UiO Department of Physics
University of Oslo

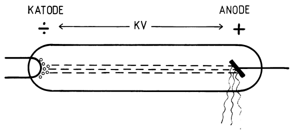
Oslo
University Hospital

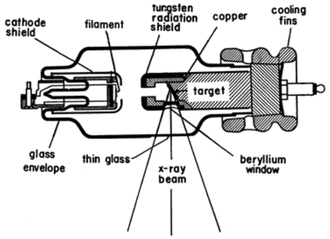
Accelerators and radiation spectra

Eirik Malinen

X-ray tube

- Electrons are released from the cathode (negative electrode) by thermionic emission – accelerated in an evacuated tube – hit the anode (target, positive electrode) – brehmsstrahlung is generated:





X-ray tube and radiation

- Target and filament: often tungsten
- X-rays: photons generated by accelerated electrons
- Maximum photon energy: $h\nu_{max} = T_0 = eV$
- Power P=V x I; unit kW
- Radiation yield:

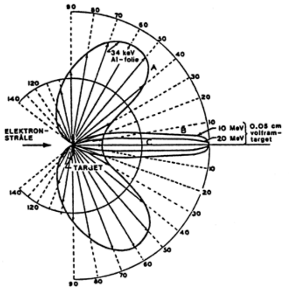
$$\frac{\text{Energy emitted as X-ray radiation}}{\text{Total electron kinetic energy}} \sim 0.1\% - 2\%$$

for 10 keV – 200 keV electrons (increasing with kinetic energy) in tungsten

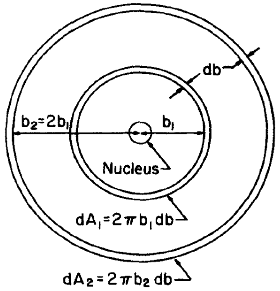
---

X-rays – directional dependence

- The direction of brehmsstrahlung photons depend strongly on the electron energy



Brehmsstrahlung photons

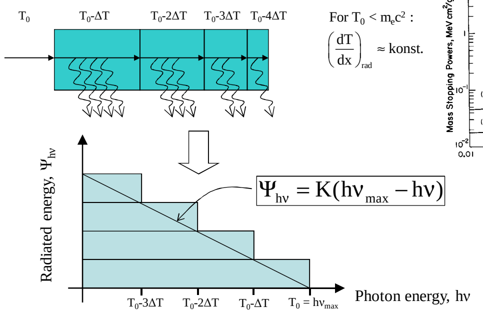


- Assume cross section prop. to $dA=2\pi bdb$
- At $b=b_1 \rightarrow dA=2\pi b_1 db$
- At $b=2b_1 \rightarrow dA=4\pi b_1 db$
- Twice as many γ's at $2b_1$
- BUT, photon energy prop. to $1/b \rightarrow h\nu_2=2h\nu_1$

  $\rightarrow \Psi_{h\nu}$ = constant

Kramer's rule 1

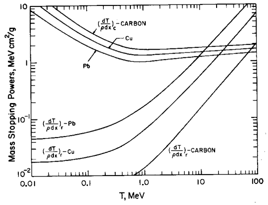
- Unfiltered (energy fluence-) photon spectrum is obtained from Kramer's rule:



For $T_0 < m_e c^2$:

$$\left(\frac{dT}{dx}\right)_{rad} \approx \text{konst.}$$

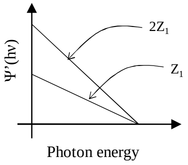
$$\boxed{\Psi_{h\nu} = K(h\nu_{max} - h\nu)}$$

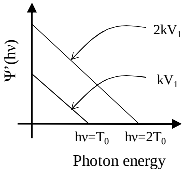


Kramer's rule 2

- Spectral distribution of bremsstrahlung: dependence on atomic number (left) and voltage (right)

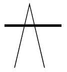
$$\Psi_{h\nu} = K(h\nu_{max} - h\nu)$$





Filtered X-rays

- Filtering modifies spectrum, both in intensity and characterization



- Each photon is attenuated with a probability $e^{-\mu x}$
- Low energetic ("soft") X-ray radiation most attenuated
- X-ray spectrum becomes more homogenous the harder the filtering

Spectrometry

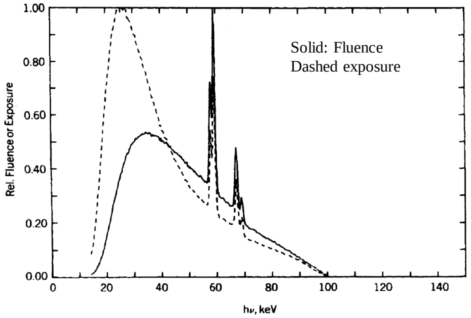
- Measurement of radiation spectra
- Pulse- height analysis by:
- \- Scintillation counter, (NaI(Tl)):

