UiO : Department of Physics
University of Oslo
Oslo University Hospital

Cavity Theory and Interface Effects

Eirik Malinen

Definition of absorbed dose

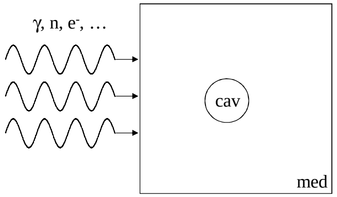
$$D = \frac{d\varepsilon}{dm}$$

D is the expectation value of the energy imparted to matter per unit mass at a point

- Is this an unambiguous definition?
- Two different media in the same radiation field will not receive the same dose

D is the expectation value of the energy imparted to matter per unit mass at a point *in a given medium*

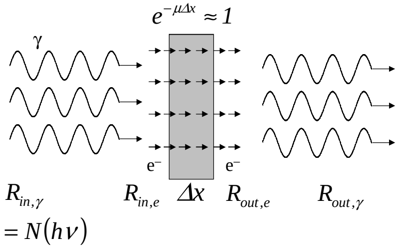
Cavity



- Cavity theory, narrow sense: convert “dose to detector” to “dose to medium”
- Cavity theory, broad sense: dose distribution in inhomogeneous media

---

Absorbed dose in γ irradiated thin foil, CPE



$R_{in,\gamma}$: Radiant incoming energy = N(hν) for monoenergetic photons

Energy transferred:

$$\varepsilon_{tr} = R_{in,\gamma} + R_{in,e} - R_{out,\gamma} - R_{out,e} \overset{CPE}{=} R_{in,\gamma} - R_{out,\gamma}$$
$$= N(h\nu)\mu_{tr}\Delta x$$

Absorbed dose in γ irradiated thin foil, CPE

Absorbed dose (no brehmsstrahlung)

$$D = K = \frac{\varepsilon_{tr}}{m} = \frac{N(h\nu)\mu_{tr}\Delta x}{m} = \frac{\Psi A\mu_{tr}\Delta x}{\rho A\Delta x} = \Psi\left(\frac{\mu_{tr}}{\rho}\right)$$

$$\mu_{tr} = \mu\frac{\overline{T}}{h\nu}$$

If brehmsstrahlung:

$$D = K_c = \Psi\left(\frac{\mu_{en}}{\rho}\right)$$

Energy *loss* from electrons

- Stopping power:

$$S = \frac{dT}{dx} = S_{col} + S_{rad} = \rho n\int_{E_{min}}^{E_{max}} E\left(\frac{d\sigma_{tot}}{dE}\right)dE$$

n: number of electrons per gram

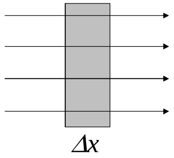
- Collision stopping power: S_{col}

$$S_{col} = \rho n\int_{E_{min}}^{E_{max}} E\left(\frac{d\sigma_{col}}{dE}\right)dE$$

- Restricted stopping power: L_Δ

$$L_\Delta = \rho n\int_{E_{min}}^{\Delta} E\left(\frac{d\sigma_{col}}{dE}\right)dE$$

Absorbed dose in thin foil, electrons



$$\Delta x << \Re$$

Range of charged particle

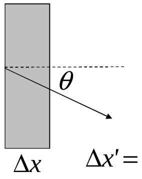
Energy loss <*ΔT*> → energy imparted *ε* ?

→ Brehmsstrahlung, δ rays, path lengthening

Brehmsstrahlung: S_{rad}

Path lengthening due to multiple scattering

$$\overline{cos(\theta)} = cos\left(\sqrt{\rho\Delta x\frac{d\overline{\theta^2}}{dx}}\right)$$

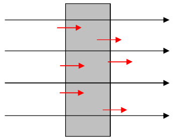


$$\Delta x' = \frac{\Delta x}{cos\,\theta}$$

Scattering power: $\frac{d\overline{\theta^2}}{dx}$

δ rays

- Energetic, secondary electrons
- Significant range compared to foil thickness
- Results from high energy transfers (included in S_{col})



Maximum energy transfer:

$$E_{max} = 2m_e c^2 \frac{\beta}{1-\beta^2}$$

Heavy ions

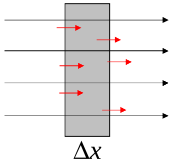
$$E_{max} = T/2$$

Electrons

δ rays

Energy imparted for charged particles:

$$\varepsilon = \underset{\substack{\uparrow\\ \text{primary}}}{R_{in,p}} + R_{in,\delta} - R_{out,p} - R_{out,\delta}$$



