Radiation and radiation dosimetry
Spring 2006
Introduction

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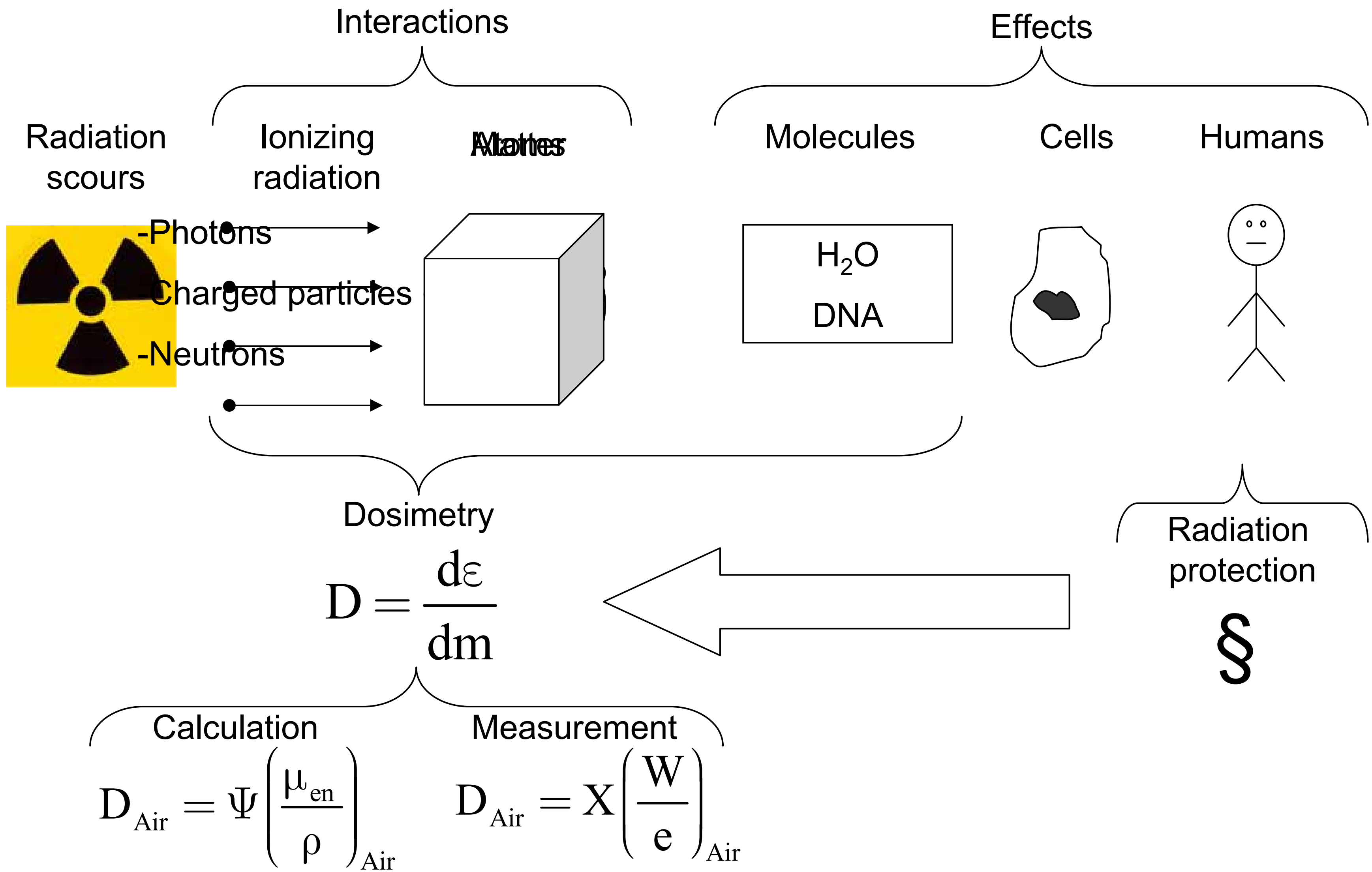
Contents

- Interaction between ionizing radiation and matter
- Radioactive and non-radioactive radiation sources
- Calculations and measurement of radiation doses (dosimetry)
- The effect of radiation on relevant substances like water and important biological molecules
- The understanding of:
  - the biological effects of ionizing radiation
  - measurement principles and methods
  - the principles of radiation protection