  Light is emitted by irradiation – intensity ("height") of light pulse proportional with quantum energy – number of pulses at each pulse height gives intensity of the given energy interval
- \- Semiconductor (Ge(Li)):

  Short current trough p-n-junction at irradiation – height of pulse proportional with quantum energy. Must be cooled with liquid N_2

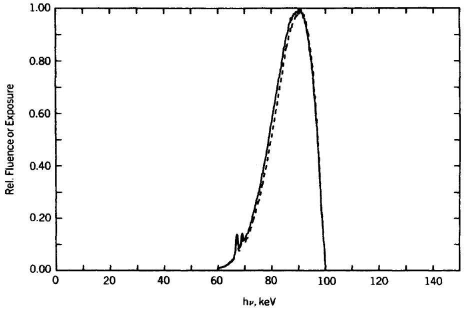
X-ray spectrum 1

- 100 kV, 2.0 mm Al filter
- Average energy ~ 45 keV

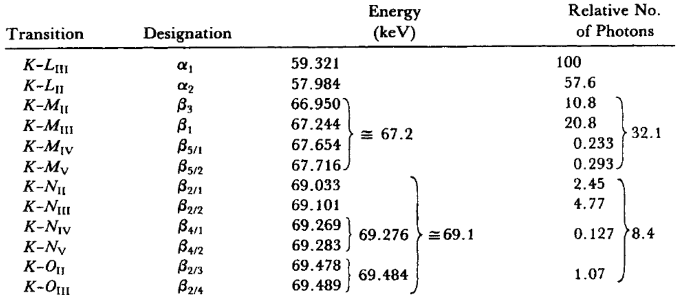


X-ray spectrum 2

- 100 kV, 4.0 mm Al + 0.5 mm Cu + 2mm Sn filter
- Average energy ~ 90 keV



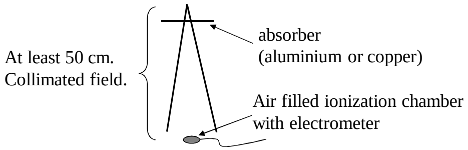
Fluorescence, Tungsten



Transition	Designation	Energy (keV)			Relative No. of Photons	
$K\text{-}L_{III}$	α_1	59.321			100	
$K\text{-}L_{II}$	α_2	57.984			57.6	
$K\text{-}M_{II}$	β_3	66.950	≅ 67.2		10.8	32.1
$K\text{-}M_{III}$	β_1	67.244			20.8	
$K\text{-}M_{IV}$	$\beta_{5/1}$	67.654			0.233	
$K\text{-}M_{V}$	$\beta_{5/2}$	67.716			0.293	
$K\text{-}N_{II}$	$\beta_{2/1}$	69.033		≅69.1	2.45	8.4
$K\text{-}N_{III}$	$\beta_{2/2}$	69.101			4.77	
$K\text{-}N_{IV}$	$\beta_{4/1}$	69.269	69.276		0.127	
$K\text{-}N_{V}$	$\beta_{4/2}$	69.283				
$K\text{-}O_{II}$	$\beta_{2/3}$	69.478	69.484		1.07	
$K\text{-}O_{III}$	$\beta_{2/4}$	69.489				

X-ray quality

- X-ray spectra gives most detailed characterization
- But: spectrometry is expensive and time consuming
- Half value layer (HVL) is recommended :



At least 50 cm.
Collimated field.

absorber
(aluminium or copper)

Air filled ionization chamber
with electrometer

- HVL: thickness of absorber which reduces the exposure (~absorbed dose to air) with 50 %

Half value layer

- Exponential attenuation of monoenergetic photons:

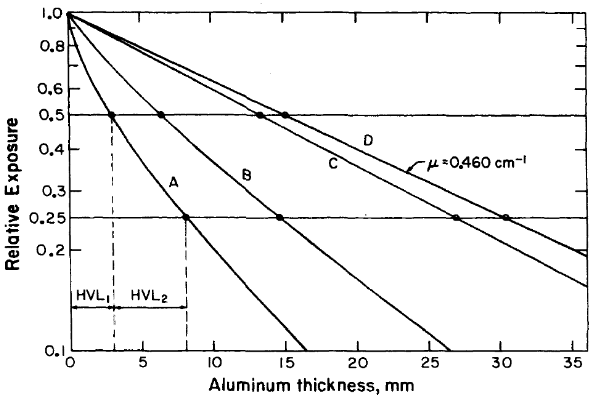
$$N = N_0 e^{-\mu x}$$

$$N = \frac{N_0}{2} = N_0 e^{-\mu \mathrm{HVL}}$$

$$\Rightarrow \mathrm{HVL} = \frac{\ln 2}{\mu}$$

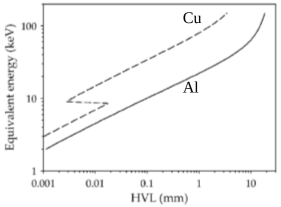
- X-ray quality often given as HVL in Cu or Al

