}$ ^{206}Pb (stab.)

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^{90}Sr (29y) $\xrightarrow{\beta^-}$ ^{90}Y (2.7d) $\xrightarrow{\beta^-}$ ^{90}Zr (stable)

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^{137}Cs (30y) $\xrightarrow{\beta^-}$ ^{137m}Ba (2.6m) $\xrightarrow{\gamma}$ ^{137}Ba (stable)

Mother/daughter relations

- We have two genetically connected radionuclides, 1 and 2
- Nuclide 1 ⇒ nuclide 2 ⇒ stable

We want an expression of disintegration rates as function of time and start-conditions.

Assume that nuclide 1 is the first.
Then we have:

$N_1 = N_{1,0}e^{-\lambda_1 t}$

in the time interval dt, the increase in N2 is:

$dN_2 = (\lambda_1 N_1 - \lambda_2 N_2)dt$

or:

$$\frac{dN_2}{dt} + \lambda_2 N_2 - \lambda_1 N_{1,0}e^{-\lambda_1 t} = 0$$

Mother/daughter relations

Solve this differential equation:

$$N2 \equiv uv, \Rightarrow \frac{dN_2}{dt} = v\frac{du}{dt} + u\frac{dv}{dt}$$

$$v\frac{du}{dt} + u\frac{dv}{dt} + \lambda_2 uv - \lambda_1 N_{1,0} e^{-\lambda_1 t} = 0$$

Demand: $u\left(\frac{dv}{dt} + \lambda_2 v\right) = 0$

Gives: $v = e^{-\lambda_2 t}$

$$\frac{du}{dt} e^{-\lambda_2 t} - \lambda_1 N_{1,0} e^{-\lambda_1 t} = 0 \quad \text{or}$$

$$\frac{du}{dt} = \lambda_1 N_{1,0} e^{-(\lambda_1 - \lambda_2)t}$$

INTEGRATE

$$u = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0} e^{-(\lambda_1 - \lambda_2)t} + C$$

Mother/daughter relations.

$$N_2 = uv = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0} e^{-\lambda_1 t} + C e^{-\lambda_2 t}$$

C must be determined

$$N_{2,0} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0} + C$$

$$C = N_{2,0} - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0}$$

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0} e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0} e^{-\lambda_2 t} + N_{2,0} e^{-\lambda_2 t}$$

$$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + N_{2,0} e^{-\lambda_2 t}$$

$$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0} e^{-\lambda_1 t} (1 - e^{-(\lambda_2 - \lambda_1)t}) + N_{2,0} e^{-\lambda_2 t}$$

$$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1 (1 - e^{-(\lambda_2 - \lambda_1)t}) + N_{2,0} e^{-\lambda_2 t}$$

$$D_2 = \lambda_2 N_2 = \frac{\lambda_2}{\lambda_2 - \lambda_1} \underbrace{D_{1,0} e^{-\lambda_1 t}}_{D_1} (1 - e^{-(\lambda_2 - \lambda_1)t}) + D_{2,0} e^{-\lambda_2 t}$$

Mother/daughter relations.

Frequently, $N_{2,0}$ and $D_{2,0}$ are 0:

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1 (1 - e^{-(\lambda_2 - \lambda_1)t})$$

$$D_2 = \frac{\lambda_2}{\lambda_2 - \lambda_1} D_1 (1 - e^{-(\lambda_2 - \lambda_1)t})$$

Saturation factor

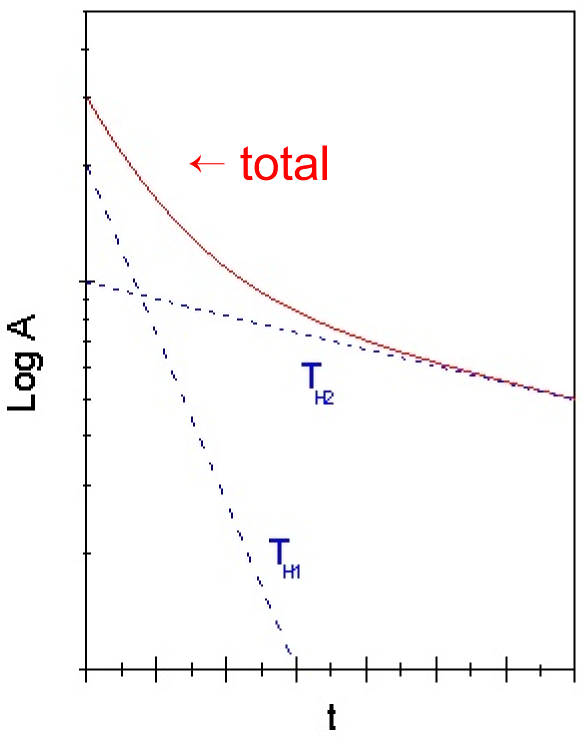
If $\lambda_1 << \lambda_2$:

