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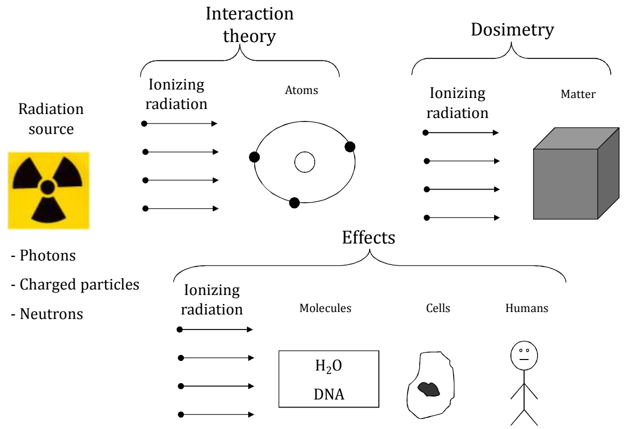
Interaction theory – Photons

Eirik Malinen

Introduction



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INTRODUCTION TO RADIOLOGICAL PHYSICS AND RADIATION DOSIMETRY

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Objectives

- To understand primary and secondary effects of ionizing radiation
- How radiation doses are calculated and measured
- To understand the principles of radiation protection, their origin and applications

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Contents FYSKJM4710

- Interactions between ionizing radiation and matter
- Radioactive and non-radioactive sources
- Calculations and measurement of absorbed doses (dosimetry)
- Radiation chemistry
- Biological effects of ionizing radiation
- Principles of radiation protection

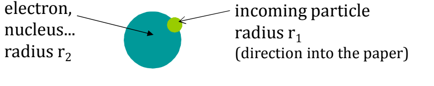
- X-ray contrast: only a matter of differences in density?

Relevant issues

- X-ray and CT investigations
- Radiotherapy
- Positron emission tomography
- Radiation protection
- Radiation Biology

Cross section 1

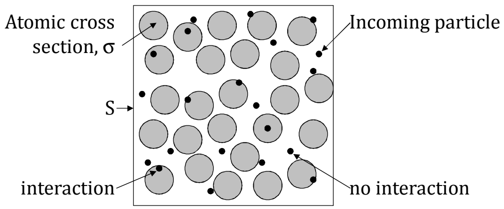
- Cross section s: "target area", effective target covering a certain area
- Proportional to the interaction strength between an incoming particle and the target particle
- Consider two discs, one target and one incoming:



- s is the total area: $\pi(r_1^2 + r_2^2)$

Cross section 2

- N particles move towards an area S with n atoms



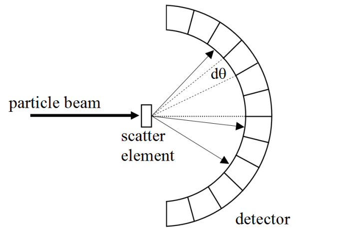
- Probability of interaction: p = nσ/S
- Number of interacting particles: Np= Nnσ/S

Cross section 3

- Separate between *electronic* and *atomic* cross section
- The cross section depends on:
  - Type of target (nucleus, electron, ..)
  - Type of and energy of incoming particle (photon, electron…)
- Cross section calculated with quantum mechanics
  - here visualized in a classical window

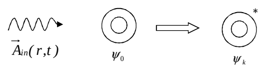
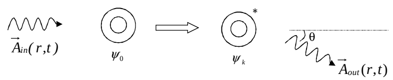
Cross section 4

- *Differential cross section* with respect to scattering angle



$$\frac{d\sigma}{d\Omega} = \frac{number\ of\ particles\ scattered\ into\ d\Omega}{number\ of\ particles\ per\ unit\ area}\frac{1}{d\Omega}$$

Photon interactions

- Photon represented by a plane wave $\vec{A}_{in}(r,t) \sim e^{i(\vec{p}_{in}\cdot\vec{r}-\omega_{in}t)}$ in quantum mechanical calculations
- In principle, two different processes:
  - Absorption 
  - Scattering 
- Scattering: coherent (elastic) og incoherent (inelastic)

