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Interaction Between Ionizing Radiation And Matter, Problems Photons

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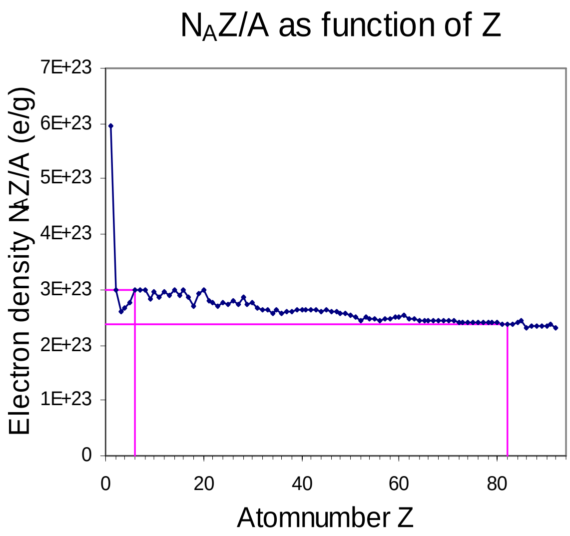
Problem 7.1

- Is the mass Compton attenuation/energy transfer coefficient larger in carbon or lead?

Solution: ${}_e\sigma \tilde{\propto} Z^0 \quad \left(\mathrm{cm}^2/_{\mathrm{electron}}\right)$ ←Independent of Z

$$\frac{{}_a\sigma}{\rho} = \frac{N_A Z}{A} \cdot {}_e\sigma \quad \left(\mathrm{cm}^2/_{\mathrm{g}}\right)$$

$$\frac{{}_a\sigma_{tr}}{\rho} = \frac{{}_a\sigma}{\rho} \frac{\bar{T}}{h\nu} \quad \left(\mathrm{cm}^2/_{\mathrm{g}}\right)$$



Carbon: $N_A Z/A=0.49954 N_A$
Lead: $N_A Z/A=0.39575 N_A$

$N_A=6.0022\ 10^{23}$

Problem 7.2

- Why is Rayleigh scattering not plotted in Fig. 7.16*a,b*, although quite significant in Fig. 7.13*a,b*?

Solution: $$\frac{{}_a\sigma_R}{\rho} \tilde{\propto} \frac{Z}{(h\nu)^2} \quad \left(\mathrm{cm}^2/\mathrm{g}\right)$$

$$\underline{\frac{{}_a\sigma_{R\,tr}}{\rho} = 0 \quad \left(\mathrm{cm}^2/\mathrm{g}\right)}$$

Problem 7.3

- On the basis of the K-N theory, what is the ratio of the Compton interaction cross section per atom for lead and carbon?

Solution: $${}_e\sigma \tilde{\propto} Z^0 \quad \left(\mathrm{cm}^2/\mathrm{electron}\right)$$

$${}_a\sigma = Z \cdot {}_e\sigma \quad \left(\mathrm{cm}^2/\mathrm{atom}\right)$$

$$\Downarrow$$

$$\frac{{}_{Pb}\sigma}{{}_C\sigma} = \frac{Z_{Pb} \cdot {}_e\sigma}{Z_C \cdot {}_e\sigma} = \frac{82}{6}$$

Problem 7.4

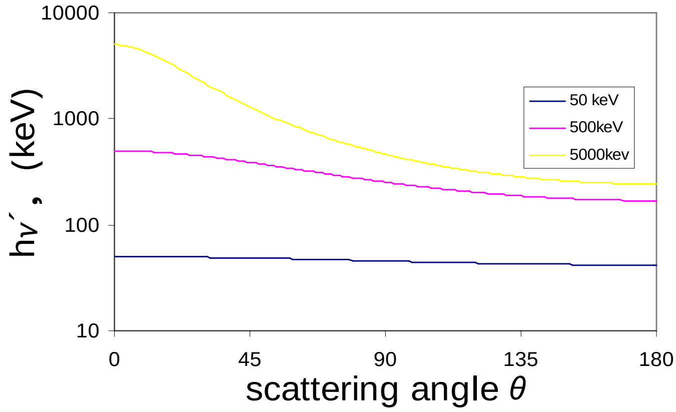
- Calculate the energy of the Compton-scattered photon at θ= 0°, 45 °, 90 ° and 180 for hν = 50 keV, 500 keV and 5 MeV.

Solution:

$$h\nu' = \frac{h\nu}{1+\left(\frac{h\nu}{m_e c^2}\right)(1-\cos\theta)}$$

hν′:

hν\θ	0°	45°	90°	180°	
50	50	48.60	45.54	41.82	keV
500	500	388.6	252.7	169.1	keV
5000	5000	1293	463.6	243.1	keV



Problem 7.5

- What are the corresponding energies and angels of the recoiling electrons for the cases in problem 4?

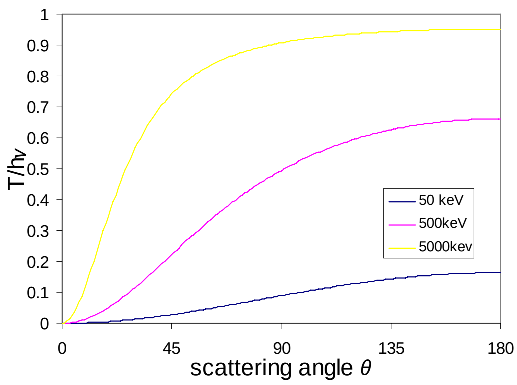
Solution: $$T = h\nu - h\nu', \quad \cot\varphi = \left(1+\frac{h\nu}{m_e c^2}\right)\tan\left(\frac{\theta}{2}\right)$$

T:

hν\θ	0°	45°	90°	180°	
50	0	1.393	4.456	8.183	keV
500	0	111.4	247.3	330.9	keV
5000	0	3707	4536	4757	keV

φ:

hν\θ	0°	45°	90°	180°	
50	90	65.55	42.33	0	keV
500	90	50.67	26.81	0	keV
5000	90	12.62	5.298	0	keV



Problem 7.6

- Calculate for 1-MeV photons the total K-N cross section from Eq.(7.15), and derive the Compton mass attenuation coefficient for copper in cm^2/g and m^2/kg.