$$N_2 = \frac{\lambda_1}{\lambda_2} N_1 (1 - e^{-\lambda_2 t})$$

$$D_2 = D_1 (1 - e^{-\lambda_2 t})$$

- Saturation factor:
  - 0,999 after 10 daughter nuclide halflives
  - Then $\lambda_1 N_1 = \lambda_2 N_2$ og $D_1 = D_2$
- With more steps in the chain and $T_{½}(1) >> T_{½}(2)$:
  - $\lambda_1 N_1 = \lambda_2 N_2 = ...\lambda_n N_{n1}$ and $D_1 = D_2 .... = D_n$

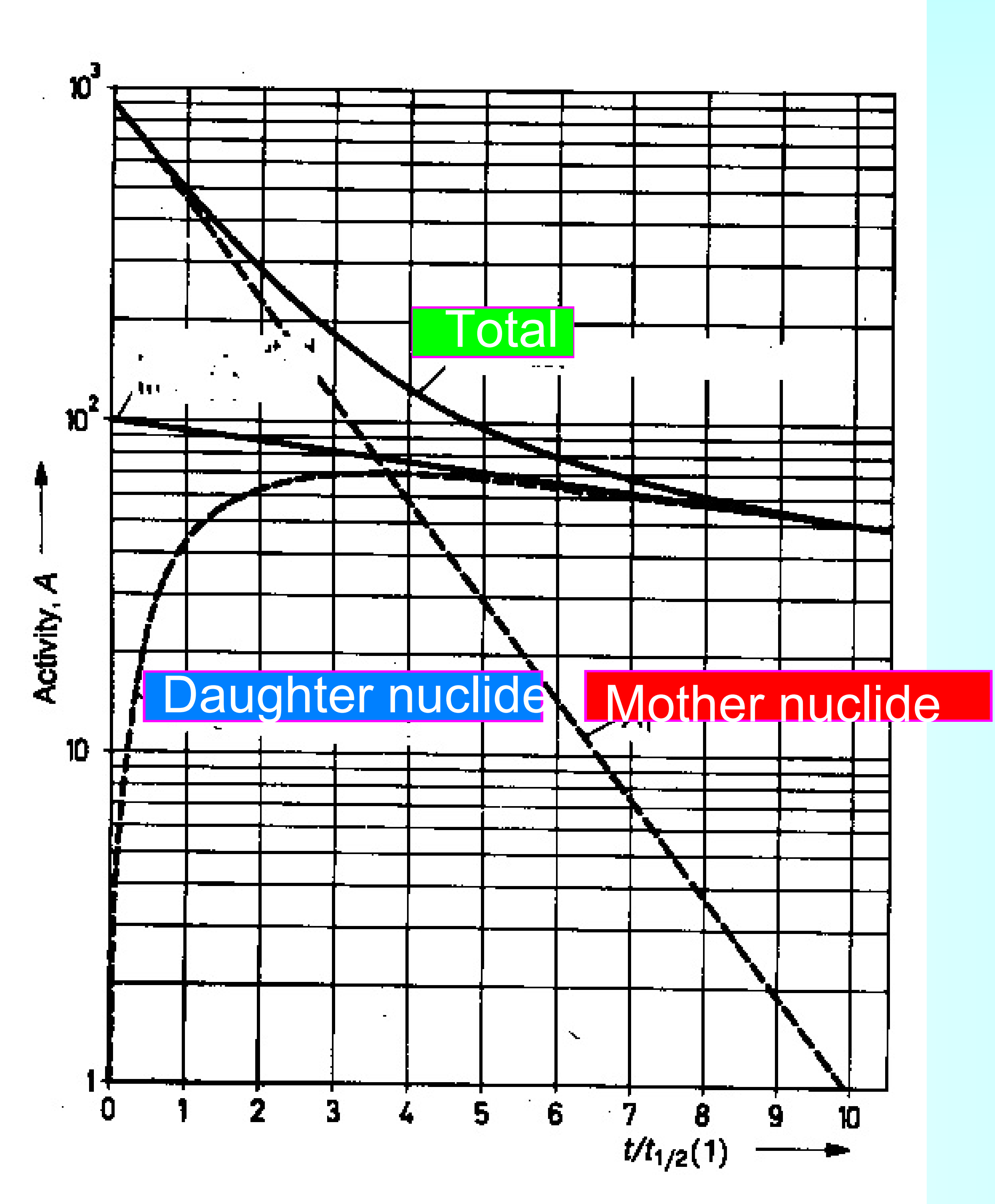
Genetic independence $T_{½}(1)<<T_{½}(2)$