Learning objectives

- To understand primary and secondary effects of ionizing radiation
- How radiation doses are calculated and measured
- The understanding of the principles of radiation protection, their origin and applications
- This will provide a tool for evaluating possible dangers in the use of ionizing radiation

Overview



$$D = \frac{d\varepsilon}{dm}$$

$$D_{Air} = \Psi \left(\frac{\mu_{en}}{\rho} \right)_{Air}$$

$$D_{Air} = X \left(\frac{W}{e} \right)_{Air}$$

Interaction Between Ionizing Radiation And Matter, Part 1 Photons

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Photon Interaction

- Five interaction processes between photons and matter:
  - Rayleigh scattering
  - Compton scattering
  - Photoelectric effect
  - Pair- and triplet -production
  - Photon-nuclear reactions
- Probability of interaction described by cross section
- Scattering and energy transfer described kinematics
- Joint gives the possibility to calculate radiation doses

Scattering (Rayleigh scattering, Compton scattering)

Absorption (Photoelectric effect, Pair- and triplet -production, Photon-nuclear reactions)

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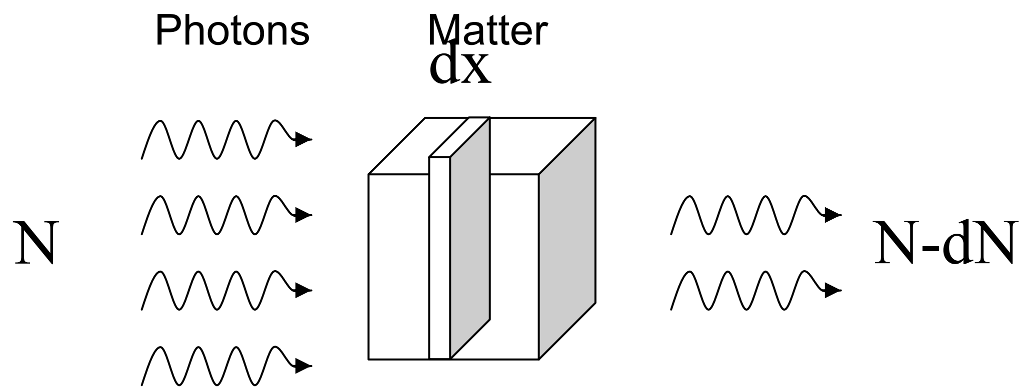
Rayleigh/Coherent scattering

- Scattering of photons without loss of energy
- Photons absorbed by the atom, then emitted with a small angel
- Depend on photon energy, hν, and atomic structure
- The atomic cross section of coherent scattering:

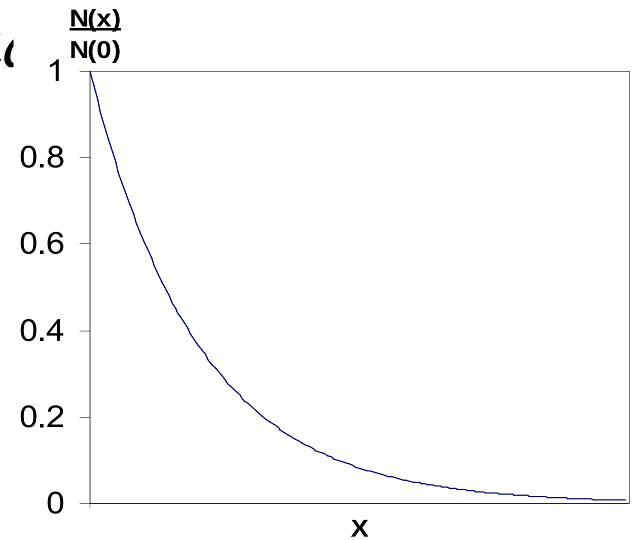
$$\sigma_R \propto \left(\frac{Z}{h\nu}\right)^2$$

- Special case hν → 0 : Thomson scattering

Photon attenuation



- Probability of a photon interaction: μdx
- Number of photons interacting : $N\mu$



$$dN = N\mu dx \quad \Rightarrow \quad \int \frac{dN}{N} = \int \mu dx$$

$$\Rightarrow \quad N = N_0 e^{-\mu x}$$

Average pathlength

- The probability of a photon not interacting: $e^{-\mu x}$
- Normalized probability:

$$p_{ni} = Ce^{-\mu x}, \;\Rightarrow\; \int_0^\infty Ce^{-\mu x} \equiv 1, \;\Rightarrow\; \qquad p_{ni} = \mu e^{-\mu x}$$