Attenuation of X-ray spectra



Equivalent photon energy

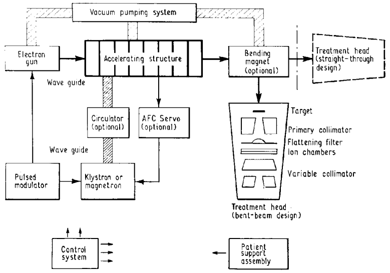
- : “the quantum energy of a monoenergetic beam having the same HVL as the beam being specified”



Beam (kV)	Filter	HVL (mm Al)	HVL (mm Cu)	Equivalent energy (keV)
60	1.5 mm Al	1.61	0.051	26.6 ± 0.2
100	1.5 mm Al	2.95	0.10	33.6 ± 0.3
220 (b)	1.5 mm Al	6.55	0.35	50.8 ± 2.3
160	0.5 mm Cu	13.1	1.12	83.8 ± 1.2
220 (a)	1.5 mm Al, 0.5 mm Cu	15.0	1.65	99.3 ± 0.1

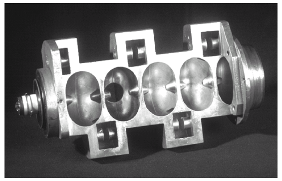
Linear accelerator 1

- Acceleration of charged particles in strong microwave field:



Linear accelerator 2

- Effective accelerating potential ~ MV
- Electrons have almost light speed after acceleration in one “cavity”:

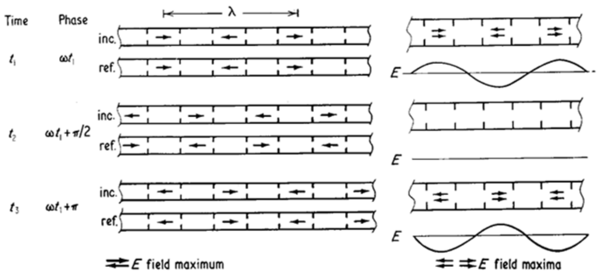


Acceleration tube
Effective potential: 6 MV

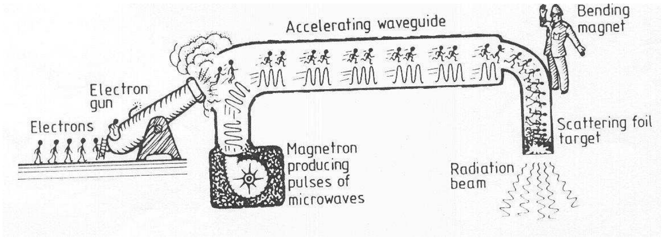
- Electrons can hit a target (ex. Tungsten) – high energy bremsstrahlung generated

Linear accelerator 3

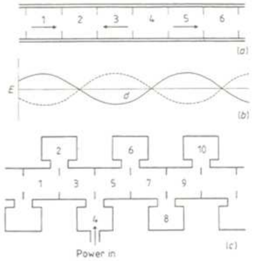
- Electrons “surf” on the electric field waves
- Wave amplitude decides the effective potential



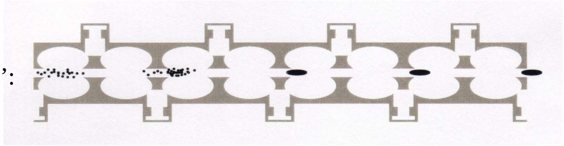
Linear accelerator 4



Linear accelerator 5

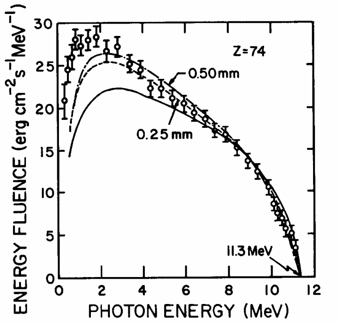


Side-coupled cavities:



’bunching’:

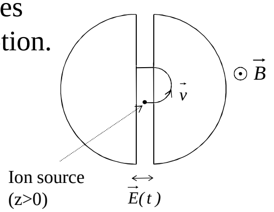
Linear accelerator – photon spectrum



11.3 MeV electrons on 1.5 mm tungsten target.
Lines: model using different target thickness

Other principles: cyclotron

- Acceleration of charged particles which are kept in a circular motion.
- Two-part “D” structure
- Time dependent voltage between the two “D”s
- Two accelerations per cycle
  - period synchronized with time dependent voltage
- Not a good principle for acceleration of electrons and other light particles