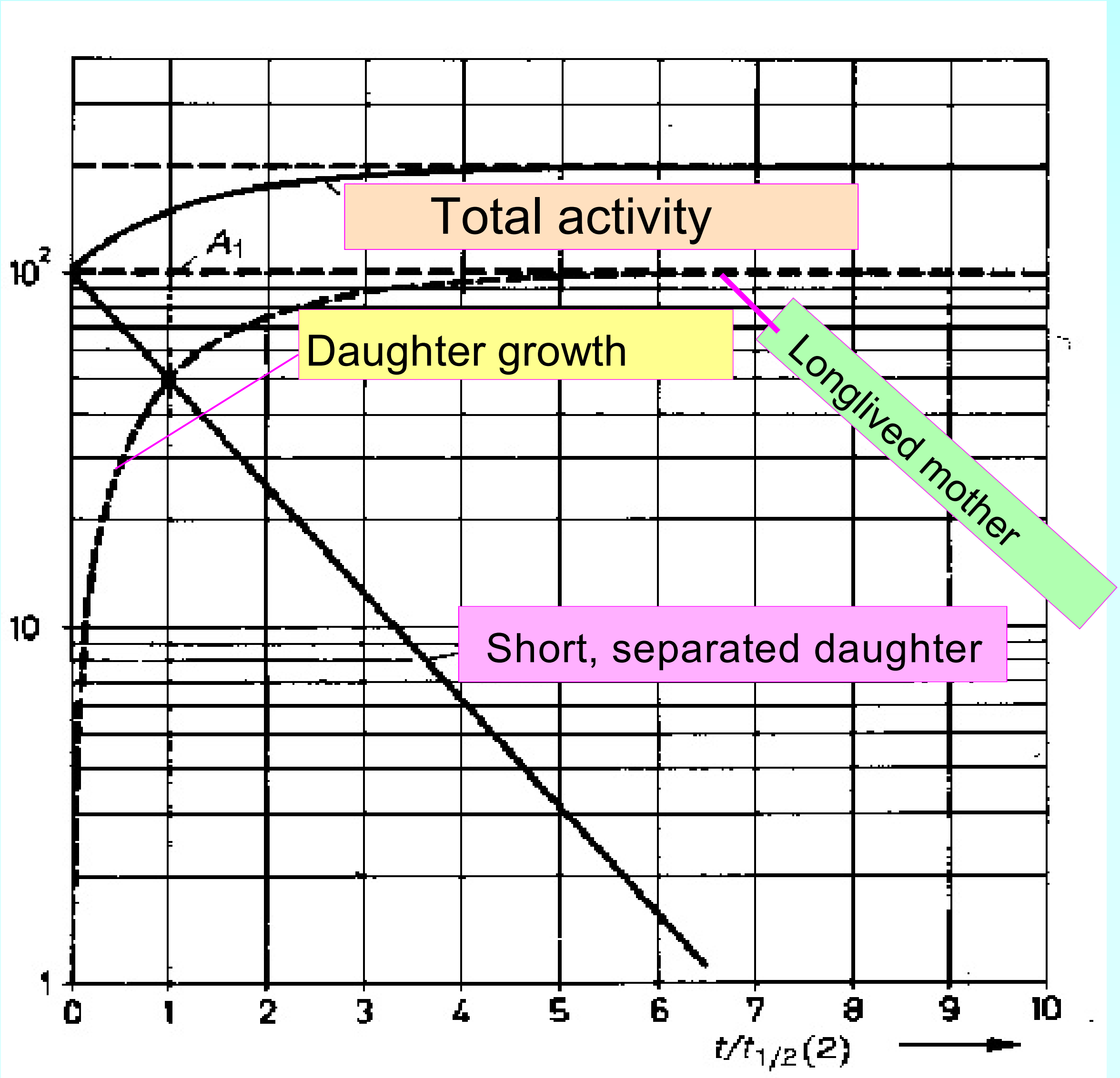
Mother/daughter relations, three cases

- Short mother, long daughter ($\lambda_1 >> \lambda_2$), $T_{½}(1) << T_{½}(2)$
  - No equilibrium
- Long mother, shorter daughter ($\lambda_1 < \lambda_2$), $T_{½}(1) > T_{½}(2)$
  - Transient equilibrium may occur
- Very long mother,short daughter ($\lambda_1 << \lambda_2$), $T_{½}(1) >> T_{½}(2)$
  - Secular equilibrium may occur