δ particle equilibrium

$$R_{in,\delta} = R_{out,\delta} \quad \Rightarrow \quad \varepsilon = R_{in,p} - R_{out,p}$$

δPE requirements: homogeneous medium and $\Re_\delta << \Re_p$

δPE always present under CPE

δ rays

- Since β is low for heavy charged particles in the MeV-region, E_{max} is low
- β=0.1 (e.g. 38 MeV α-particles) gives E_{max}=10 keV
- Range of 10 keV electrons in water: 2.5 μm
- →δ-electrons deposit their energy locally, and δ-equilibrium may often be present
- Range of 1 MeV electrons: 0.5 cm
- →δ-equilibrium may not obtained for high energy electron beam

---

Absorbed dose

$$D = \frac{\varepsilon}{m}$$

- Under δPE (foil sandwiched, short $\Re_\delta$), no path lengthening, no brehmstrahlung:

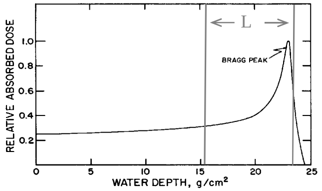
$$\varepsilon = R_{in,p} - R_{out,p} = \Delta R_p = NS\Delta x$$

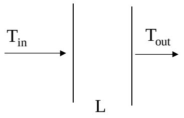
$$\Rightarrow D = \frac{NS\Delta x}{\rho V} = \frac{NS\Delta x}{\rho A \Delta x} = \frac{N}{A}\frac{S}{\rho} \quad , \quad \Phi = \underset{\uparrow}{\frac{N}{A}}$$

$$\boxed{D = \Phi\left(\frac{S}{\rho}\right)}$$

Fluence of primary electrons

Absorbed dose, thick foil, heavy particles





- The average dose may be found by:
  - Calculating the residual range: $\Re_{res} = \Re_{in}$-L
  - Find the energy T_{out} corresponding to $\Re_{res}$
  - Imparted energy is: $\Delta T = T_{in}-T_{out}$
  - Dose: $D=\frac{N\Delta T}{m}=\Phi\frac{\Delta T}{\rho L}$

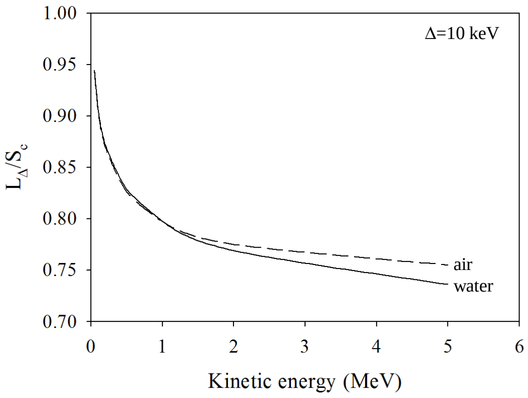
Foil placed in vacuum

δ rays with $T > \varDelta$ lost from foil (δPE absent):

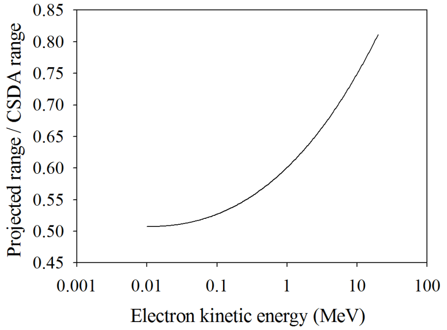
$$\varepsilon = R_{in,p} - R_{out,p} - R_{out,\delta} = N\left[\rho n \int_{E_{min}}^{\varDelta} E\frac{d\sigma}{dE}dE\right]$$

$$\boxed{D = \varPhi\left(\frac{L_{\varDelta}}{\rho}\right)}$$

$\underline{L_{\Delta}}$



Range and projected range, electrons



Spectrum of charged particles, δPE present

$\Phi_T dT$: number of primary electrons cm^{-2} in $[T, T+dT]$

Minimum energy: 0

Maximum energy: $T_{\max}$

$$\Rightarrow dD = \Phi_T dT\left(\frac{S}{\rho}\right) \quad \Rightarrow D = \int_0^{T_{\max}} \Phi_T dT\left(\frac{S}{\rho}\right)$$