Average pathlength: $\langle x \rangle = \int_0^\infty x\mu e^{-\mu x} = \frac{1}{\mu}$

Attenuation - Cross section

- μ: Denote the number of photons with a single energy E and direction which interacts per length unit
- $\mu = p/dx$, probability of per length unit; macroscopic
- σ: Cross section target area; surface proportional with the probability of interaction
- $\sigma = p/n_v dx$, σ: cross section, probability per atom density n_v, and length unit; microscopic
- $\mu = \rho(N_A/A)\sigma$, ρ: mass density, (N_A/A): number of atoms per mass unit

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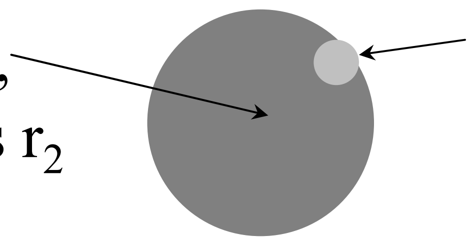
Cross section

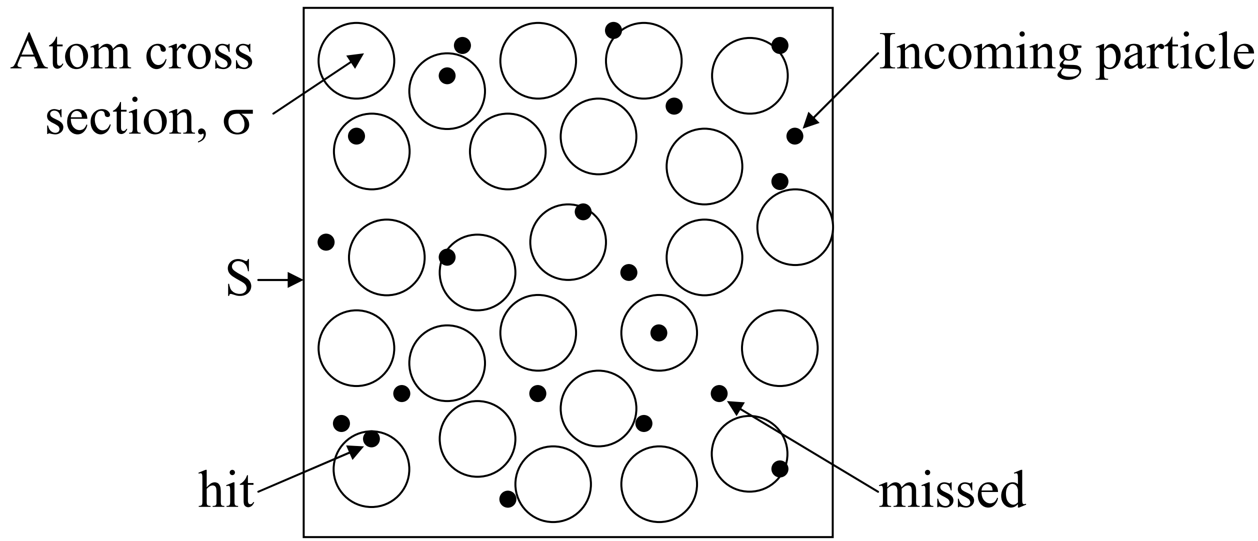
- Look at two spheres:



- σ equals total area: $\pi(r_1+r_2)^2$

Cross section (2)

- N particles move towards an area S with n atoms



- Probability of interaction: $p=S_{cs}/S = n\sigma/S$
- Number of interacting particles: $Np= Nn\sigma/S$

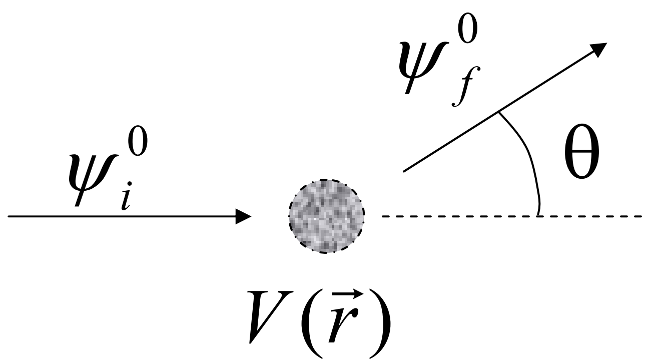
Cross section (3)

