

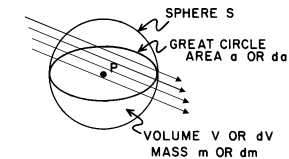
## Description of radiation fields

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## Ionizing radiation field

- Field of ionizing particles, where the particles may have a directional- and energy distribution
- Radiation field striking a small sphere:



- Number of particles  $N$  striking the sphere is proportional to dose

## Fluence

- Fluence  $F$ : number of particles  $dN$  striking the sphere per unit area  $da$ :

$$\Phi = \frac{dN}{da} \quad (da \text{ is the great circle area})$$

- The small sphere defines a point in space
- Fluence is as an expectation value;  $N$  is in reality a stochastic quantity
- For a radiation field through a medium, the fluence varies due to absorption, scattering and creation of new particles  $\rightarrow \Phi = \vec{\Phi}(\mathbf{r})$

## Fluence 2

- The fluence may vary in time – the fluence rate is defined as:

$$\Phi_t = \frac{d\Phi}{dt} = \frac{d^2N}{dt da}$$

- Thus

$$\Phi = \int_0^{t_0} \Phi_t dt$$

- For a time-independent field:

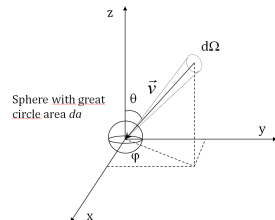
$$\Phi = \Phi_t \Delta t$$

### Fluence 3

- The radiation field may have an energy and directional dependence. The differential fluence is:

$$\Phi_T = \frac{d\Phi}{dT}, \quad \Phi_\Omega = \frac{d\Phi}{d\Omega} \quad (d\Omega = \sin\theta d\theta d\phi)$$

- $\Phi_T$  is the number of particles per energy and area in the energy interval  $[T, T+dT]$  striking the sphere



### Energy fluence

- How much energy 'strikes' the sphere?
- The energy fluence is defined as:

$$\Psi = \int_0^{T_{max}} T \Phi_T dT$$

- For a monoenergetic field:

$$\Psi = T \Phi = T \frac{dN}{da}$$

- Differentiated:

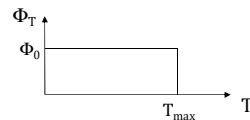
$$\Psi_T = \frac{d\Psi}{dT} = T \Phi_T, \quad \Psi_\Omega = \frac{d\Psi}{d\Omega} = \int_0^{T_{max}} T \frac{d\Phi_T}{d\Omega} dT$$

### Fluence vs energy fluence

- Differential fluence with respect to energy is constant up to  $T_{max}$ :

$$\Phi_T = \Phi_0 \Rightarrow \Phi = \int_0^{T_{max}} \Phi_T dT$$

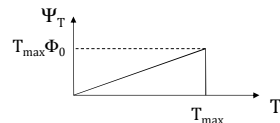
$$\Rightarrow \Phi = T_{max} \Phi_0$$



- Differential energy fluence is:

$$\Psi_T = T \Phi_T \Rightarrow \Psi = \int_0^{T_{max}} \Psi_T dT = \int_0^{T_{max}} T \Phi_T dT$$

$$\Rightarrow \Psi = \frac{1}{2} T_{max}^2 \Phi_0$$



### Average particle energy in field

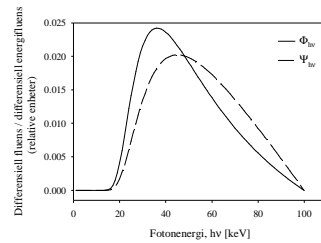
- Differential fluence and energy fluence are distribution functions
- Average energy defined as:

$$\langle T \rangle_\Phi = \frac{\int_0^{T_{max}} T \Phi_T dT}{\int_0^{T_{max}} \Phi_T dT} = \frac{\Psi}{\Phi}$$

$$\langle T \rangle_\Psi = \frac{\int_0^{T_{max}} T \Psi_T dT}{\int_0^{T_{max}} \Psi_T dT} = \frac{\int_0^{T_{max}} T^2 \Phi_T dT}{\int_0^{T_{max}} T \Phi_T dT} \neq \langle T \rangle_\Phi$$

## Fluence vs energy fluence 2

- X-ray spectrum is either differential fluence or differential energy fluence
- Problem: is often given as "intensity"



## Fluence vs energy fluence 3

- In our example:
  - $\langle T \rangle_{\Phi} \approx 48 \text{ keV}$
  - $\langle T \rangle_{\Psi} \approx 54 \text{ keV}$
- Always ask what the unit of the ordinate is in X-ray (or e.g.  $e^-$ ) spectra!