$$\boxed{D = \int_0^{T_{\max}} \Phi_T \left(\frac{S}{\rho}\right) dT}$$

Partial δPE

Electron beams: constant fluence of secondary, low energy electrons with T < Δ

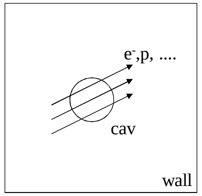
Energetic secondary electrons added to total fluence:

$$D = \int_{\Delta}^{T_{max}} \Phi_T^{p+\delta}\left(\frac{L_\Delta}{\rho}\right) dT$$

$$\Phi_T^{p+\delta} \ ?$$

Particles either assigned to radiation field or to energy imparted

Bragg-Gray cavity theory



$$D_{cav} = \Phi\left(\frac{S}{\rho}\right)_{cav}$$

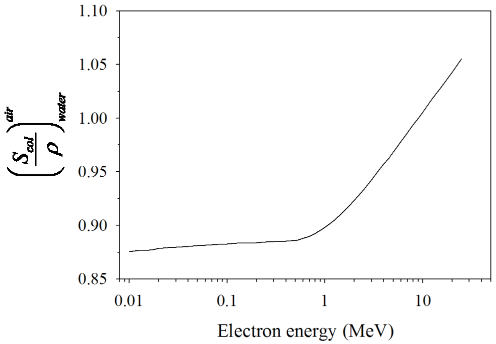
$$D_{wall} = \Phi\left(\frac{S}{\rho}\right)_{wall}$$

$$\Rightarrow \boxed{\frac{D_{cav}}{D_{wall}} = \left(\frac{S}{\rho}\right)_{wall}^{cav}}$$

B-G conditions:

1. Charged particle fluence is not perturbed by cavity
2. Absorbed dose entirely due to charged particles

Bragg-Gray cavity theory



Bragg-Gray-Laurence

Laurence: incorporated slowing down spectrum of charged particles generated in the wall

$$D = \int_0^{T_0} \Phi_T \left(\frac{S}{\rho}\right)_{wall} dT \overset{CPE}{=} n_0 T_0 \longleftarrow$$

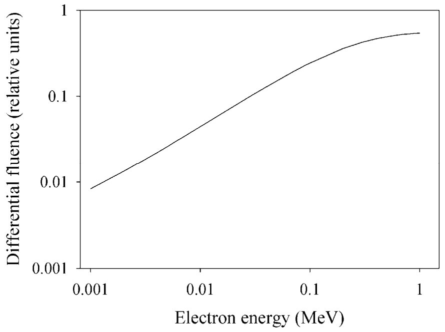
Photons give rise to monoenergetic electrons with kinetic energy T_0

$$\Rightarrow \int_0^{T_0} \Phi_T \left(\frac{S}{\rho}\right)_{wall} dT = n_0 \int_0^{T_0} dT$$

$$\Rightarrow \int_0^{T_0} \left[\Phi_T \left(\frac{S}{\rho}\right)_{wall} - n_0 \right] dT = 0 \quad \Rightarrow \boxed{\Phi_T = \frac{n_0}{\left(\frac{S}{\rho}\right)_{wall}}}$$

Bragg-Gray-Laurence

Slowing down spectrum of primary electrons in water



Bragg-Gray-Laurence

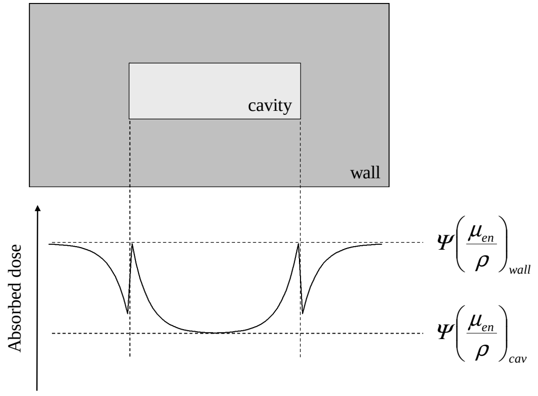
The total fluence:

$$\Phi = \int_0^{T_0} \Phi_T dT = n_0 \int_0^{T_0} \frac{dT}{(S/\rho)} = n_0 \Re_{CSDA}$$

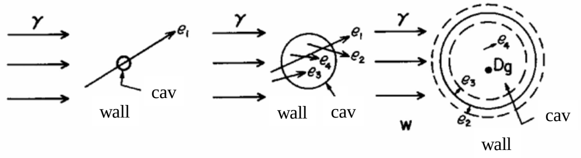
Dose to cavity:

$$\overline{D}_{cav} = \int_0^{T_0} \Phi_T \left(\frac{S}{\rho}\right)_{cav} dT = n_0 \int_0^{T_0} \frac{\left(\frac{S}{\rho}\right)_{cav}}{\left(\frac{S}{\rho}\right)_{wall}} dT = \boxed{n_0 \int_0^{T_0} \left(\frac{S}{\rho}\right)_{wall}^{cav} dT}$$

