12

Neutron Interactions with Matter

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12.1 INTRODUCTION

How neutrons interact with matter affects the ways in which assays can be performed with neutrons. Neutron interactions with the assay material affect the interpretation of neutron measurements and limit the amount of fissile material the assay instrument can contain safely. A neutron detector is based on some neutron interaction with the material in the detector. Also, neutron interactions with shielding materials are necessary to protect radiation workers.

This chapter provides fundamental information about neutron interactions that are important to nuclear material measurements. The first section describes the interactions on the microscopic level where individual neutrons interact with other particles and nuclei. The concepts are then extended to macroscopic interactions with bulk compound materials.

12.2 MICROSCOPIC INTERACTIONS

12.2.1 The Cross-Section Concept

The probability of a particular event occurring between a neutron and a nucleus is expressed through the concept of the cross section. If a large number of neutrons of the same energy are directed into a thin layer of material, some may pass through with no interaction, others may have interactions that change their directions and energies, and still others may fail to emerge from the sample. There is a probability for each of these events. For example, the probability of a neutron not emerging from a sample (that is, of being absorbed or captured) is the ratio of the number of neutrons that do not emerge to the number originally incident on the layer. The cross section for being absorbed is the probability of neutrons being absorbed divided by the areal atom density (the number of target atoms per unit area of the layer). The cross section thus has the dimensions of area; it must be a small fraction of a square centimeter because of the large number of atoms involved. Because this type of cross section describes the probability of neutron interaction with a single nucleus, it is called the microscopic cross section and is given the symbol σ. (A macroscopic cross section for use with bulk matter is defined in Section 12.3.)

Another approach to understanding the concept of the microscopic cross section is to consider the probability of a single neutron attempting to pass through a thin layer of

material that has an area A and contains N target nuclei, each of cross-sectional area s. The sum of all the areas of the nuclei is Ns. The probability of a single neutron hitting one of these nuclei is roughly the ratio of the total target area Ns to the area of the layer A. In other words, the probability of a single neutron having a collision with a nucleus is Ns/A or (N/A)s, the areal target density times s. On the atomic level, however, cross sections for neutron interactions are not simply the geometrical cross-sectional area of the target. By replacing this s by the σ of the preceding paragraph, σ might be thought of as an effective cross-sectional area for the interaction. The cross section for the interaction retains the dimensions of area that s had.

The physical cross-sectional area s of a heavy nucleus is about 2×10^{-24} cm^2. Interaction cross sections for most nuclei are typically between 10^{-27} and 10^{-21} cm^2. To avoid the inconvenience of working with such small numbers, a different unit of area is used: the barn, denoted by the symbol b. It is defined to be 10^{-24} cm^2, so that the physical cross-sectional area of a heavy nucleus is about 2 b. Many neutron interaction cross sections range between 0.001 and 1000 b.

Each type of event has its own probability and cross section. The probability of each type of event is independent of the probabilities of the others, so the total probability of any event occurring is the sum of the individual probabilities. Similarly, the sum of all the individual cross sections is the total cross section.

12.2.2 The Energy-Velocity Relationship for Neutrons

Cross-section magnitudes are strong functions of neutron energy, as discussed in Section 12.2.4. As a preliminary to that discussion, this section describes the relationship between neutron energy and velocity. This connection is important not only for understanding cross sections but also for estimating the time that neutrons are present in regions such as those found in assay instruments.

The classical expression for kinetic energy, $E = mv^2/2$, is sufficiently accurate because even a kinetic energy of 100 MeV is still only about one-tenth of the rest-mass energy of a neutron (939.55 MeV). For velocity v in meters per second and kinetic energy E in MeV,

$$E = 5.227 \times 10^{-15} v^2 \tag{12-1}$$

and

$$v = 1.383 \times 10^7 E^{1/2}. \tag{12-2}$$

Figure 12.1 shows a graph of these equations for ready use. The graph shows, for example, that a 1-MeV neutron has a speed of 1.383×10^7 m/s and therefore will cross a 15-cm sample region in a typical assay instrument in about 11 ns. A thermal neutron with an energy of 0.025 eV (see Section 12.2.3) has a speed of 2187 m/s and will cross the same 15-cm region in about 70 μs.

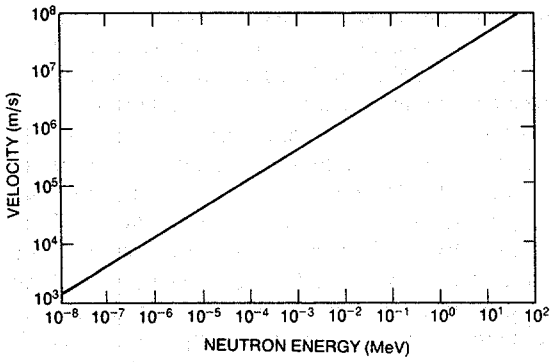

Fig. 12.1 *Graph showing the relationship between a neutron's speed and its kinetic energy.*

12.2.3 Types of Interactions

A neutron can have many types of interactions with a nucleus. Figure 12.2 shows the types of interactions and their cross sections. Each category of interaction in the figure consists of all those linked below it. The total cross section σ_t expresses the probability of any interaction taking place.

A simple notation can be used to give a concise indication of an interaction of interest. If a neutron n impinges on a target nucleus T, forming a resultant nucleus R and the release of an outgoing particle g, this interaction is shown as T(n,g)R. The heavy nuclei are shown outside the parentheses. To denote a type of interaction without regard for the nuclei involved, only the portion in parentheses is shown. An example of an (n,p) reaction is ^{5}B(n,p)^{5}Be.

An interaction may be one of two major types: scattering or absorption. When a neutron is scattered by a nucleus, its speed and direction change but the nucleus is left with the same number of protons and neutrons it had before the interaction. The nucleus will have some recoil velocity and it may be left in an excited state that will lead to the eventual release of radiation. When a neutron is absorbed by a nucleus, a wide range of radiations can be emitted or fission can be induced.