Coherent (Rayleigh) scattering

- Scattering without loss of energy: hν=hν'
- Photon is absorbed by atom, thereby emitted at a small deflection angle
- Depends on atomic structure and photon energy
- Atomic cross section:

$$\sigma_R \propto \left(\frac{Z}{h\nu}\right)^2$$

Compton scattering – kinematics

- Conservation of energy and momentum:

$$h\nu = h\nu' + T$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p\cos\varphi \quad , \quad \frac{h\nu'}{c}\sin\theta = p\sin\varphi$$

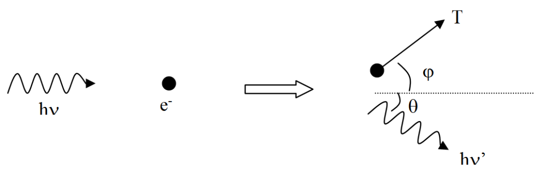
$$(pc)^2 = T^2 + 2Tm_ec^2$$

→

$$h\nu' = \frac{h\nu}{1+\frac{h\nu}{m_ec^2}(1-\cos\theta)} \quad , \quad \cot\varphi = \left(1+\frac{h\nu}{m_ec^2}\right)\tan\left(\frac{\theta}{2}\right)$$

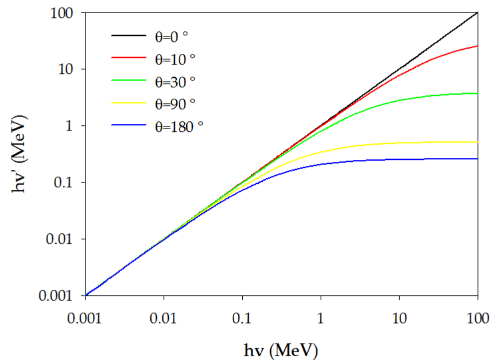
Incoherent (Compton) scattering

- Scattering with loss of energy: hν'<hν
- Photon-electron scattering; electron may be assumed free (i.e. unbound)



- Thomson scattering: low energy limit, hν→0

Compton scattering – kinematics



Compton scattering – example

- An X-ray unit is to be installed, with the beam direction towards the ground. Employees in the floor above the unit are worried. Maximum X-ray energy is 250 keV. What is the maximum energy of the backscattered photons?

$$\theta = 180° \Rightarrow h\nu' = \frac{h\nu}{1+\frac{h\nu}{m_e c^2}(1-\cos\theta)} = \frac{h\nu}{1+\frac{2h\nu}{m_e c^2}}$$

$$h\nu = 250\ keV \Rightarrow h\nu' = \frac{250}{1+\frac{2\times 250}{511}} = \underline{\underline{126\ keV}}$$

Compton scattering – cross section 1

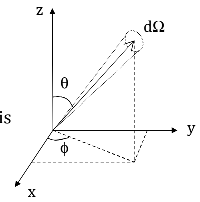
- Klein and Nishina derived the cross section for Compton scattering, assuming free electron
- Differential cross section:

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{r_0^2}{2}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu'}{\nu}+\frac{\nu}{\nu'}-\sin^2\theta\right)$$

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

r_0: classical electron radius

incoming photon along z-axis



Compton scattering – cross section 2

- Cylinder symmetry results in:

$$\left(\frac{d\sigma}{d\theta}\right) = \pi r_0^2\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu'}{\nu}+\frac{\nu}{\nu'}-\sin^2\theta\right)\sin\theta$$

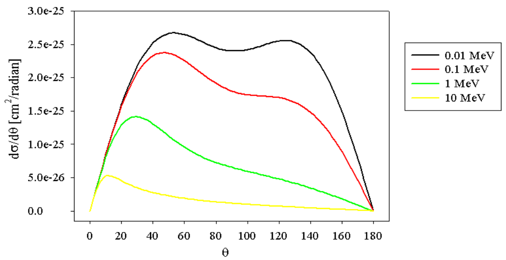
- ~ probability of finding a scattered photon in the interval [θ, θ+dθ]
- Total electronic cross section:

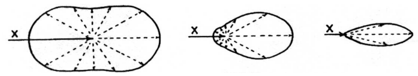
$${}_e\sigma = \int_0^{\pi} \pi r_0^2\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu'}{\nu}+\frac{\nu}{\nu'}-\sin^2\theta\right)\sin\theta\, d\theta$$