Burlin cavity theory



Burlin cavity theory



Small: Bragg-Gray Intermediate: Burlin Large: CPE

Burlin cavity theory

Cavity with dimensions << electron range: B-G theory:

$$\frac{D_{cav}}{D_{wall}} \approx \left(\frac{S}{\rho}\right)_{wall}^{cav}$$

Cavity with dimensions >> electron range: CPE-theory:

$$\frac{D_{cav}}{D_{wall}} = \left(\frac{\mu_{en}}{\rho}\right)_{wall}^{cav}$$

Burlin cavity theory

General theory for intermediate sized cavities:

$$\frac{D_{cav}}{D_{wall}} = d\left(\frac{S}{\rho}\right)_{wall}^{cav} + (1-d)\left(\frac{\mu_{en}}{\rho}\right)_{wall}^{cav}$$

d: average attenuation of electrons generated in the wall crossing the cavity

$$d = \frac{\int_0^L e^{-\beta x}dx}{\int_0^L dx} = \frac{1-e^{-\beta L}}{\beta L} \quad \Rightarrow \quad 1-d = \frac{\beta L + e^{-\beta L} - 1}{\beta L}$$

Burlin cavity theory

β: effective electron attenuation coefficient

Empirical expression:

$$e^{-\beta t_{max}} \approx 0.04$$

t_{max}: depth at which 1 % of electrons can travel

$t_{max}/\Re_{CSDA} \approx 0.9$ low Z

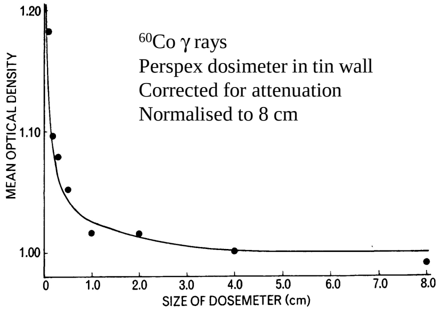
$t_{max}/ \Re_{CSDA} \approx 0.8$ intermediate Z

$t_{max}/ \Re_{CSDA} \approx 0.7$ high Z

Burlin cavity theory - assumptions

- Wall and cavity homogenous
- No significant γ attenuation
- CPE exists
- Spectrum of δ rays equal in wall and cavity
- Electrons generated in wall are exponentially attenuated within cavity
- Electrons generated in cavity increase exponentially

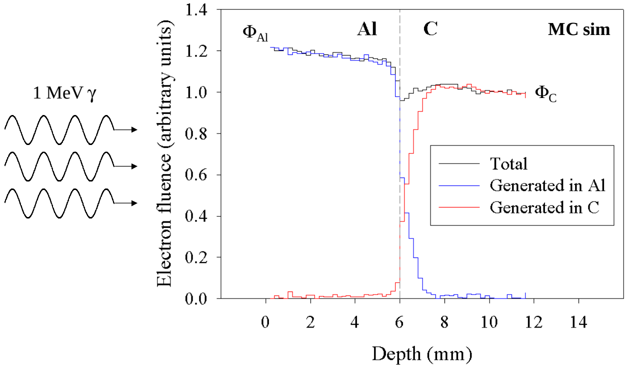
Burlin cavity theory – experiment vs theory



^{60}Co γ rays
Perspex dosimeter in tin wall
Corrected for attenuation
Normalised to 8 cm

Interface dosimetry

Interface dosimetry



Interface dosimetry

Total equilibrium fluence, secondary electrons, CPE:

$$\Phi = n_0 \Re_{CSDA}$$

n_0: number of electrons generated per gram

→

$$D = n_0 \overline{T} \overset{CPE}{=} \Psi \frac{\mu_{en}}{\rho} = Nh\nu \frac{\mu_{en}}{\rho} \approx n_0 \frac{\mu_{en}}{\mu} h\nu$$

$$\Rightarrow n_0 = N \frac{\mu}{\rho}$$

Interface dosimetry

Therefore, fluence ratio, medium 1 and 2 becomes:

$$\frac{\Phi_1}{\Phi_2} = \left(\frac{\mu}{\rho}\right)_2^1 \left(\Re_{CSDA}\right)_2^1$$