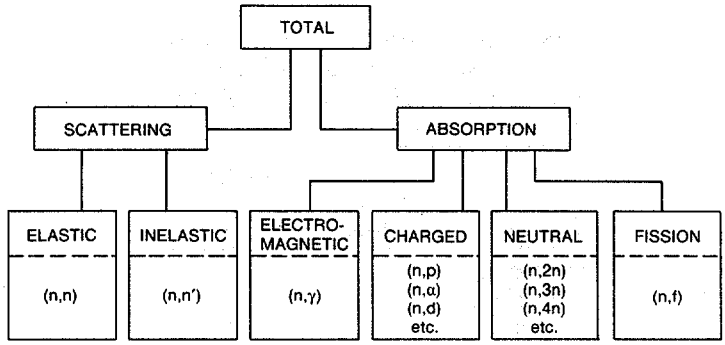

Fig. 12.2 *Various categories of neutron interactions. The letters separated by commas in the parentheses show the incoming and outgoing particles.*

Scattering events can be subdivided into elastic and inelastic scattering. In elastic scattering, the total kinetic energy of the neutron and nucleus is unchanged by the interaction. During the interaction, a fraction of the neutron's kinetic energy is transferred to the nucleus. For a neutron of kinetic energy E encountering a nucleus of atomic weight A, the average energy loss is $2EA/(A+1)^2$. This expression shows that in order to reduce the speed of neutrons (that is, to moderate them) with the fewest number of elastic collisions, target nuclei with small A should be used. By using hydrogen, with $A = 1$, the average energy loss has its largest value of E/2. A neutron with 2 MeV of kinetic energy will (on the average) have 1 MeV left after one elastic collision with a hydrogen nucleus, 0.5 MeV after a second such collision, and so on. To achieve a kinetic energy of only 0.025 eV would take a total of about 27 such collisions. (A neutron of energy 0.025 eV is roughly in thermal equilibrium with its surrounding medium and is considered a "thermal neutron." From the relation $E = kT$ where k is Boltzmann's constant, an energy E of 0.025 eV corresponds to a temperature T of 20°C.) In general, after n elastic collisions, the neutron's energy is expected to change from E_0 to $E_n = E_0[(A^2+1)/(A+1)^2]^n$. To reach E_n from E_0 thus requires $n = \log(E_n/E_0)/\log[(A^2+1)/(A+1)^2]$ collisions, on the average. Table 12-1 gives examples of the number of collisions required to "thermalize" a 2-MeV neutron in some materials.

Inelastic scattering is similar to elastic scattering except that the nucleus undergoes an internal rearrangement into an excited state from which it eventually releases radiation. The total kinetic energy of the outgoing neutron and nucleus is less than the kinetic energy of the incoming neutron; part of the original kinetic energy is used to place the nucleus into the excited state. It is no longer easy to write an expression for the average energy loss because it depends on the energy levels within the nucleus. But the net effect on the neutron is again to reduce its speed and change its direction. If all the excited states of the nucleus are too high in energy to be reached with the energy available from the incoming neutron, inelastic scattering is impossible. In particular, the hydrogen nucleus does not have excited states, so only elastic scattering can occur in that case. In general, scattering moderates or reduces the energy of neutrons and provides the basis for some neutron detectors (for example, proton recoil detectors).

Table 12-1. Average number of collisions required to reduce a neutron's energy from 2 MeV to 0.025 eV by elastic scattering

Element	Atomic Weight	Number of Collisions
Hydrogen	1	27
Deuterium	2	31
Helium	4	48
Beryllium	9	92
Carbon	12	119
Uranium	238	2175

Instead of being scattered by a nucleus, the neutron may be absorbed or captured. A variety of emissions may follow, as shown in Figure 12.2. The nucleus may rearrange its internal structure and release one or more gamma rays. Charged particles may also be emitted; the more common ones are protons, deuterons, and alpha particles. The nucleus may also rid itself of excess neutrons. The emission of only one neutron is indistinguishable from a scattering event. If more than one neutron is emitted, the number of neutrons now moving through the material is larger than the number present before the interaction; the number is said to have been multiplied. Finally, there may be a fission event, leading to two or more fission fragments (nuclei of intermediate atomic weight) and more neutrons (see Chapter 11).

Many safeguards instruments have neutron detectors that use an absorption reaction as the basis of the detection technique. The lack of an electric charge on the neutron makes direct detection difficult, so the neutron is first absorbed by a nucleus, which then emits a charged particle (such as a proton or deuteron). Helium-3, uranium-235 and boron-10 are commonly used in detectors because they have large absorption cross sections for the production of charged particles with low-speed neutrons.

When moderation alone is desired, absorption should be avoided. For example, hydrogen is a better moderator than deuterium (that is, it requires fewer collisions to achieve a particular low speed), but it also has a larger absorption cross section for neutrons. The net effect is that deuterium will yield more thermal neutrons than hydrogen and may be the preferred moderating material.

The cross sections associated with the various interactions described above can be designated by the following notation:

σ_t = total cross section ($\sigma_s + \sigma_a$)
σ_s = total scattering cross section ($\sigma_{el} + \sigma_i$)
σ_{el} or $\sigma_{n,n}$ = elastic scattering cross section
σ_i or $\sigma_{n,n'}$ = inelastic scattering cross section
σ_a or σ_c = absorption or capture cross section
σ_{ne} = nonelastic cross section, $\sigma_t - \sigma_{el}$
$\sigma_{n,\gamma}$ = radiative capture cross section
σ_f or $\sigma_{n,f}$ = fission cross section
$\sigma_{n,p}$ = (n,p) reaction cross section.

12.2.4 Energy Dependence of Cross Sections

All of the cross sections described above vary with neutron energy and with the target nucleus, sometimes in a dramatic way. This section gives some generalizations about the energy dependence of cross sections and shows data (Ref. 1) for a few important nuclei.

Figure 12.3 is the total cross section for ^{239}Pu for incident neutrons of 0.001-eV to 10-MeV energy. Note that as a general rule the cross section decreases with increasing energy. At low energies, below 1 MeV, the elastic cross section is nearly constant, whereas the inelastic scattering cross section and absorption cross sections are proportional to the reciprocal of the neutron's speed (that is, 1/v). So at low energies the total cross section can be nearly constant or decreasing with energy, depending on which type of event dominates. For example, in ^{239}Pu the inelastic cross section dominates and the total cross section decreases as 1/v. Similar behavior is observed for most light and

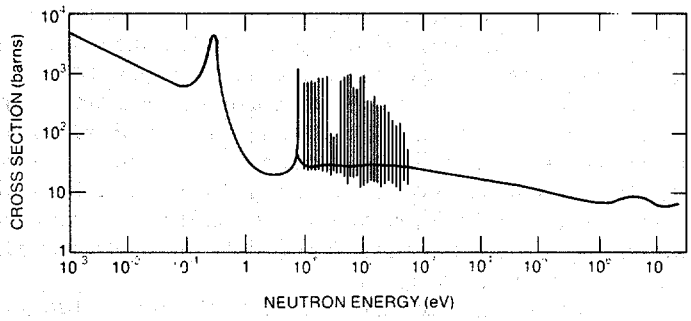

Fig. 12.3 Total neutron cross section of ^{239}Pu.

intermediate weight nuclei as well. Figures 12.4 and 12.5 illustrate the low-energy total cross-section behavior of boron and cadmium. The unusually high absorption cross sections of these two materials make them useful as thermal-neutron poisons.