- Atomic cross section: ${}_a\sigma = Z_e\sigma$

Compton scattering – cross section 3

- Scattered photons are more forwardly directed with increasing photon energy:





10 keV 200 keV 2 MeV

Compton scattering – cross section 3

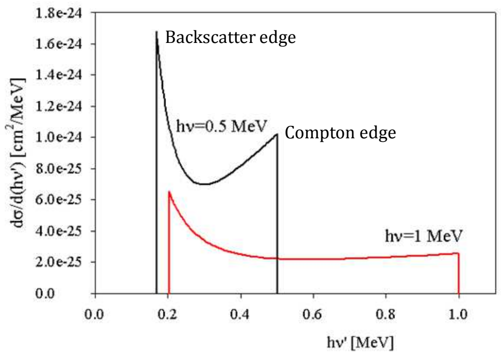
- Cross section may be modified with respect to energy:

$$\frac{d\sigma}{d(h\nu')} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d(h\nu')} = \frac{d\sigma}{d\Omega}2\pi\sin\theta\frac{d\theta}{d(h\nu')}$$

$$h\nu' = \frac{h\nu}{1+\frac{h\nu}{m_ec^2}(1-\cos\theta)}$$

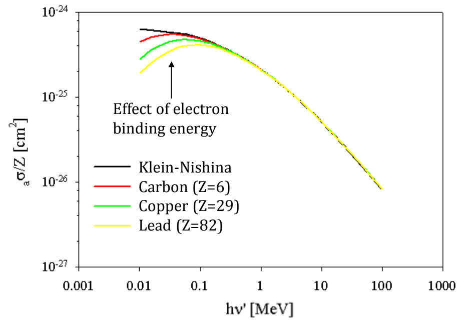
$$\Rightarrow \frac{d\sigma}{d(h\nu')} = \frac{\pi r_0^2 m_e c^2}{(h\nu)^2}\left[\frac{h\nu'}{h\nu}+\frac{h\nu}{h\nu'}-1+\left(1-\left(\frac{h\nu}{h\nu'}-1\right)\frac{m_ec^2}{h\nu}\right)^2\right]$$

Compton scattering – cross section 4



Compton scattering – cross section 5

- Correct atomic cross section:



Compton scattering – transferred energy 1

- The energy transferred to an electron in a Compton process:

$$T = h\nu - h\nu'$$

- The cross section for energy transfer:

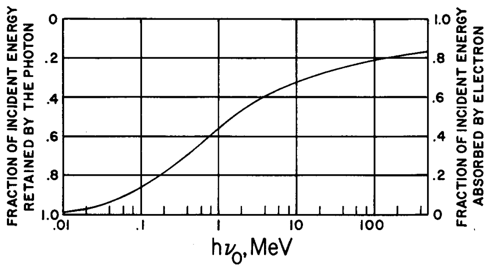
$$\frac{d\sigma_{tr}}{d\Omega} = \frac{d\sigma}{d\Omega}\frac{T}{h\nu} = \frac{d\sigma}{d\Omega}\frac{h\nu - h\nu'}{h\nu}$$

- Mean energy transferred:

$$\bar{T} = \frac{\int T\frac{d\sigma}{d\Omega}d\Omega}{\int \frac{d\sigma}{d\Omega}d\Omega} = \frac{\int \frac{h\nu-h\nu'}{h\nu}\frac{d\sigma}{d\Omega}d\Omega}{\sigma} = \frac{\sigma_{tr}}{\sigma}\times h\nu$$

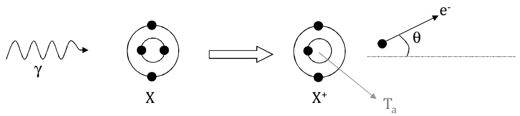
Compton scattering – transferred energy 2

- The fraction of incident energy transferred:

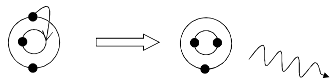


Photoelectric effect 1

- Photon is absorbed by atom/molecule; the result is an excitation or ionization



- Atom may deexcite and emit characteristic radiation:



