Description of radiation fields

Lesson FYSKJM4710 Eirik Malinen

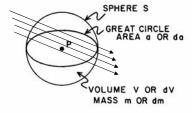
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Ionizing radiation field

- Field of ionizing particles, where the particles may have a directional- and energy distribution
- Radiation field striking a small sphere:



 Number of particles N striking the sphere is proportional to dose





Fluence

• Fluens Φ : number of particles dN striking the sphere per unit area da:

$$\Phi = \frac{dN}{da}$$
 (da is the great circle area)

- The small sphere defines a point in space
- Fluence is as an expectation value; *N* is in reality a stochastic quantity
- For a radiation field through a medium, the fluence varies due to absorption, scattering and creation of new particles $\rightarrow \Phi = \Phi(r)$

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Fluence 2

• The fluence may vary in time – the fluence rate is defined as:

$$\Phi_{t} = \frac{d\Phi}{dt} = \frac{d^{2}N}{dtda}$$

• The radiation field may have a an energy and directional dependence. The differential fluence is:

$$\Phi_{\rm T} = \frac{{\rm d}\Phi}{{\rm d}{\rm T}}$$
, $\Phi_{\Omega} = \frac{{\rm d}\Phi}{{\rm d}\Omega}$ (${\rm d}\Omega = \sin\theta {\rm d}\theta {\rm d}\phi$)

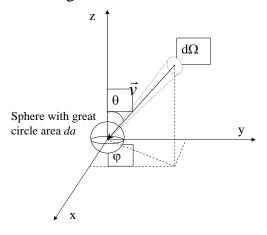
• Φ_T is the number of particles per energy and area in the energy interval [T, T+dT] striking the sphere





Solid angle

• The solid angle Ω is defined as:



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Energy fluence

- How much energy 'strikes' the sphere?
- The energy fluence is defined as:

$$\Psi = \int_{0}^{T_{max}} T\Phi_{T} dT$$

• For a monoenergetic field:

$$\Psi = T\Phi = T\frac{dN}{da}$$

• Differentiated:

$$\Psi_{\rm T} = \frac{d\Psi}{dT} = T\Phi_{\rm T} \ , \ \Psi_{\Omega} = \frac{d\Psi}{d\Omega} = \int\limits_{0}^{T_{max}} T \frac{d\Phi_{\rm T}}{d\Omega} dT$$

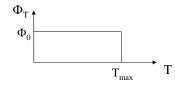




Fluence vs energy fluence

Differential fluence with respect to energy is constant up to T_{max} :

$$\begin{split} \Phi_{T} &= \Phi_{0} \implies \Phi = \int\limits_{0}^{T_{max}} \Phi_{T} dT \\ \Rightarrow &\underline{\Phi = T_{max} \Phi_{0}} \end{split}$$

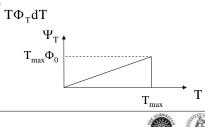


Differential energy fluence is:

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$$\Psi_{T} = T\Phi_{T} \implies \Psi = \int_{0}^{T_{max}} \Psi_{T} dT = \int_{0}^{T_{max}} T\Phi_{T} dT$$

$$\Rightarrow \underbrace{\Psi = \frac{1}{2} T_{max}^{2} \Phi_{0}}_{T_{max}} \Phi_{0}$$



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Average particle energy in field

- Differential fluence and energy fluence are distribution functions
- Average energy defined as:

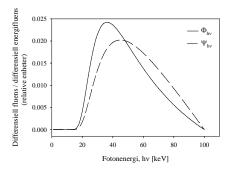
$$\begin{split} &<\boldsymbol{T}>_{\boldsymbol{\Phi}} = \frac{\int\limits_{0}^{T_{max}}\boldsymbol{T}\boldsymbol{\Phi}_{T}d\boldsymbol{T}}{\int\limits_{0}^{T_{max}}\boldsymbol{\Phi}_{T}d\boldsymbol{T}} = \frac{\boldsymbol{\Psi}}{\boldsymbol{\Phi}} \\ &<\boldsymbol{T}>_{\boldsymbol{\Psi}} = \frac{\int\limits_{0}^{T_{max}}\boldsymbol{T}\boldsymbol{\Psi}_{T}d\boldsymbol{T}}{\int\limits_{0}^{T_{max}}\boldsymbol{\Psi}_{T}d\boldsymbol{T}} = \frac{\int\limits_{0}^{T_{max}}\boldsymbol{T}^{2}\boldsymbol{\Phi}_{T}d\boldsymbol{T}}{\int\limits_{0}^{T_{max}}\boldsymbol{\Psi}_{T}d\boldsymbol{T}} \neq <\boldsymbol{T}>_{\boldsymbol{\Phi}} \end{split}$$





Fluence vs energy fluence 2

- X-ray spectrum is either differential fluence or differential energy fluence
- Problem: is often given as "intensity"



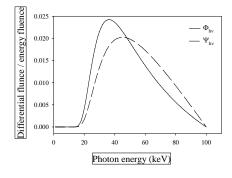
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Fluence vs energy fluence 3

- X-ray spectrum is either differential fluence or differential energy fluence
- Problem: is often given as "intensity"







Fluence vs energy fluence 4

• In our example:

$$<$$
 T $>_{\Phi} \approx 48 \text{ keV}$
 $<$ T $>_{\Psi} \approx 54 \text{ keV}$

• Always ask what the unit of the ordinate is in X-ray (or e.g. e⁻) spectra!