At higher energies the cross section may have large peaks superimposed on the 1/v trend. These peaks are called resonances and occur at neutron energies where reactions with nuclei are enhanced. For example, a resonance will occur if the target nucleus and the captured neutron form a "compound" nucleus, and the energy contributed by the neutron is close to that of an excited state of the compound nucleus.

In heavy nuclei, large and narrow resonances appear for neutron energies in the eV range. For energies in the keV region the resonances can be too close together to resolve. In the MeV region the resonances are more sparse and very broad, and the cross sections become smooth and rolling. For light nuclei, resonances appear only in the MeV region and are broad and relatively small. For nuclei with intermediate weights (such as

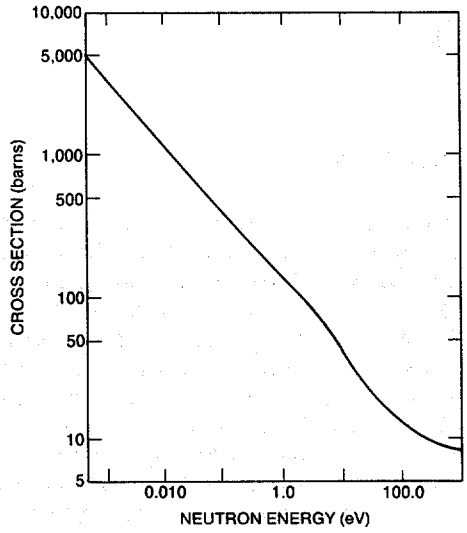

Fig. 12.4 Low-energy total neutron cross section of boron (Ref. 1).

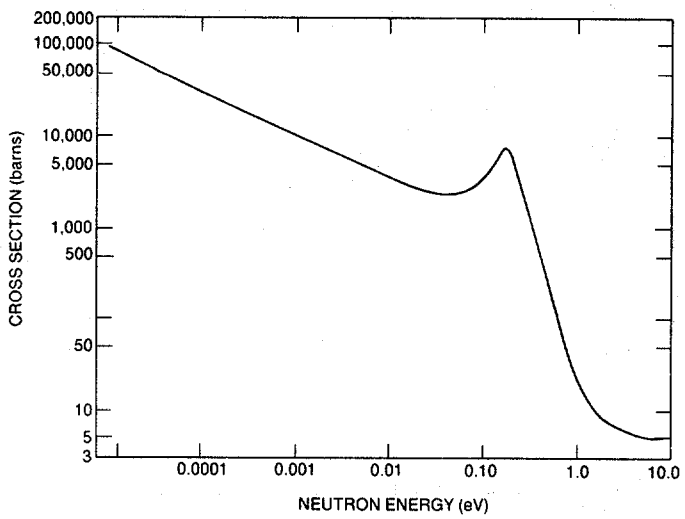

Fig. 12.5 *Low-energy total neutron cross section of cadmium (Ref. 1).*

cadmium, nickel, iron), resonances can be found below 1 keV. These resonances have heights and widths between those of light and heavy nuclei.

Some exceptions to the general trends exist in ^{1}H and ^{2}H where there are no resonances at all and in nuclei with "magic" numbers of protons or neutrons where the behavior may be similar to that of light nuclei despite the actual atomic weight. In practice, it is necessary to rely on tables of cross sections for the nuclei of interest because there is no convenient way to calculate cross sections. Some microscopic cross sections are included in the second table in Section 12.3.

Some neutron-induced fission cross sections important for nondestructive assay are shown in Figure 12.6. The fissile isotopes ^{235}U and ^{239}Pu have large cross sections (about 1000 b) for fission by thermal or near-thermal neutrons. For fission by fast neutrons (10 keV to 10 MeV), these cross sections are reduced to 1 to 2 b. The fertile isotopes ^{238}U and ^{240}Pu have negligible fission cross sections for low-energy neutrons but exhibit a "threshold" near 1-MeV neutron energy. Above 1 MeV the fission cross sections of the fertile isotopes are comparable to those of the fissile isotopes.

12.3 MACROSCOPIC INTERACTIONS

12.3.1 Macroscopic Cross Sections

Although study of the interactions of a neutron with a single nucleus on the microscopic scale provides a basis for understanding the interaction process, measurements are actually performed with thick samples that often contain a mixture of elements. These additional features are described by using the macroscopic cross sections appropriate for bulk materials.

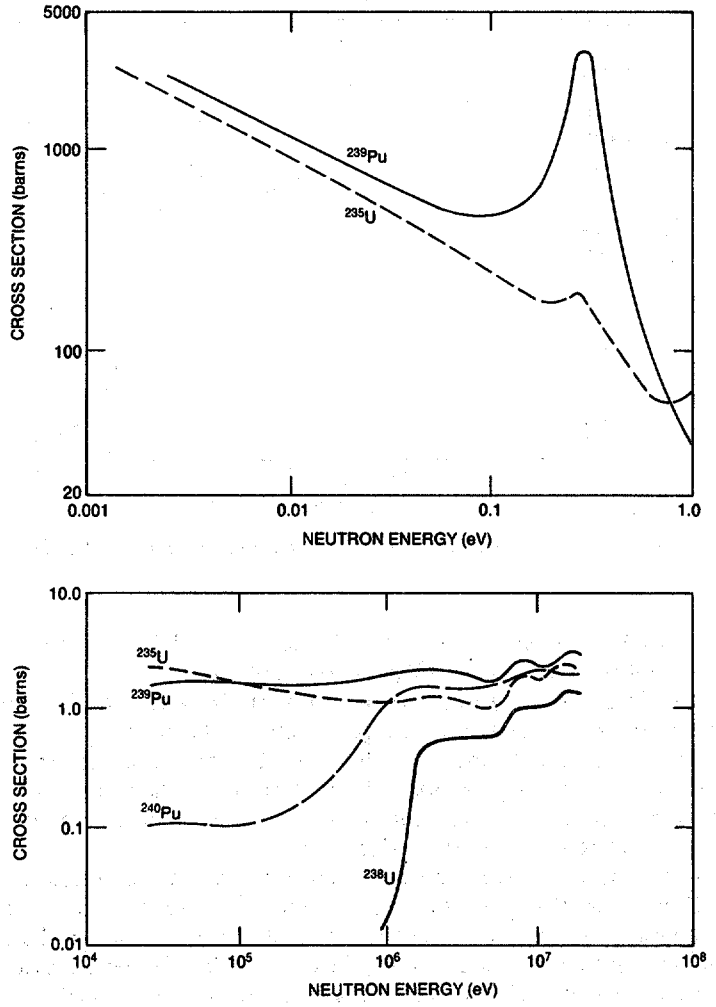

Fig. 12.6 Fission cross sections for some important fissile (^{235}U, ^{239}Pu) and fertile (^{238}U, ^{240}Pu) isotopes (Ref. 1).