Genetically dependent nuclides $T_{½}(1) << T_{½}(2)$

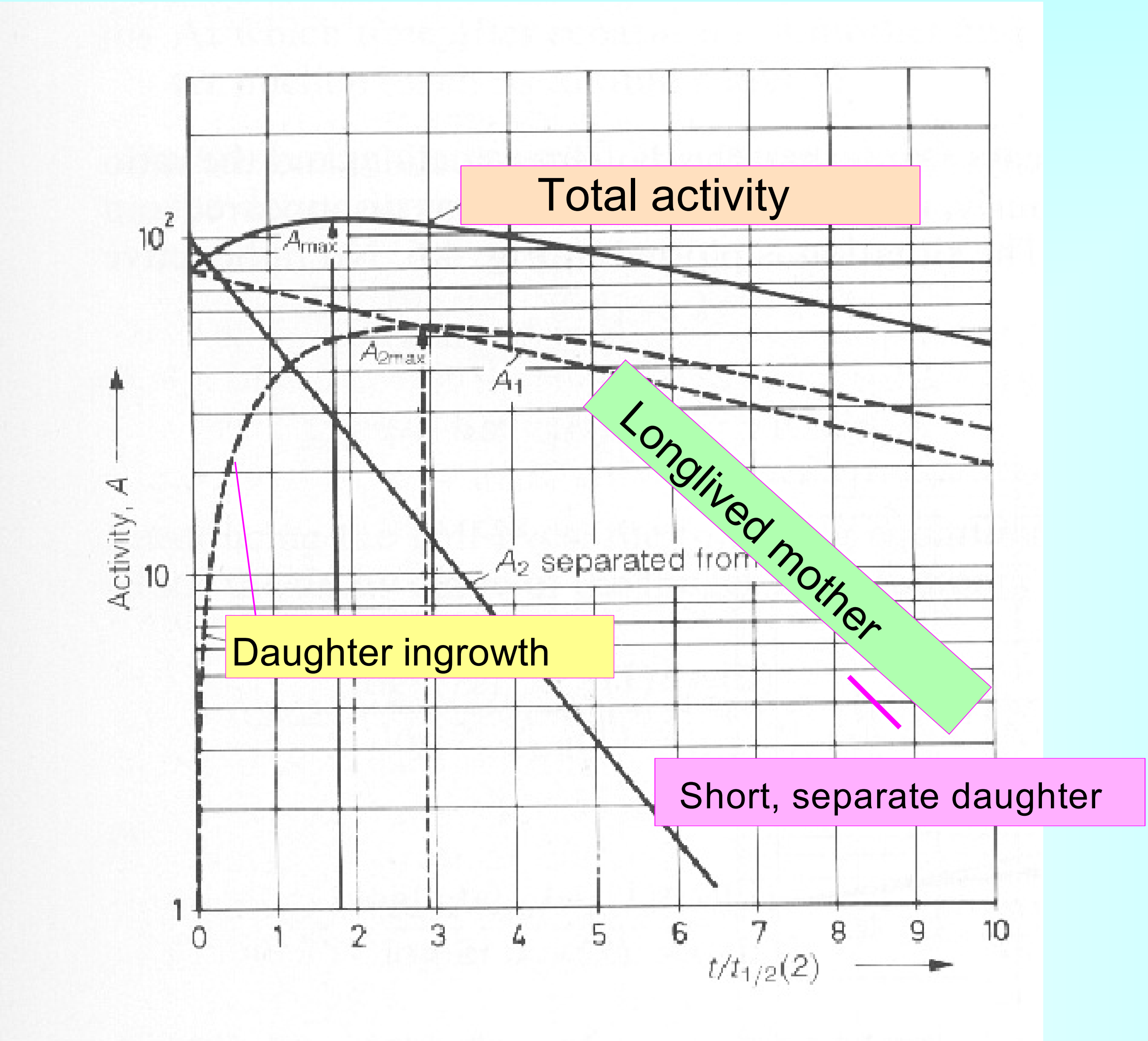


$T_{½}(1) >> T_{½}(2)$



- Equilibrium after approx.10 $T_{½}$