Cyclotron 2

- Particle is kept in circular trajectory with B-field, and accelerated by time depending potential (kV/MHz)
- Potential V gives: $T = zV = \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{2zZ}{m}$
- Combined with the Lorentz force: $(\vec{F} = z\vec{v} \times \vec{B})$

$$|F| = zvB = ma = \frac{mv^2}{r} \Rightarrow v^2 = \left(\frac{zBr}{m}\right)^2$$

$$\frac{2zV}{m} = \left(\frac{zBr}{m}\right)^2 \Rightarrow r^2 = \frac{2mV}{zB^2}$$

  - Stronger magnetic field: implicitly higher acceleration

Cyclotron 3

- The period Γ of a charged particle in circular motion is:

$$\Gamma = \frac{2\pi r}{v} \quad \left(v = \frac{zBr}{m} \right)$$

$$\Rightarrow \Gamma = \underline{\underline{\frac{2\pi m}{zB}}}$$

- m is relativistic mass:

$$m = \gamma m_0 \ , \ \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad , \quad \beta = v/c$$

- When the speed increases, m increases and Γ thus increases

Cyclotron 4

- Energy considerations:

$$T_a = T_b + zV \quad \left(V = \int \vec{E} \cdot \vec{dl} \ , \ T = (\gamma\text{-}1)m_0c^2 \right)$$

$$\Rightarrow (\gamma_a\text{-}1)m_0c^2 = (\gamma_b\text{-}1)m_0c^2 + zV$$

$$\Rightarrow \gamma_a = \gamma_b + \frac{zV}{m_0c^2}$$

$$\Rightarrow \Gamma = \frac{2\pi m}{zB} = \frac{2\pi\gamma_a m_0}{zB} = \frac{2\pi m_0}{zB}\left(\gamma_b + \frac{zV}{m_0c^2} \right)$$

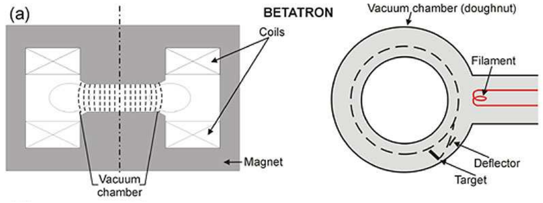
Cyclotron 5

- Increase in period: ~zV/m_0c^2
- Example: zV = 100 keV
- Proton: zV/m_pc^2 ~ 0.01 %
- Electron: zV/m_ec^2 ~ 20 % → close to 50 % rise in one round → Time dependent E-field will have the wrong direction relative to velocity of electron
- The E-field frequency can be synchronized with the rise in period → synchrocyclotron / synchrotron

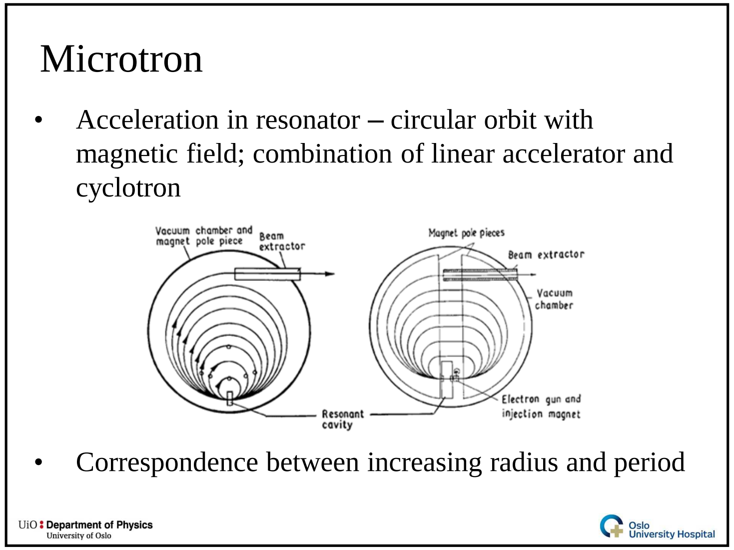
---

Betatron

- Charged particle (electron) accelerated in doughnut shaped unit:



- Time dependent magnetic (and electric) field to accelerate electrons in circular trajectory



Microtron

- Acceleration in resonator – circular orbit with magnetic field; combination of linear accelerator and cyclotron

- Correspondence between increasing radius and period

UiO Department of Physics
University of Oslo

Oslo University Hospital