The definition of the macroscopic cross section arises from the transmission of a parallel beam of neutrons through a thick sample. The thick sample can be considered to be a series of atomic layers; for each layer we can apply the results found with the microscopic cross-section concept. By integrating through enough atomic layers to reach a depth x in the sample, the intensity I(x) of the uncollided neutron beam is

$$I(x) = I_0 e^{-N\sigma_t x} \tag{12-3}$$

where I_0 is the intensity of the beam before it enters the sample, N is the atom density, and σ_t is the total cross section. Figure 12.7 shows the uncollided intensity remaining in a parallel beam as it passes through a thick layer of matter. Note that the fraction transmitted without collisions, $I(x)/I_0$, depends on the energy of the neutrons through the energy dependence of the microscopic total cross-section σ_t.

An expression similar to Equation 12-3 is used for gamma-ray attenuation. In that case, low-energy gamma rays are very likely to be absorbed and thus removed not only from the parallel beam but from the material entirely. With neutrons at low energies, elastic scattering is the most likely event. Although Equation 12-3 gives the intensity of the neutrons that have had no interaction up to a depth x, the actual number of neutrons present that can be detected may be much larger because of multiple scattering, multiplication, or finite detector acceptance angle.

The total macroscopic cross section is $\Sigma_t = N\sigma_t$. Σ_t has dimensions of cm^{-1} (see Equation 12-3 above) and is analogous to the linear attenuation coefficient for gamma rays. If only a particular type of interaction is of interest, a macroscopic cross section for it alone can be defined using its microscopic cross section in place of the total cross section. For quantitative calculations, the concept of macroscopic cross section is less used than the analogous gamma-ray linear attenuation coefficient because of the complications of multiple scattering and other effects mentioned in the previous paragraph.

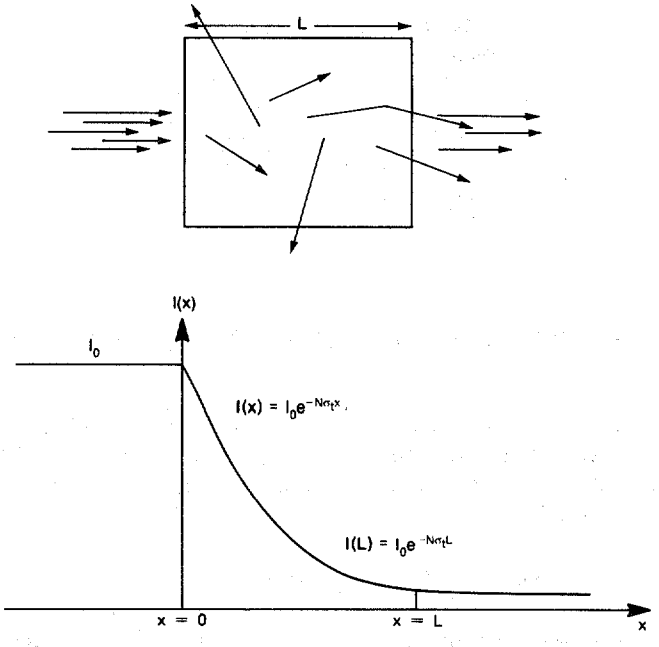

Fig. 12.7 *The intensity of a parallel beam of uncollided neutrons decreases exponentially as it passes through a thick layer of matter.*

If the sample is a compound instead of a simple element, the total macroscopic cross section is the sum of the macroscopic cross sections of the individual elements:

$$\Sigma = \Sigma_1 + \Sigma_2 + \Sigma_3 \ldots \quad . \tag{12-4}$$

The atom density N_i of each element i is given by

$$N_i = \rho N_a n_i / M \tag{12-5}$$

where ρ is the density of the compound; M is the molecular weight of the compound; N_a is Avogadro's number, 6.022×10^{23} atoms/mole; and n_i is the number of atoms of element i in one molecule. From Equations 12-4 and 12-5 the general form of the macroscopic cross section can be written as

$$\Sigma = \frac{\rho N_a}{M}(n_1\sigma_1 + n_2\sigma_2 + n_3\sigma_3 + \ldots) \quad . \tag{12-6}$$

As an illustration of these equations, the total macroscopic cross section for 1-MeV neutrons in $^{nat}UO_2$ (density 10 g/cm^3, molecular weight 270) is calculated from the data in Table 12-2.

Table 12-2. Nuclear data for $^{nat}UO_2$

Isotope	n_i	σ_t at 1 MeV (b)
^{235}U	0.007	6.84
^{238}U	0.993	7.10
^{16}O	2.000	8.22

$$\Sigma_t = \frac{(10)(0.6022)}{270}[(0.007)(6.84) + (0.993)(7.10) + 2(8.22)] = 0.525 \text{ cm}^{-1} \quad . \tag{12-7}$$

Powers of 10^{24} and 10^{-24} have been cancelled in Avogadro's number and in the cross-section values. These cross-section values were taken from Table 12-3 (Ref. 2), which is a compilation of microscopic and macroscopic cross sections at two neutron energies, 0.025 eV (thermal) and 1 MeV.

12.3.2 Mean Free Path and Reaction Rate

A very descriptive feature of the transmission of neutrons through bulk matter is the mean-free-path length, which is the mean distance a neutron travels between interac-

tions. It can be calculated from Equation 12-3 with $N\sigma_t$ replaced by Σ_t. The mean-free-path length λ is

$$\lambda = 1/\Sigma_t \quad , \tag{12-8}$$

the reciprocal of the macroscopic cross section. For the case of 1-MeV neutrons in UO_2 calculated above, a macroscopic cross section of 0.525 cm^{-1} implies a mean-free-path length of 1.91 cm.

The mean-free-path length has many qualitative applications in assay instruments and shielding. (a) If the mean-free-path length of neutrons emitted by a sample in a passive assay instrument is long compared to the dimensions of the sample, it is likely that most of the neutrons will escape from the sample and enter the detection region. (b) If the number of collisions required to thermalize a neutron is known, the necessary moderator thickness of a shield can be estimated. (c) If the thickness of a shield is many times the mean-free-path length of a neutron trying to penetrate the shield, then the shield fulfills its purpose. (Because the mean-free-path length is a function of the neutron's energy, the actual calculation is not so simple.)