- The daughter nuclide may be chemically isolated, and reappears.

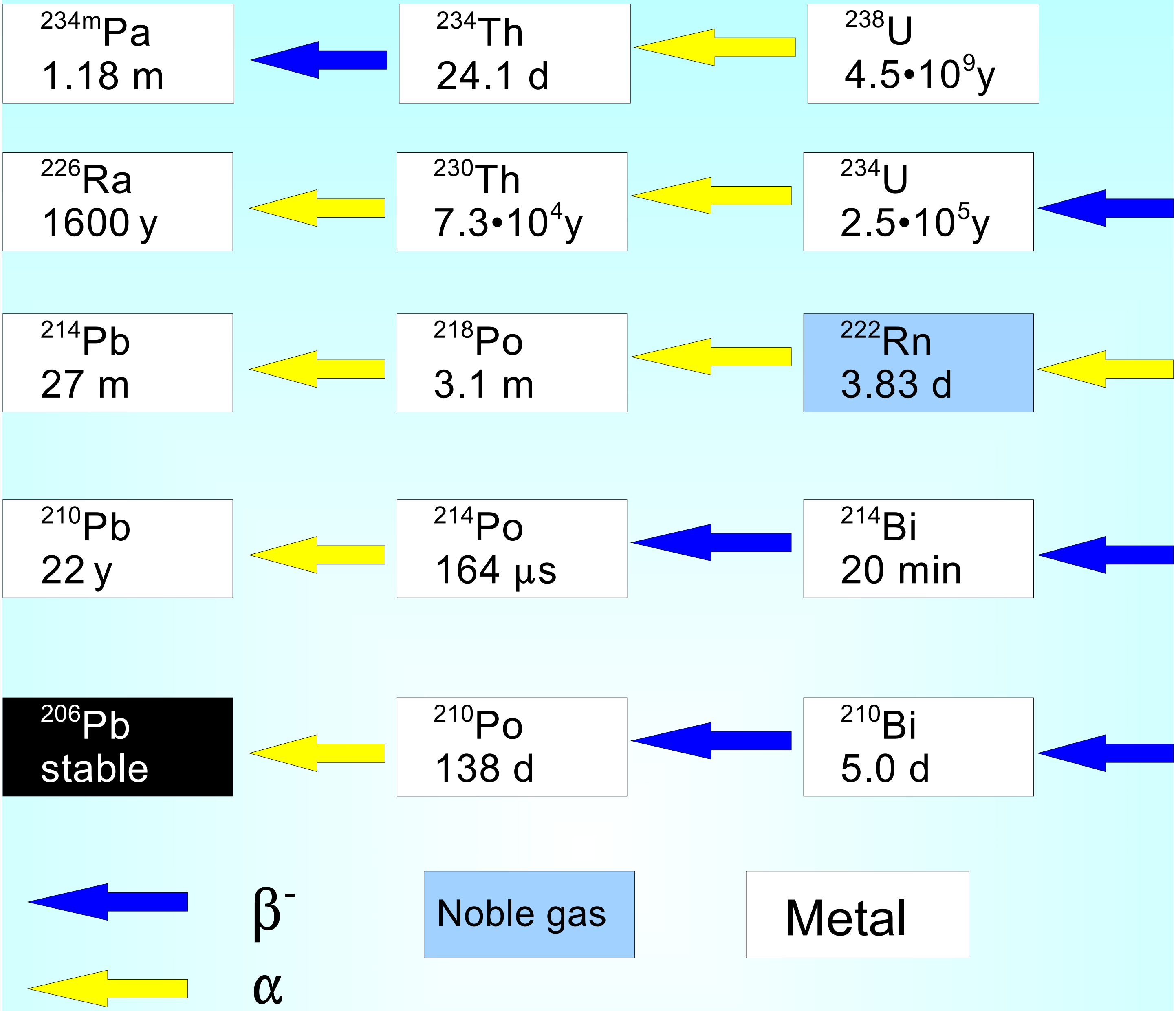
$T_{½}(1)>T_{½}(2)$, transient equilibrium