- The cross section of an interaction depend on:
  - Type of target (nucleus, electron, ..)
  - Type of incoming particle
  - Energy of the incoming particle
  - Distance between target and particle
- Cross section calculated with quantum mechanics - visualized in a classical window

Calculating Rayleigh Cross section

- A wave beam interacts with a weak potential $V(\vec{r})$
- The Hamiltonian is: $H = \frac{1}{2m} p^2 + V(\vec{r}) = H_0 + V(\vec{r})$
- Free particle wave function: Initial: $\psi_i^0 = \sqrt{V_0}\, e^{i(\vec{p}_i \vec{r} - E_i t)/\hbar}$

Final: $\psi_f^0 = \sqrt{V_0}\, e^{i(\vec{p}_f \vec{r} - E_f t)/\hbar}$



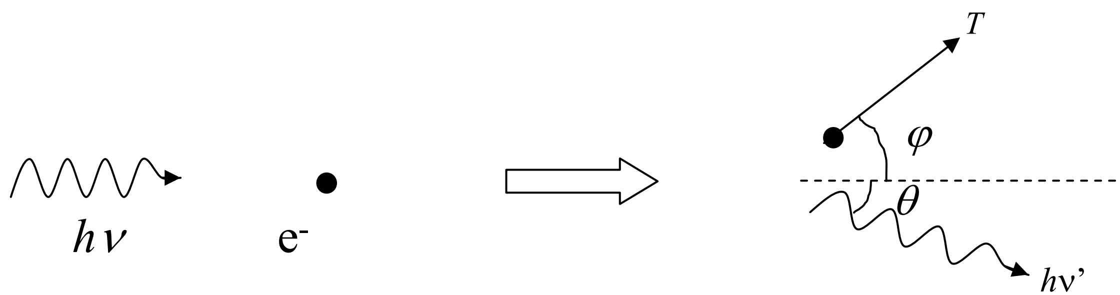
$$\frac{d\sigma}{d\Omega} = \left| \frac{m}{2\pi\hbar^2} \int d^3 r V(\vec{r}) e^{i(\vec{p}_i - \vec{p}_f)\vec{r}/\hbar} \right|^2$$

What is the probability of elastic scattering by an angel θ of a photon of energy hν?

$$\xrightarrow[V = Ze^2/4\pi\varepsilon_0 r]{Coulombfield} \left(\frac{Ze^2}{16\pi\varepsilon_0 h\nu} \right)^2 \frac{1}{\sin^4(\theta/2)}$$

Incoherent scattering

- The photon energy loss due to the interaction is significant
- The interaction is a photon-electron scattering, assuming the electron being free (binding energy neglect able)
- Also called Compton scattering



Compton scattering(1)

- Kinematics:

$$h\nu = h\nu' + T$$ Energy conservation

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\varphi + p\cos\theta$$

Momentum conservation

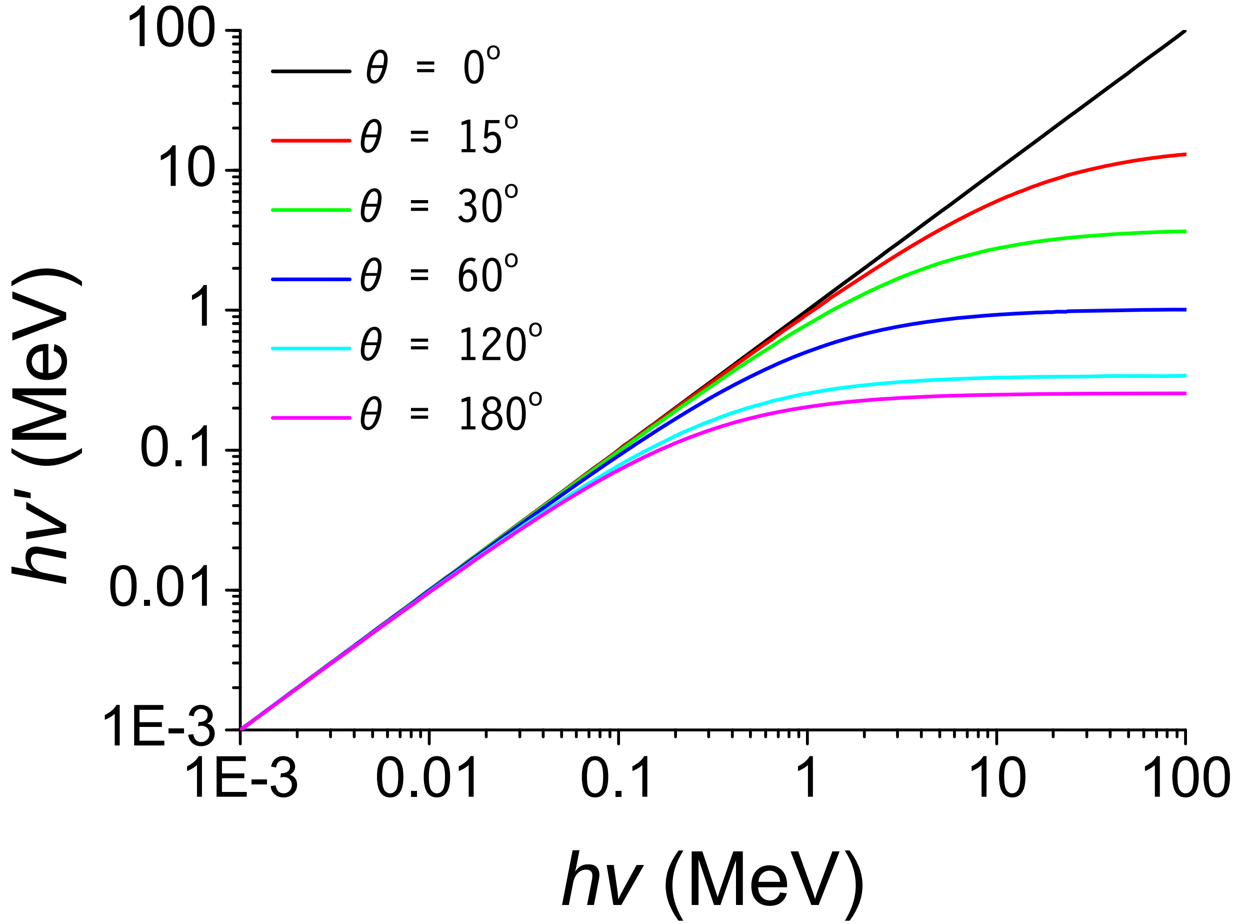
$$\frac{h\nu'}{c}\sin\theta = p\sin\varphi$$

$$(pc)^2 = T^2 + 2Tm_ec^2$$ "Law of invariance"

- Solution:

$$h\nu' = \frac{h\nu}{1+\left(\frac{h\nu}{m_ec^2}\right)(1-\cos\theta)}, \quad T = h\nu - h\nu', \quad \cot\varphi = \left(1+\frac{h\nu}{m_ec^2}\right)\tan\left(\frac{\theta}{2}\right)$$

Compton scattering(2)

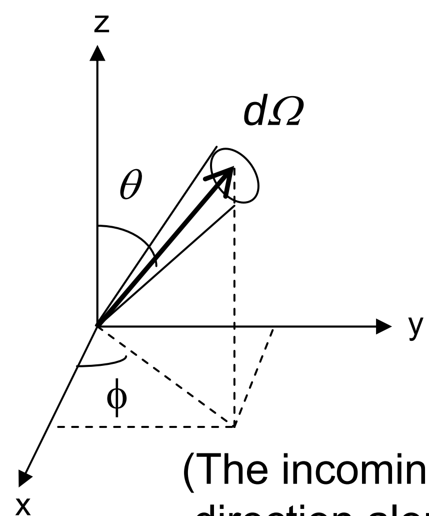


Compton scattering-Cross section

- Klein and Nishina derived the cross section of the Compton scattering
- The differential cross section for photon scattering at angel θ, per unit solid angel and per electron, may be written as.

$$\left(\frac{d_e\sigma}{d\Omega_\theta}\right)=\frac{r_0^2}{2}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'}+\frac{\nu'}{\nu}-\sin^2\theta\right)$$

$$d\Omega_\theta=\sin\theta d\theta d\phi,\ \ r_0=\frac{e^2}{m_ec^2}$$



(The incoming photon direction along the z-axis)

r_0: classic electron radius; distance between two electrons with potential energy equal electron rest mass energy; $m_ec^2 = e^2/4\pi\varepsilon_0 r_0$