A closely related concept is that of the reaction rate. When traveling with a speed v, a neutron has an average time between interactions of λ/v. The reaction rate is the frequency with which interactions occur: v/λ, or $v\Sigma_t$. In uranium oxide, for example, a 1-MeV neutron will have a reaction rate of 7.26×10^8 per second (from Equations 12-2 and 12-7). This does not mean, however, that in one second there will be that many reactions; with each collision the neutron's energy decreases and the cross section changes, thereby altering the instantaneous reaction rate.

The paths of neutrons in matter can be simulated with Monte Carlo calculations. Figure 12.8 shows a few paths for neutrons with 1 MeV of energy entering cylinders of different materials. The mean-free-path length depends on both the type of material and the energy of the neutron. After each collision, the energy is decreased and the mean-free-path length is affected accordingly. Figure 12.8 shows that a cylinder of polyethylene is more effective in preventing the transmission of neutrons than a cylinder of heavy metal. A neutron loses most of its energy by colliding with the light elements in polyethylene and then the mean-free-path length becomes small as the cross sections increase. An important effect of polyethylene is that it seemingly retains a large fraction of the neutrons near a certain depth; these neutrons have had enough collisions to lose nearly all their kinetic energy. If a thermal-neutron detector is placed in this region, the chance of detecting neutrons is optimized.

12.4 EFFECTS OF MODERATION IN BULK MATTER

It is often a design goal to reduce or moderate the speed of neutrons in the sample region or the detector region or both. Recalling the general 1/v trend of interaction cross sections (Figures 12.3 through 12.6), the purpose of the reduction in speeds is to increase the probability of an interaction. In other regions, it may be desirable to hinder interactions by choosing materials that are poor moderators or by adding low-energy neutron absorbers to remove neutrons once they become moderated.

Table 12-3. Neutron cross sections of common materials[a]

Material	Atomic or Molecular Weight	Density (g/cm^3)	Cross Sections[b] E = 0.0253 eV σ_t (b)	σ_a (b)	Σ_t (cm^{-1})	Σ_a (cm^{-1})	E = 1 MeV σ_t (b)	σ_a (b)	Σ_t (cm^{-1})	Σ_a (cm^{-1})
Al	27	2.7	1.61	0.232	0.097	0.014	2.37	0.000	0.143	0.000
B	10	2.3	3845	3843	533	532	2.68	0.189	0.371	0.0262
B	11	2.3	5.28	0.005	0.665	0.0006	2.13	0.000	0.268	0.000
Be	9	9.0	6.35	0.010	3.82	0.0060	3.25	0.003	1.96	0.0018
C	12	1.9	4.95	0.003	0.472	0.0003	2.58	0.000	0.246	0.000
Nat Ca	40.08	1.55	3.46	0.433	0.081	0.101	1.14	0.004	0.027	0.0001
Cd	112	8.7	2470	2462	115.5	115.2	6.50	0.058	0.304	0.0027
Nat Cl	35.45	Gas	50.2	33.4	Gas	Gas	2.30	0.0005	Gas	Gas
Nat Cu	63.55	8.94	12.5	3.80	1.06	0.322	3.40	0.011	0.288	0.0009
F	19	Gas	3.72	0.010	Gas	Gas	3.15	0.000	Gas	Gas
Fe	56	7.9	14.07	2.56	1.19	0.217	5.19	0.003	0.441	0.0003
Nat Gd	157.25	7.95	49 153	48 981	1496	1491	7.33	0.223	0.223	0.0068
H	1	Gas	30.62	0.33	Gas	Gas	4.26	0.000	Gas	Gas
H	2	Gas	4.25	0.000	Gas	Gas	2.87	0.000	Gas	Gas
He	3	Gas	5337	5336	Gas	Gas	2.87	0.879	Gas	Gas
He	4	Gas	0.86	0.000	Gas	Gas	7.08	0.000	Gas	Gas
Li	6	0.534	938	937	50.3	50.2	1.28	0.230	0.069	0.0123
Li	7	0.534	1.16	0.036	0.053	0.0017	1.57	0.000	0.072	0.0000
Nat Mg	24.31	1.74	3.47	0.063	0.150	0.0027	2.66	0.001	0.115	0.0000
Mn	55	7.2	14.5	13.2	1.14	1.04	3.17	0.003	0.250	0.0002
N	14	Gas	12.22	1.9	Gas	Gas	2.39	0.021	Gas	Gas
Na	23	0.971	3.92	0.529	0.100	0.0134	3.17	0.000	0.081	0.0000
Ni	59	8.9	23.08	4.58	2.10	0.416	3.66	0.0008	0.322	0.0001
O	16	Gas	3.87	0.000	Gas	Gas	8.22	0.000	Gas	Gas
Pb	204	11.34	11.40	0.18	0.381	0.0060	4.39	0.0033	0.147	0.0001

[a]Ref. 2.

[b]A zero value means zero to the number of figures shown.

Table 12-3. Neutron cross sections of common materials[a] (continued)

Material	Atomic or Molecular Weight	Density (g/cm^3)	Cross Sections[b] E = 0.0253 eV σ_t (b)	σ_a (b)	Σ_t (cm^{-1})	Σ_a (cm^{-1})	E = 1 MeV σ_t (b)	σ_a (b)	Σ_t (cm^{-1})	Σ_a (cm^{-1})
Pu	238.05	19.6	599.3	562.0	29.72	27.87	6.66	0.190	0.330	0.0094
Pu	239.05	19.6	1021	270	50.4	13.3	7.01	0.026	0.346	0.0013
Pu	240.05	19.6	294	293	14.5	14.4	7.15	0.108	0.352	0.0053
Pu	241.06	19.6	1390	362	68.1	17.7	7.98	0.117	0.391	0.0057
Pu	242.06	19.6	26.7	18.9	1.30	0.922	7.31	0.098	0.357	0.0048
Nat Si	28.09	2.42	2.24	0.161	0.116	0.0084	4.43	0.001	0.230	0.0001
Th	232	11.3	20.4	7.50	0.598	0.220	7.00	0.135	0.205	0.0040
U	233.04	19.1	587	45.8	29.0	2.26	6.78	0.069	0.335	0.0034
U	234.04	19.1	116	103	5.70	5.07	8.02	0.363	0.394	0.0178
U	235.04	19.1	703	96.9	34.3	4.74	6.84	0.117	0.335	0.0057
U	236.05	19.1	13.3	5.16	0.648	0.251	7.73	0.363	0.377	0.0177
U	237.05	19.1	487.5	476.4	23.6	23.1	6.72	0.135	0.326	0.0066
U	238.05	19.1	11.63	2.71	0.562	0.131	7.10	0.123	0.343	0.0059
Nat U	238.03	19.1	16.49	3.39	0.797	0.1637	7.01	0.120	0.343	0.0058
Nat W	183.85	19.3	23.08	18.05	1.459	1.141	6.95	0.057	0.439	0.0036
CH_2	14	0.94			2.68	0.027			0.449	0.0000
H_2O	18	1.0			2.18	0.022			0.560	0.0000
D_2O	20	1.1			0.410	0.000			0.420	0.0000
Average Fission Products of:										
^{235}U	117		4496	4486			7.43	0.00036		
^{239}Pu	119		2087	2086			7.48	0.00093		

[a]Ref. 2.