Solution: $${}_e\sigma = 2\pi r_0^2 \left\{ \frac{1+\alpha}{\alpha^2}\left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha}\right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}$$

$$\alpha = h\nu / m_e c^2 = 1.96,\ r_0 = 2.818\cdot 10^{-13} cm,\ {}_e\sigma = \underline{\underline{2.112\cdot 10^{-25} cm^2 / electron}}$$

$$\frac{{}_a\sigma}{\rho} = \frac{N_A Z}{A}\cdot {}_e\sigma,\quad \frac{N_A Z_{Cu}}{A_{Cu}} = \frac{6.0022\ \cdot 10^{23} amu/g \cdot 29 electron}{63.55 amu} = 2.748\cdot 10^{23} electron/g$$

$$\left(\frac{{}_a\sigma}{\rho}\right) = 2.748\cdot 10^{23} electron/g \cdot 2.112\cdot 10^{-25} cm^2/electron$$

$$= \underline{\underline{0.05804 cm^2/g}} = 0.05804\cdot (10^{-2} m)^2 / 10^{-3} kg = \underline{\underline{0.005804 m^2/kg}}$$

Problem 7.7

- What is the maximum energy, what is the average energy, of the Compton recoil electrons generated by 20-keV and 20-MeV γ-rays?

Solution: $$T_{\max} = h\nu - h\nu'_{\min} = h\nu - \frac{h\nu}{1+\left(\frac{h\nu}{m_e c^2}\right)(1-\cos(180))} = \frac{h\nu}{\frac{m_e c^2}{2h\nu}+1}$$

$$T_{\max,20keV} = \underline{\underline{1.452 keV}}$$

$$T_{\max,20MeV} = \underline{\underline{19.75 MeV}}$$

Based on Eq.(7.20) page 134

$$\overline{T}_{20keV} = \underline{\underline{0.7210 keV}}$$

$$\overline{T}_{20MeV} = \underline{\underline{14.53 MeV}}$$

Problem 7.8

- Calculate the energy of a photoelectron ejected from the K-shell in tin by a 40-keV photon. Calculate τ_{tr}/ρ; you may estimate from fig. 7.15.

Solution: Tin: K-edge 29.20keV

$T = h\nu - T_b = (40-29.20)\text{keV} = 10.80\text{keV}$

$$\frac{{}_a\tau_{tr}}{\rho} = \frac{{}_a\tau}{\rho}\left(\frac{h\nu - P_K Y_K h\bar{\nu}_K - (1-P_K) P_L Y_L h\bar{\nu}_L}{h\nu}\right) \quad \left(\text{cm}^2/\text{g}\right)$$

Tin Z= 50 and from Fig 7.15: $P_L Y_L h\bar{\nu}_L \approx P_L Y_L (E_b)_{L1} \approx 0,\ P_K Y_K h\bar{\nu}_K = 19 keV$

$$\left(\frac{{}_a\tau}{\rho}\right)_{\text{Sn,40keV}} = 18.9\text{cm}^2/\text{g} \Rightarrow \left(\frac{{}_a\tau_{tr}}{\rho}\right)_{\text{Sn,40keV}} = 18.9\text{cm}^2/\text{g}\left(\frac{40-19-0}{40}\right) = \underline{\underline{9.92\text{cm}^2/\text{g}}}$$

Problem 7.9

- What is the average energy of the charged particles resulting from pair production in (a) the nuclear field (b) the electron field, for photons of hν = 2 and 20 MeV?

Solution: a) $\quad pair: \quad h\nu_{\min} = 2m_e c^2 \qquad \bar{T} = \dfrac{h\nu - 2m_e c^2}{2}$

b) $\quad trip: \quad h\nu_{\min} = 4m_e c^2 \qquad \bar{T} = \dfrac{h\nu - 2m_e c^2}{3}$

$$h\nu = 2\text{MeV}: \quad \bar{T}_{\text{pair}} = 0.489\,\text{MeV}, \quad \bar{T}_{\text{trip}} = 0$$

$$h\nu = 20\text{MeV}: \quad \bar{T}_{\text{pair}} = 9.49\,\text{MeV}, \quad \bar{T}_{\text{trip}} = 6.33\,\text{MeV}$$

Problem 7.10

- A narrow beam containing 10^{20} photons at 6 MeV impinges perpendicularly on a layer of lead 12 mm thick, having a density 11.3 g/cm^3. How many interactions of each type (photoelectric, Compton, pair, Rayleigh) occur in the lead?

  Solution:

  Number of interactions

  $\Delta N_{tot} = N_0(1-e^{-\Delta x\,\mu})$

  $\Delta N'_{tot} = N_0 \Delta x\,\mu = N_0 \Delta x\,(\tau_{p.el}+\sigma_C+\kappa_{pair}+\sigma_R)$

  $= \Delta N'_{p.el}+\Delta N'_C+\Delta N'_{pair}+\Delta N'_R > \Delta N_{tot}$

  $\Delta N_{p.el} = \Delta N'_{p.el} \cdot \Delta N_{tot} / \Delta N'_{tot}$

Photoelectric	Compton	Pair	Rayleigh
9.999E+17	1.792E+19	2.541E+19	5.375E+16