Compton scattering-Cross section(2)

- The cylinder symmetry gives:

$$\left(\frac{d_e\sigma}{d\theta}\right) = \pi r_0^2 \left(\frac{\nu'}{\nu}\right)^2 \left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin^2\theta\right)\sin\theta$$

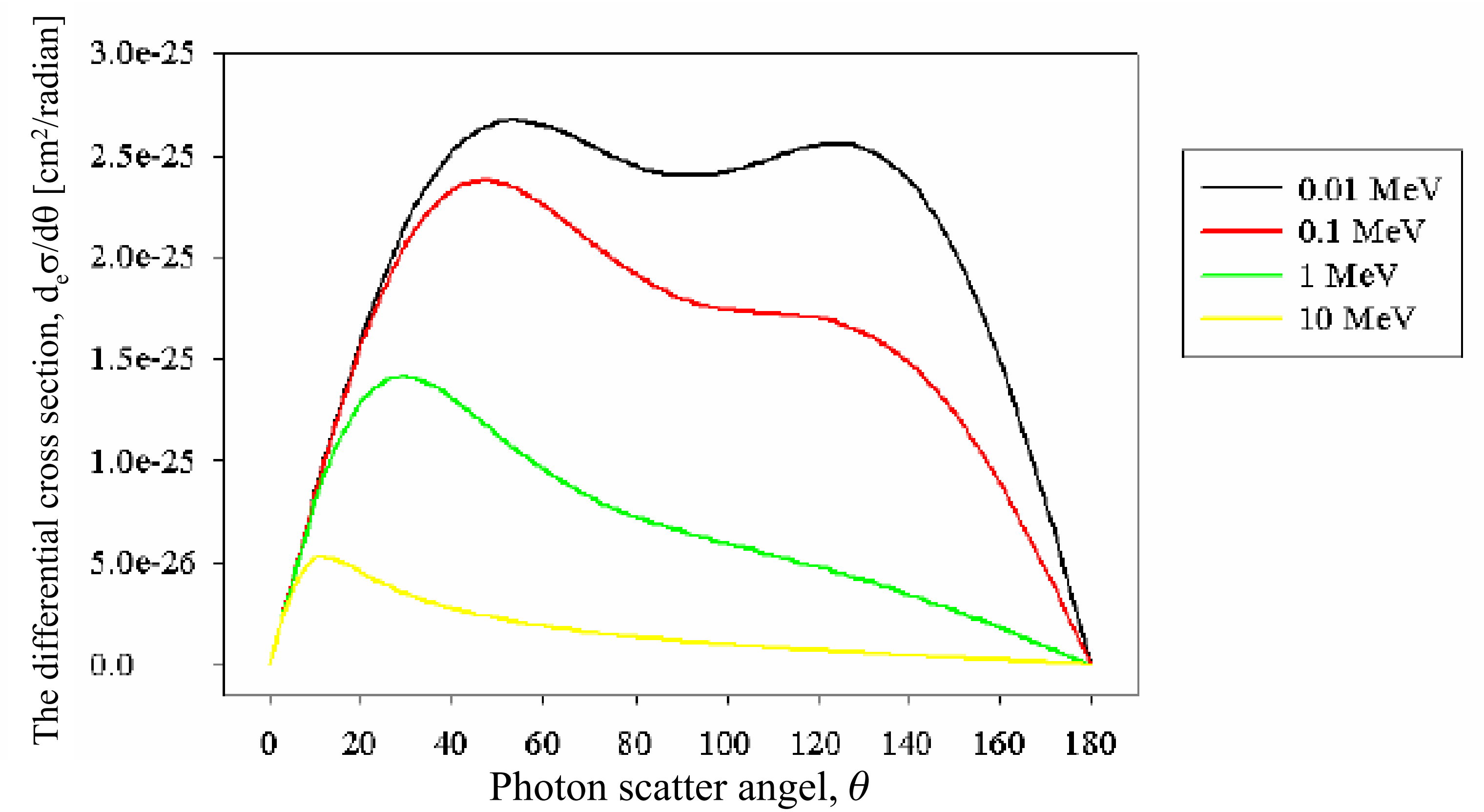
- Denotes the probability of finding a scattered photon inside the angel interval [θ+dθ] after the interaction with the electron
- The total cross section per electron $_e\sigma$ is:

$$_e\sigma = \int_0^{\pi} \pi r_0^2 \left(\frac{\nu'}{\nu}\right)^2 \left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin^2\theta\right)\sin\theta d\theta$$