[b]A zero value means zero to the number of figures shown.

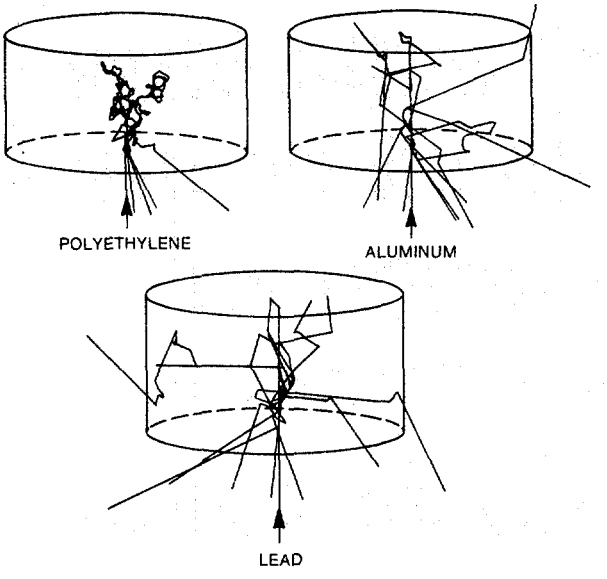

Fig. 12.8 *Neutrons with 1 MeV of kinetic energy are shown entering cylinders of material from the bottom and then being scattering or absorbed. The paths were calculated using a Monte Carlo technique.*

For example, in the assay of plutonium, moderation is not a desirable effect in the sample region. High-speed neutrons are more able to penetrate the sample and they have lower fission cross sections so that multiplication is less than with low-speed neutrons. On the other hand, in the detector region, moderation increases the detection efficiency for detectors such as ^{3}He proportional counters. By placing hydrogenous material (such as polyethylene) around the detectors, the neutrons can be counted with more efficiency. Also needed is a filter that will let high-speed neutrons enter the detector region where they can become moderated but will not let the moderated neutrons return to the sample region where they could produce additional fissions. A layer of material with a large absorption cross section for slow neutrons (such as cadmium, Figure 12.5) placed between the sample region and the detector region is effective in this regard.

A standard basis for comparing moderating abilities of different materials is the moderating power. If one material has a larger moderating power than another, less of that material is needed to achieve the same degree of moderation. Two factors are important: (1) the probability of a scattering interaction and (2) the average change in kinetic energy of the neutron after such an interaction. To be an effective moderator, both the probability of an interaction and the average energy loss in one scatter should be high. The moderating power is defined as $\xi\Sigma_s$, where Σ_s is the macroscopic scattering cross section and ξ is the average logarithmic energy decrement in a scatter. This decrement is $\ln(E_{before}) - \ln(E_{after})$. When elastic collisions in an element with atomic weight A dominate the scattering process, the decrement becomes

$$\xi = 1 - \frac{(A-1)^2}{2A} \ln \frac{(A+1)}{(A-1)} \quad . \tag{12-9}$$

For $A > 2$, ξ can be approximated by $2/(A+0.67)$ (Ref. 3). The moderating power of a compound is given by

$$\xi\Sigma_s = \frac{\rho N_a}{M}(n_1\sigma_1\xi_1 + n_2\sigma_2\xi_2 + ...) \quad , \tag{12-10}$$

where ρ is the density of the compound, M is its molecular weight, N_a is Avogadro's number, n_i is the number of atoms of element i in one molecule, σ_i is the microscopic scattering cross section for element i, and ξ_i is the logarithmic energy decrement for element i.

A material with a large moderating power might nevertheless be useless as a practical moderator if it has a large absorption cross section. Such a moderator would effectively reduce the speeds of those neutrons that are not absorbed, but the fraction of neutrons that survive may be too small to be used in a practical manner. A more comprehensive measure of moderating materials is the moderating ratio, $\xi\Sigma_s/\Sigma_a$. A large moderating ratio is desirable; it implies not only a good moderator but also a poor absorber. For a compound, the moderating ratio is given by Equation 12-10 with each σ_i replaced by σ_s/σ_a for element i.

Table 12-4 gives the moderating powers and ratios for some common moderator materials for neutrons in the l-eV to 100-keV energy range (Ref. 4). Ordinary water has a higher moderating power than heavy water because the atomic weight of hydrogen is half that of deuterium. But the hydrogen nucleus (a proton) can absorb a neutron and create deuterium much more readily than a deuterium nucleus can absorb a neutron and create tritium. This difference in absorption cross sections gives heavy water a much more favorable moderating ratio. However, because of its availability and low cost, ordinary water is often preferred. The solid materials given in the table have a higher moderating ratio than ordinary water and can have fabrication advantages. Polyethylene is commonly selected as a moderator because of its high moderating power and moderating ratio.

Table 12-4. Moderating powers and ratios of selected materials (Ref. 4)

Moderator	Moderating Power (1 eV to 100 keV)	Moderating Ratio (Approximate)
Water	1.28	58
Heavy Water	0.18	21 000
Helium at STP	0.00001	45
Beryllium	0.16	130
Graphite	0.064	200
Polyethylene (CH_2)	3.26	122

12.5 EFFECTS OF MULTIPLICATION IN BULK MATTER

When a neutron interaction yields more than one neutron as a product, a multiplication event has occurred. More neutrons will be present in the material after the interaction than before. The most widely known multiplication event is fission, but other absorption interactions, such as (n,2n), can be important contributors to multiplication.

Of the neutrons in a given material at a given moment, some will eventually escape and the others will be absorbed. Additional neutrons can originate in the material as products of the absorptions. The definition of the multiplication M is the total number of neutrons that exist in the sample divided by the number of neutrons that were started. If 100 neutrons are started in the sample and an additional 59 are found to be created from multiplication events, the multiplication is 1.59. Only a fraction of the first generation of 100 neutrons produces additional neutrons through multiplication events; the others escape or are absorbed by other types of interactions. The same fraction of the second generation produce a third generation, and so on. The number of neutrons remaining in the sample steadily decreases until it is zero and the total number of neutrons produced by all the multiplication events is 59.

A related concept that is more commonly used is the multiplication factor. It relates the numbers of neutrons in successive generations. There are two categories of multiplication factors that apply to different physical sizes of the material involved.