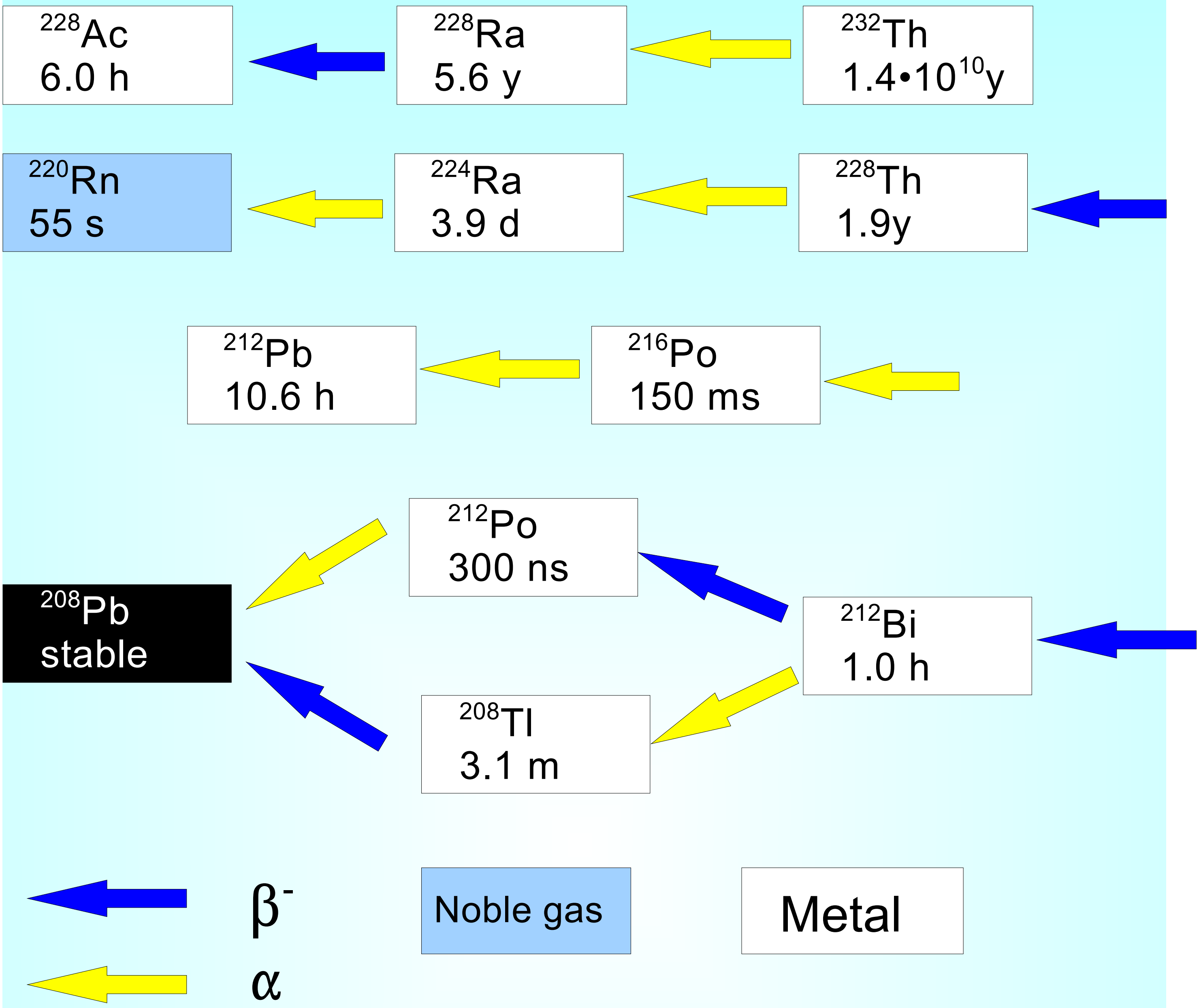
Also applicable as isotope generator.