Photoelectric effect 2

- In the kinematics, the binding energy of the ejected electron should be taken into account:

$$T = h\nu - E_b - T_a \approx h\nu - E_b$$

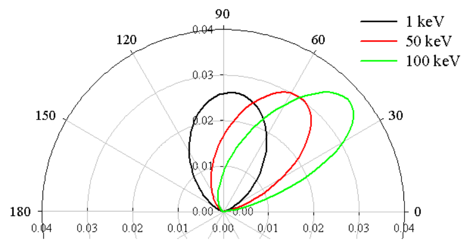
- Assuming E_b=0, the atomic cross section is:

$$\frac{d\tau}{d_e W} = 2\sqrt{2} r_0^{\,2} \alpha^4 Z^5 \left(\frac{m_e c^2}{h\nu}\right)^{7/2} \sin^2\theta \left(1 + 4\sqrt{\frac{2h\nu}{m_e c^2}}\cos\theta\right)$$

α: The fine-structure constant
Solid angle $_e\Omega$ gives the direction of the ejected electron

Photoelectric effect 2

Photoelectric cross section (dσ/dθ)/s



Characteristic radiation

- Energy of characteristic radiation depends on elektronic structure and transition probabilities
- ”K- and L-shell” vacancies ↔ $h\nu_K$ and $h\nu_L$
- Isotropic emission
- Fraction of photoelectric interactions: $P_K\ [h\nu > (E_b)_K]$ and $P_L\ [(E_b)_L < h\nu < (E_b)_K]$
- Probability for emission: Y_K og Y_L (*flourescence yield*)
- Energy emitted from the atom: $P_K Y_K h\nu_K + (1-P_K) P_L Y_L h\nu_L$

Photoelectric cross section

- General formula:

$$\tau \propto \frac{Z^n}{(h\nu)^m}\ ,\ 4<n<5\ ,\ 1<m<3$$

- Fraction of energy transferred to photoelectron:

$$\frac{T}{h\nu} = \frac{h\nu - E_b}{h\nu}$$

- However: don’t forget Auger electron(s)
- Cross section for energy transfer to photoelectron:

$$\tau_{tr} = \tau \frac{\left(h\nu - P_K Y_K h\nu_K - (1-P_K) P_L Y_L h\nu_L\right)}{h\nu}$$

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Auger effect

- Energy release by ejection of losely bound electron
- Energy of emitted electron equal to deexcitation energy
- Low Z: Auger dominates
- High Z: characteristic radiation dominates

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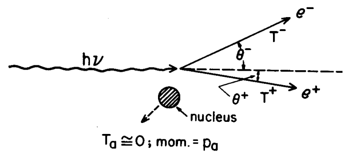
Pair production 1

- Photon absorption in the nuclear electromagnetic field where an electron-positron pair is created



- Triplet production: in the electromagnetic field of an electron

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Pair production 2

- Conservation of energy:

$$h\nu = 2m_e c^2 + T^+ + T^-$$

- Average kinetic energy after absorption:

$$\bar{T} = \frac{h\nu - 2m_e c^2}{2}$$

- Estimated electron/positron scattering angle:

$$\bar{\theta} \approx \frac{m_e c^2}{\bar{T}}$$

- Total cross section:

$$\kappa \approx \alpha r_0^2 Z^2 \bar{P}$$

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Triplet production

- In the electromagnetic field from an electron, an electron-positron pair is created
- Energy conservation:

$$h\nu = 2m_e c^2 + T^+ + T_1^- + T_2^-$$

- Average kinetic energy:

$$\bar{T} = \frac{h\nu - 2m_e c^2}{3}$$

- Primary electron is also given energy
- Threshold: $4m_0c^2$

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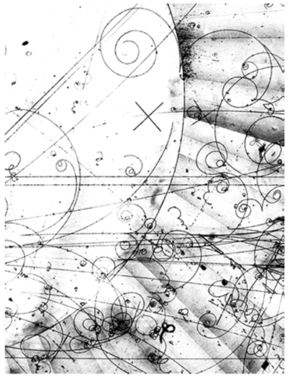
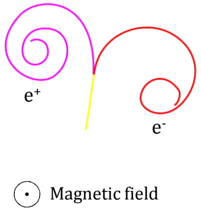
Discovery of pair production