If the material is infinite in extent, the multiplication factor is written k_∞ and is defined as the ratio of the number of neutrons in one generation to the number in the previous generation. Because of the infinite size of the material, all neutrons of a generation become absorbed. Thus k_∞ is also the ratio of the number of neutrons produced in one generation to the number absorbed in the preceding generation.

If the material is not infinite in size, some neutrons in a generation may escape through the surface and not be absorbed; these are "leakage" neutrons. The multiplication factor for this more practical situation is called k_{eff}. It is defined as the ratio of the number of neutrons produced in one generation to the number either absorbed or leaked in the preceding generation. The multiplication factor k_{eff} is the more practical ratio for safeguards work because instruments are often made small to comply with size and weight constraints.

As an example of k_{eff} and its connection with the multiplication M, consider the case described earlier. The original 100 neutrons would constitute the first generation. If the original neutrons create 37 neutrons through reactions, the 37 neutrons would be the next generation. The multiplication factor in this case is thus 0.37. With k_{eff} less than one, the number of neutrons in succeeding generations decreases, eventually reaching zero. As Table 12-5 indicates, it takes 7 generations to reduce the number of neutrons from 100 to about zero.

The number of neutrons in one generation is found by multiplying the number in the previous generation by the multiplication factor k_{eff}, which in this example is 0.37. This is a statistical process, of course, and the exact number in any generation cannot be exactly known, but for large numbers of neutrons the ratio of populations in successive generations is very nearly constant.

Table 12-5. Example of neutron population decline

Generation	Average No. of Neutrons for $k_{eff} = 0.37$
1	100
2	37
3	14
4	5
5	2
6	1
7	0
	159

The multiplication M is readily connected to the multiplication factor k_{eff} when k_{eff} is less than one. By adding together all the numbers of neutrons in all the generations, the geometric sum can be found:

$$M = 1/(1-k_{eff}), \quad k_{eff} < 1 \quad . \tag{12-11}$$

With the multiplication of 1.59 in Table 12-5, the multiplication factor is 0.37, showing that the number of neutrons is decreasing in successive generations.

As k_{eff} approaches one, the value of M becomes larger and larger, as shown in Figure 12.9. When $k_{eff} = 1$, the formula shows that there is no limit to the number of neutrons that will be produced; in practice, there is a finite number of nuclei that can produce neutrons, so the number of neutrons created is finite but extremely large. Criticality is said to be reached when $k_{eff} = 1$. If k_{eff} is larger than one, the sample is supercritical; with k_{eff} less than one, the sample is subcritical.

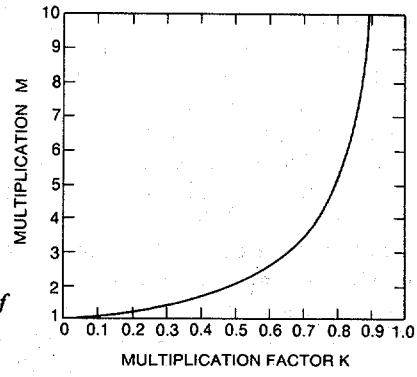

Fig. 12.9 *The multiplication M is shown as a function of the multiplication factor k_{eff}. Only subcritical values (k_{eff} less than one) are included here.*

12.6 NEUTRON SHIELDING

To protect personnel from the biological effects of neutrons and to reduce background counts, neutron shielding is often necessary. The selection and arrangement of shielding materials vary with the circumstances. Some general principles can be derived from the neutron interactions with matter described earlier in this chapter.

Thermal neutrons with energies of 0.025 eV or less are absorbed with great effectiveness by thin layers of boron or cadmium, as suggested by the large cross sections shown in Figures 12.4 and 12.5. Boron is often used in the form of boron carbide (B_4C) or boron-loaded solutions. One commonly used material is Boral, a mixture of boron carbide and aluminum, which is available in sheets of varying thickness. Cadmium has the disadvantage of emitting high-energy gamma rays after neutron capture, which may necessitate additional gamma-ray shielding. A comparison of Figures 12.4 and 12.5 shows that cadmium is more effective than boron for absorbing thermal neutrons, whereas boron is more effective for absorbing epithermal neutrons (energy range 0.1 eV to 10 eV).

High-speed neutrons are more difficult to shield against because absorption cross sections are much lower at higher energies. Thus it is first necessary to moderate high-speed neutrons through elastic or inelastic scattering interactions. Inelastic scattering or absorption may again produce potentially hazardous gamma rays; for example, neutron capture in hydrogen releases a 2.224-MeV gamma ray. An effective radiation shield consists of a combination of materials: hydrogenous or other low-A materials to moderate neutrons; thermal neutron absorbers; and high-Z materials to absorb gamma rays. Examples of hybrid shielding materials are polyethylene and lead, concrete containing scrap iron, and more exotic materials such as lithium hydride.

In safeguards work with small samples of fissionable materials or weak neutron sources, shielding may be restricted to several centimeters of polyethylene. The shielding properties of polyethylene are illustrated in Figure 12.10, which gives dose rates on the surface of a sphere of polyethylene with a ^{252}Cf source in its center. The source emits neutrons with a high-energy fission spectrum comparable to most uranium and plutonium isotopes. Also produced are fission gamma rays and additional 2.224-MeV gamma rays from neutron capture in polyethylene. Neutrons provide most of the dose for spheres less than 22 cm in radius; beyond that, source gamma rays are the major contributors, followed by gamma rays from capture reactions. By increasing the radius from 5 to 12 cm or from 20 to 37 cm, the total dose rate can be reduced by a factor of 10. A rule of thumb for neutron dose reduction is that 10 cm of polyethylene will reduce the neutron dose rate by roughly a factor of 10.