"Isotope generator”

- An isotope generator is a system where a short-lived daughter nuclide (or a nuclide further sown in the sequence) is allowed to “grow in”, whereafter it is separated from the mother activity utilising differences in chemical properties:
- Some useful examples
  - ^{99}Mo/^{99m}Tc
  - ^{68}Ge/^{68}Ga
  - ^{228}Th/..../^{212}Pb
  - ^{227}Ac/^{227}Th/^{223}Ra
  - ^{238}U/....../^{226}Ra/^{222}Rn
- The latter is a natural isotope generator used by Marie and Pierre Curie to obtain Ra from uranium-containing minerals.


234mPa 1.18 m
234Th 24.1 d
238U 4.5•10^9 y
226Ra 1600 y
230Th 7.3•10^4 y
234U 2.5•10^5 y
214Pb 27 m
218Po 3.1 m
222Rn 3.83 d
210Pb 22 y
214Po 164 µs
214Bi 20 min
206Pb stable
210Po 138 d
210Bi 5.0 d
β-
α
Noble gas
Metal

^{238}U-series



^{232}Th-series

Natural radioactivity

There are two fundamentally different sources of radionuclides in nature:

Primordial.
“Survivors” from the element synthesis ~4-5 billion years ago

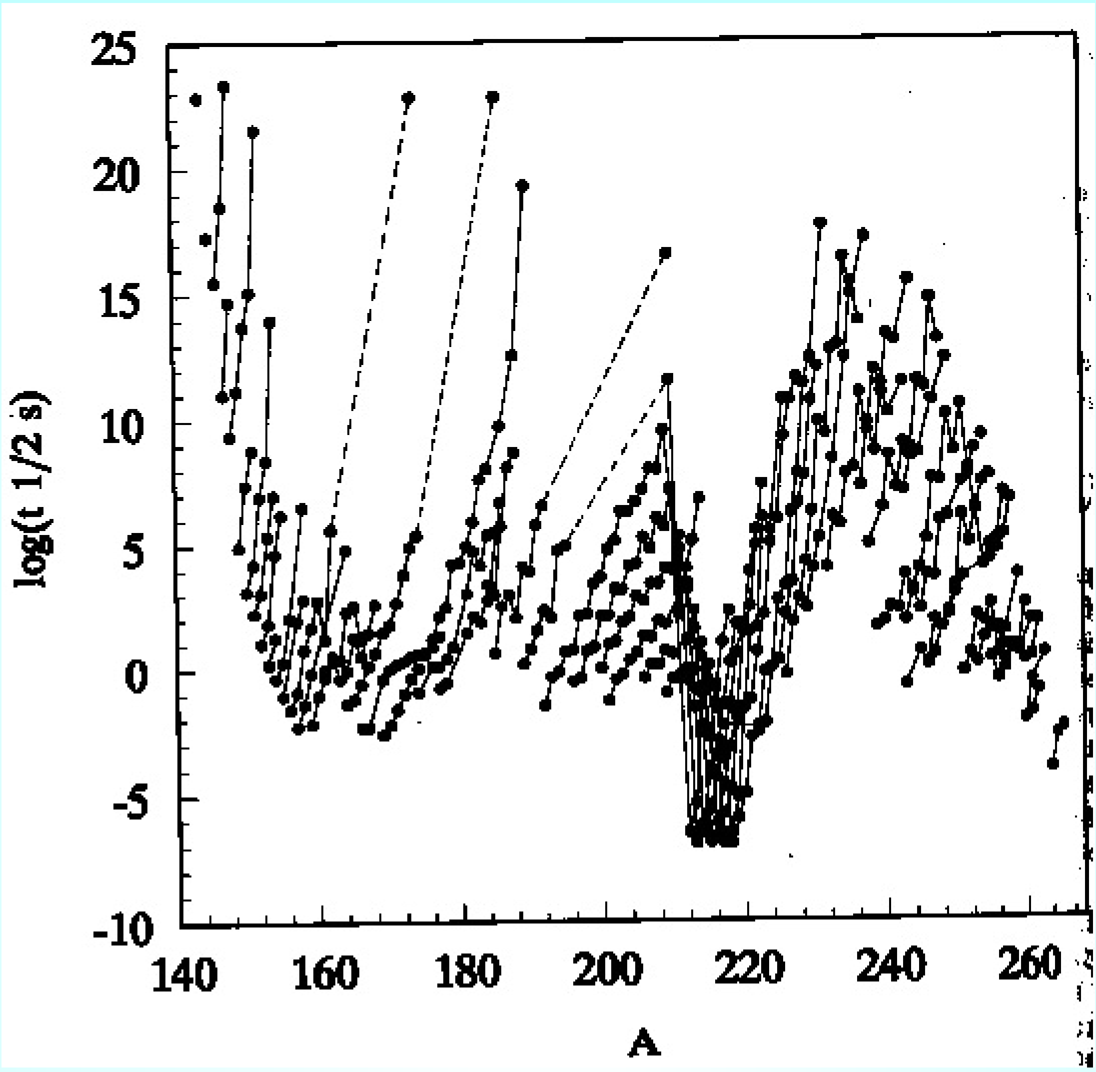
The premordial nuclides may also have daughters which are present in nature due to radioactive equilibria