1 MeV γ rays:

$\overline{T} = 0.45$ MeV , $\left(\frac{\mu}{\rho}\right)_C = 0.064$ cm^{-1} , $\left(\frac{\mu}{\rho}\right)_{Al} = 0.061$ cm^{-1}

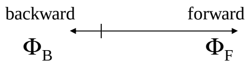
$\Re_C$ = 0.186 g/cm^2 , $\Re_{Al}$ = 0.211 MeV cm^2/g

$\Phi_{Al}/\Phi_C \approx 1.10$, against 1.14 for MC

Interface dosimetry

At the interface, transition from Φ_1 to Φ_2

Simplistic vector representation:

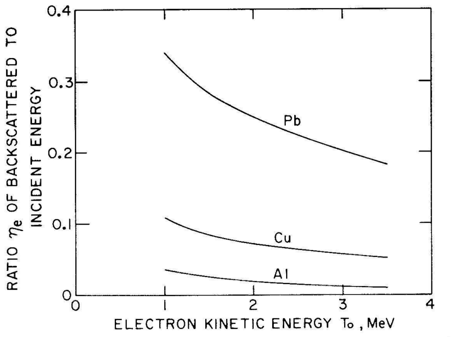


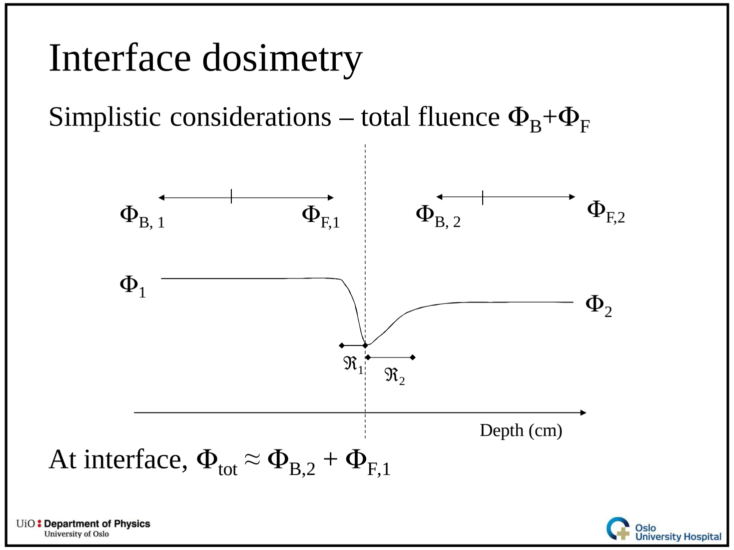
backward forward

Φ_B Φ_F

Forward/backward ratio depend on medium

Backscatter ratio





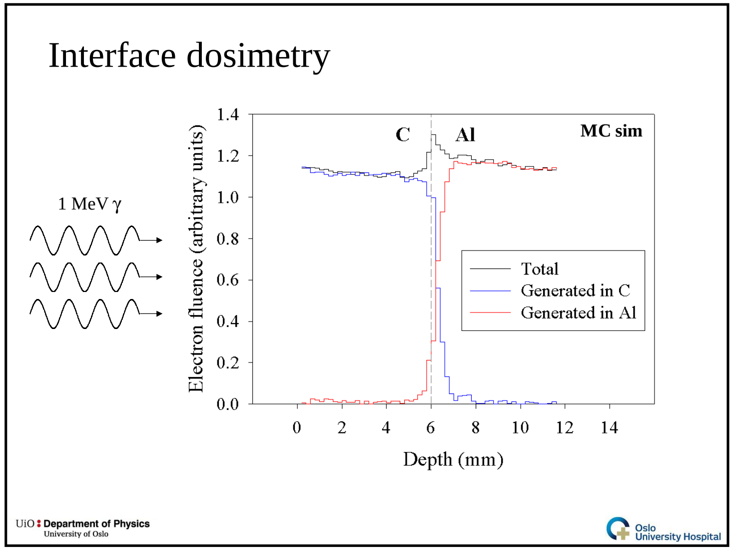
Interface dosimetry

Simplistic considerations – total fluence $\Phi_B+\Phi_F$

At interface, $\Phi_{tot} \approx \Phi_{B,2} + \Phi_{F,1}$

UiO Department of Physics
University of Oslo

Oslo University Hospital



Interface dosimetry

UiO Department of Physics
University of Oslo

Oslo University Hospital