More effective shields can sometimes be obtained by adding boron, lithium, or lead to polyethylene. The addition of boron or lithium results in a lower capture gamma-ray dose than that provided by pure polyethylene; lead effectively attenuates the source and the capture gamma-ray flux because it is a heavy element. However, because neutrons provide most of the dose up to a radius of 22 cm, the addition of these materials has little effect until the shield becomes substantially thicker than 22 cm. Figure 12.11 shows the effects on the total dose of adding other materials to polyethylene; these effects are important only for shields thicker than 30 cm. Since boron- lithium-, or lead-loaded

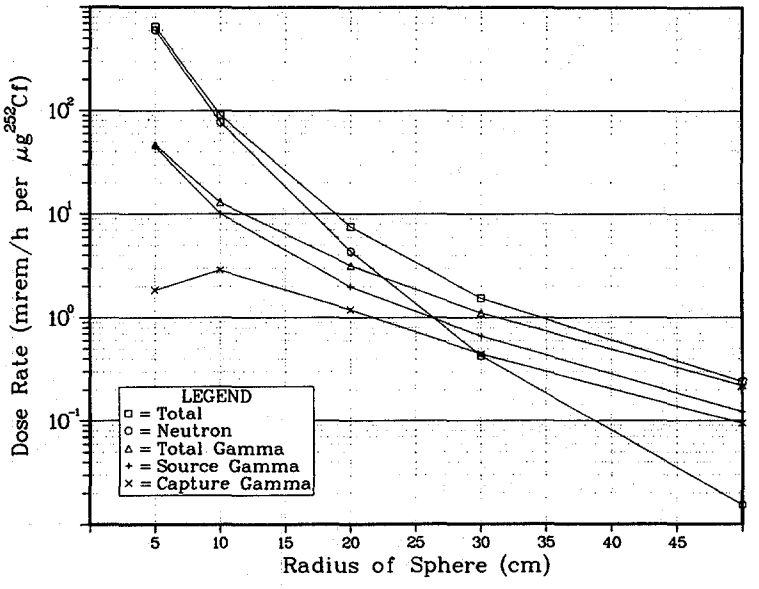

Fig. 12.10 *Various dose rates on a spherical surface from a point ^{252}Cf source in polyethylene. (Data supplied by G. E. Bosler, Los Alamos.)*

polyethylene is substantially more expensive than pure polyethylene, the additional cost is an important consideration.

12.7 TRANSPORT CALCULATIONAL TECHNIQUES

Neutron histories are difficult to determine because of the large number of different interactions possible in materials. This difficulty is further increased when the composition of matter changes frequently along the path of a neutron, as it often does throughout the volume of an assay instrument. Techniques for calculating the behavior or transport of neutrons and gamma rays in such circumstances are important for the design of assay instruments, the interpretation of measurements, and the development of shielding configurations. Two techniques for calculating the transport of neutrons in matter are described briefly in Sections 12.7.1 and 12.7.2.

12.7.1 Monte Carlo Techniques

The probability of a neutron interaction occurring is an important feature in the description of neutrons traveling through matter. Instead of trying to predict what an individual neutron may do, one can use procedures to predict what fraction of a large number of neutrons will behave in some manner of interest. Calculational techniques that, in simplistic terms, predict neutron events with "rolls of dice" (actually the generation of random numbers in a computer) are called Monte Carlo methods. The

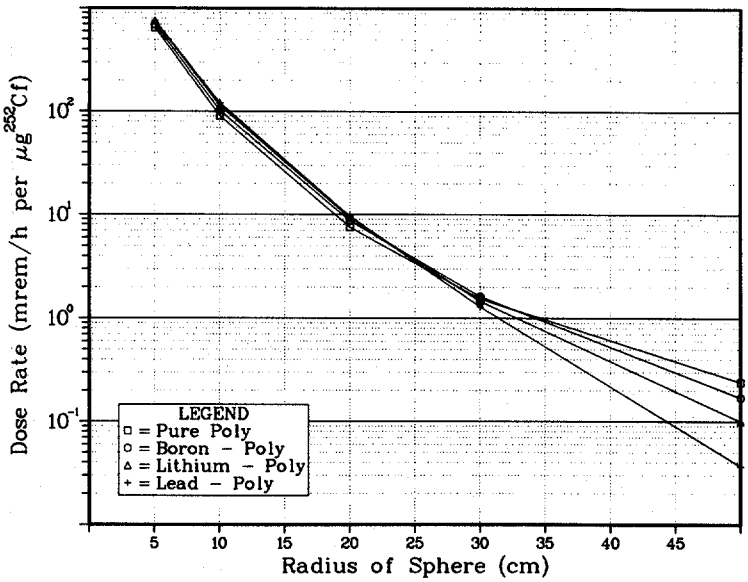

Fig. 12.11 *Total dose rate on a spherical surface from a point ^{252}Cf source in various materials. (Data supplied by G. E. Bosler, Los Alamos.)*

response of an assay system can often be calculated from the transport of many individual neutrons, despite the inclusion of a few improbable neutron histories that deviate drastically from the average behavior.

The Monte Carlo method can allow a detailed three-dimensional geometrical model to be constructed mathematically to simulate a physical situation. A neutron can be started at a selected location with a certain energy and direction. It travels distances that are consistent with the mean-free-path lengths in the materials, with random variations from the expected mean. At the end of each step in the neutron's path, a decision may be made to simulate a certain interaction, with the decision based on the cross section for the interaction with that material at that neutron energy. If an interaction is selected, the results of the interaction are simulated and its consequences followed. Eventually, a point is reached where no further interest in the neutron exists and its history is terminated. This might occur with the escape of the neutron or its moderation to very low energy. The neutron might be absorbed followed by the emission of a gamma ray of no interest, or it might undergo a multiplication event. If a multiplication event occurs, the histories of the new neutrons are followed. In principle, the history of a simulated neutron is one that might actually occur with a real neutron.

By repeating this procedure for many thousands of neutrons and by keeping tallies of how many enter a detector region, how many cause fissions, how many escape through a shield, or whatever else is of interest, an average behavior and its uncertainty are gradually deduced. Many specialized techniques may be used to get good average values with the fewest number of neutrons, but there are cases where even a fast computer cannot provide enough histories within the constraints of time and budget. Nonetheless,

Monte Carlo techniques provide essential assistance in design work by closely modeling the actual geometry of a problem and by having imaginary neutrons that simulate the motions and interactions of real ones. Examples of the results of Monte Carlo calculations are the shielding calculations in Figures 12.10 and 12.11 and the coincidence counter design calculations described in Chapters 14 and 17.

12.7.2 Discrete Ordinates Techniques

Analytical transport equations exist that describe the exact behavior of neutrons in matter. However, only approximate numerical solutions to these equations can be obtained for complicated systems. Procedures for obtaining these numeral solutions are classified as discrete ordinates techniques.

Some important differences distinguish discrete ordinates techniques from Monte Carlo techniques. Only one- or two-dimensional geometries are generally practical with a discrete ordinates process, and the neutrons are considered to be at discrete locations instead of moving freely through a three-dimensional geometry. In a two-dimensional discrete ordinates case, for example, it is as if the surface material were covered by a wire mesh and the neutrons existed only at the intersections of the wires. Furthermore, the energy of a neutron at any time must be selected from a finite set, in contrast to the continuously varying energy of a neutron in the Monte Carlo method.

Despite these disadvantages, discrete ordinates techniques can produce useful results in many cases. For problems involving large volumes and amounts of materials (such as reactor cores), the Monte Carlo technique can be too cumbersome and slow; a discrete ordinates solution might be feasible.

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