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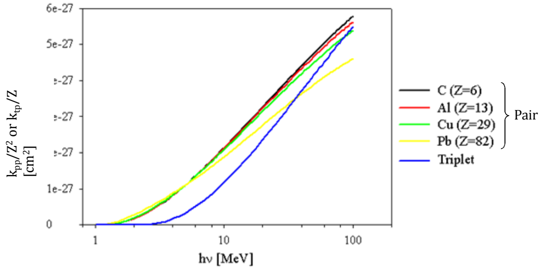
Pair- and triplet production

- Pair production dominates:



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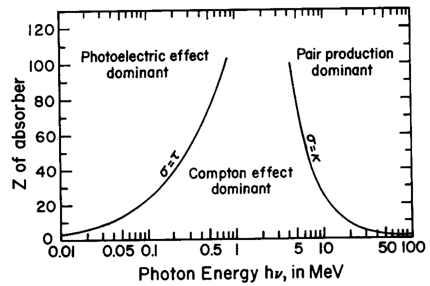
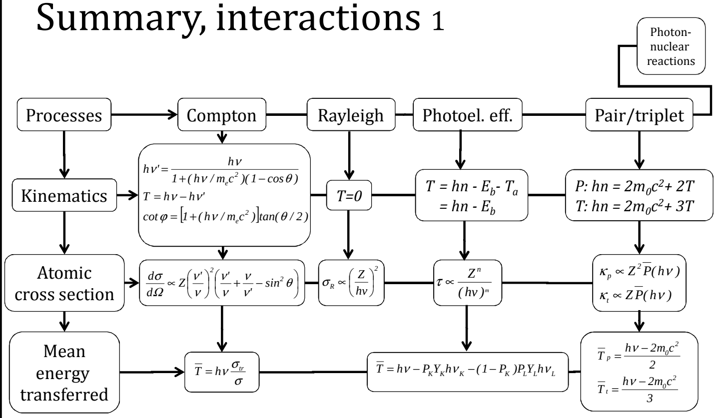
Photonuclear reactions

- Photon (energy above a few MeV) excites a nucleus
- Proton or neutron is emitted
- (γ, n) interactions may have consequences for radiation protection
- Example: Tungsten W (γ, n)

Summary, interactions 2

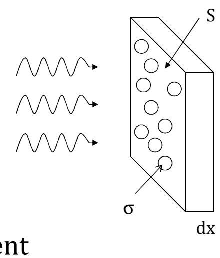


Summary, interactions 1



Attenuation coefficients 1

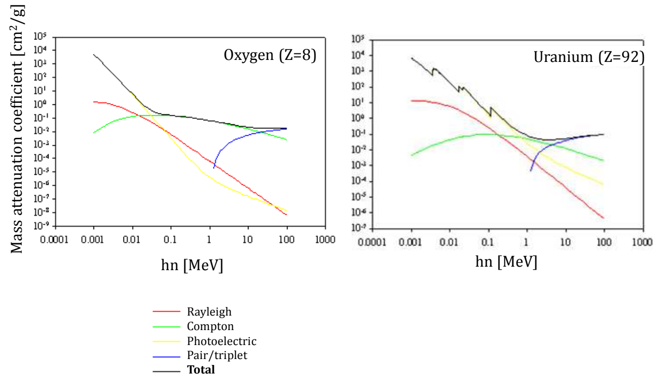
- n_V atoms per volume = $\rho(N_A/A)$
- Number of atoms:
  $n=n_V V=n_V S dx$
- Interaction probability
  $p=n\sigma/S=n_V\sigma dx$
- Probability per unit length:
  $\mu=p/dx=n_V\sigma=\rho(N_A/A)\sigma$
  μ: linear attenuation coefficient



Attenuation coefficients 2

- N_A : Avogadro's constant; 6.022×10^{23} mole^{-1}
- A: number of grams per mole
- N_A/A: number of atoms per gram
- N_AZ/A: number of electrons per gram
- Number of atoms per volume: $r(N_A/A)$
- Etc.