Cosmogenic.
Radionuclides formed continuously due to reactions induced by cosmic radiation in the stratosphere.

Primordial nuclides <Pb

Nucl.	mode	abundance	halflife
^{40}K	$\beta^-\beta^+\epsilon$	0.0117%	$1.3 \cdot 10^{9}$ y
^{87}Rb	β^-	27.8 %	$4.9 \cdot 10^{10}$ y
^{176}Lu	β^-	2.62 %	$3.8 \cdot 10^{10}$ y
^{187}Re	β^-	62.6 %	$4.2 \cdot 10^{10}$ y
^{147}Sm	α	15.0 %	$1.1 \cdot 10^{11}$ y
^{138}La	β^-	0.09 %	$1.1 \cdot 10^{11}$ y
^{190}Pt	α	0.01 %	$6.5 \cdot 10^{11}$ y
^{123}Te	ϵ	0.90 %	$1.3 \cdot 10^{13}$ y
^{115}In	β^-	95.72 %	$4.4 \cdot 10^{14}$ y
^{144}Nd	α	23.80 %	$2 \cdot 10^{15}$ y
^{186}Os	α	1.6 %	$2 \cdot 10^{15}$ y
^{174}Hf	α	0.16 %	$2 \cdot 10^{15}$ y
^{148}Sm	α	11.3 %	$7 \cdot 10^{15}$ y
^{50}V	$\beta^-\epsilon$	0.25 %	$1.4 \cdot 10^{17}$ y
^{209}Bi	α	100 %	$1.6 \cdot 10^{19}$ y
^{180m}Ta	?	0.012 %	$>10^{15}$ y

U and Th



The primordial $^{235,238}U$ and ^{232}Th arise due to a shell effect, giving low α energies and long half-lives in that region of the nucleic chart.

Long-lived cosmogenic radionuclides

Nucl.	Halflife		at m^{-2} s^{-1}
^{14}C	5715 yr	β^-	~20 000
^{3}H	12.3 yr	β^-	~ 2 500
^{10}Be	$1.5 \cdot 10^6$ yr	β^-	300
^{36}Cl	300 000 yr	β^-	60
^{39}Ar	268 yr	β^-	55
^{35}S	87 d	β^-	15
^{26}Al	710 000 yr	β^+	1.2
^{32}Si	160 yr	β^-	1.6
^{22}Na	2.6 yr	β^+	
^{55}Mn	$3.7 \cdot 10^6$ yr	ϵ	
^{81}Kr	220 000 yr	ϵ	

Short-lived cosmogenic radionuclides

Nucl.	Halfllife	Decay mode
^{7}Be	57 d	ϵ
^{24}Na	15 h	β^-
^{28}Mg	21 h	β^-
^{32}P	14 d	β^-
^{33}P	25 d	β^-
^{39}Cl	56 min	β^-

Some of these radionuclides are produced with thermal neutrons, others require more energetic particles.