Attenuation coefficients 4



Attenuation coefficients 3

- Total *mass* attenuation coeffecient:

$$\frac{\mu}{\rho} = \frac{\tau}{\rho} + \frac{\sigma}{\rho} + \frac{\kappa}{\rho} + \frac{\sigma_R}{\rho}$$

- Coefficient for energy transfer:

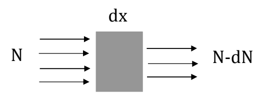
$$\frac{\mu_{tr}}{\rho} = \frac{\mu}{\rho}\frac{\overline{T}}{h\nu}$$

- Braggs rule for mixture of atoms:

$$\left(\frac{\mu}{\rho}\right)_{mix} = \sum_{i=1}^{n} f_i \left(\frac{\mu}{\rho}\right)_i \ , \ f_i = \frac{m_i}{\sum_{i=1}^{n} m_i}$$

Attenuation 1

- Beam with *N* photons impinge absorber with thickness *dx*:



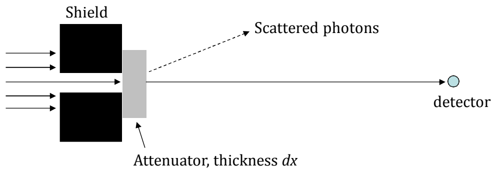
- Probability for interaction: μdx
- Number of photons interacting: $N\mu dx$

$$dN = N\mu dx \quad \Rightarrow \quad \int \frac{dN}{N} = \int \mu dx$$

$$\Rightarrow \underline{\underline{N = N_0 e^{-\mu x}}}$$

Attenuation 2

- Note that μ is the *interaction probability* per unit lenght – *not* the *absorption* probability
- $e^{-\mu x}$ corresponds to a narrow beam measurement geometry:



Attenuation 3

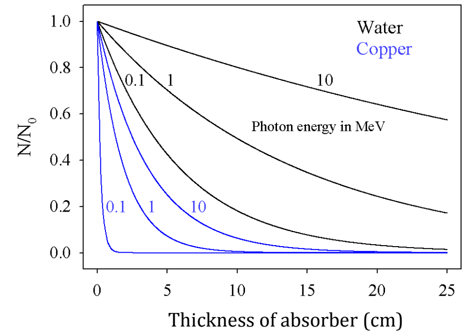
- 'Probability' for photon not interacting: $e^{-\mu x}$
- Normalized probability

$$p_{ni} = Ce^{-\mu x}\ ,\ \int_0^\infty p_{ni}\,dx \overset{!}{=} 1\ ,\ \Rightarrow p_{ni} = \mu e^{-\mu x}$$

- Mean free path:

$$\langle x \rangle = \int_0^\infty x p_{ni}\,dx = \int_0^\infty x\mu e^{-\mu x}dx = \frac{1}{\mu}$$

Attenuation 4



Attenuation - example

- 2 MeV photons

  Pb: $\mu = 0{,}516\ \text{cm}^{-1}$

  H_2O: $\mu = 0{,}049\ \text{cm}^{-1}$

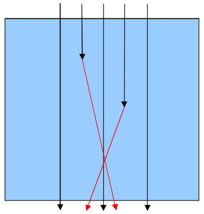
$$e^{-\mu_{H_2O}x_{H_2O}} = e^{-\mu_{Pb}x_{Pb}}$$

$$\Rightarrow \frac{x_{H_2O}}{x_{Pb}} = \frac{\mu_{Pb}}{\mu_{H_2O}}$$

- 10 times as much water necessary

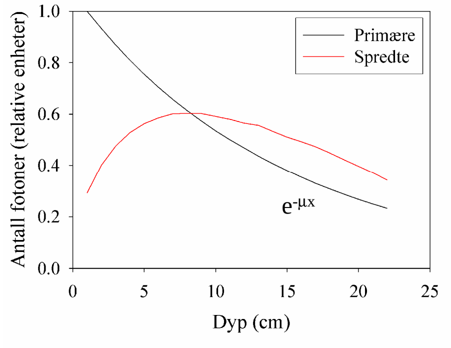
Scattered photons

- $e^{-\mu x}$: number of primary photons at a given depth
- What about the scattered photons?



- Monte Carlo simulations

Primary and scattered photons, 1 MeV



Primary and scattered photons, 100 keV