- Atomic C. scat. cross section is then: $_a\sigma = Z_e\sigma$

Compton scattering-Cross section(3)

- Scattered photons are more forward directed as initial energy increase



Compton scattering-Cross section(4)

- The photon spectra of the scattered photons:

$$\frac{d_e\sigma}{d(h\nu')} = \frac{d_e\sigma}{d\Omega_\theta}\frac{d\Omega_\theta}{d(h\nu')} = \frac{d_e\sigma}{d\Omega_\theta} 2\pi \sin\theta \frac{d\theta}{d(h\nu')}$$

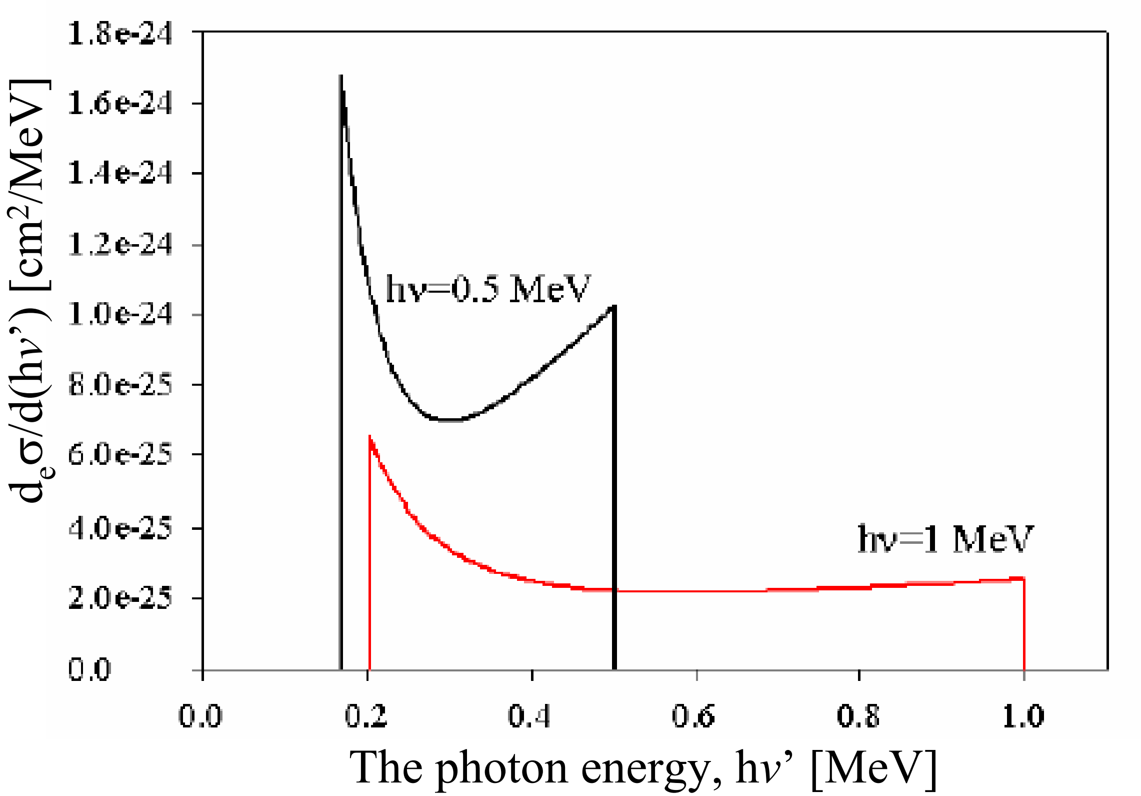
$$h\nu' = \frac{h\nu}{1+\left(\frac{h\nu}{m_e c^2}\right)(1-\cos\theta)}$$

$$\Downarrow$$

$$\frac{d_e\sigma}{d(h\nu')} = \frac{\pi r_0^2 m_e c^2}{(h\nu)^2}\left(\frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} - 1 + \left(1 - \frac{m_e c^2}{h\nu'} + \frac{m_e c^2}{h\nu}\right)^2\right)$$

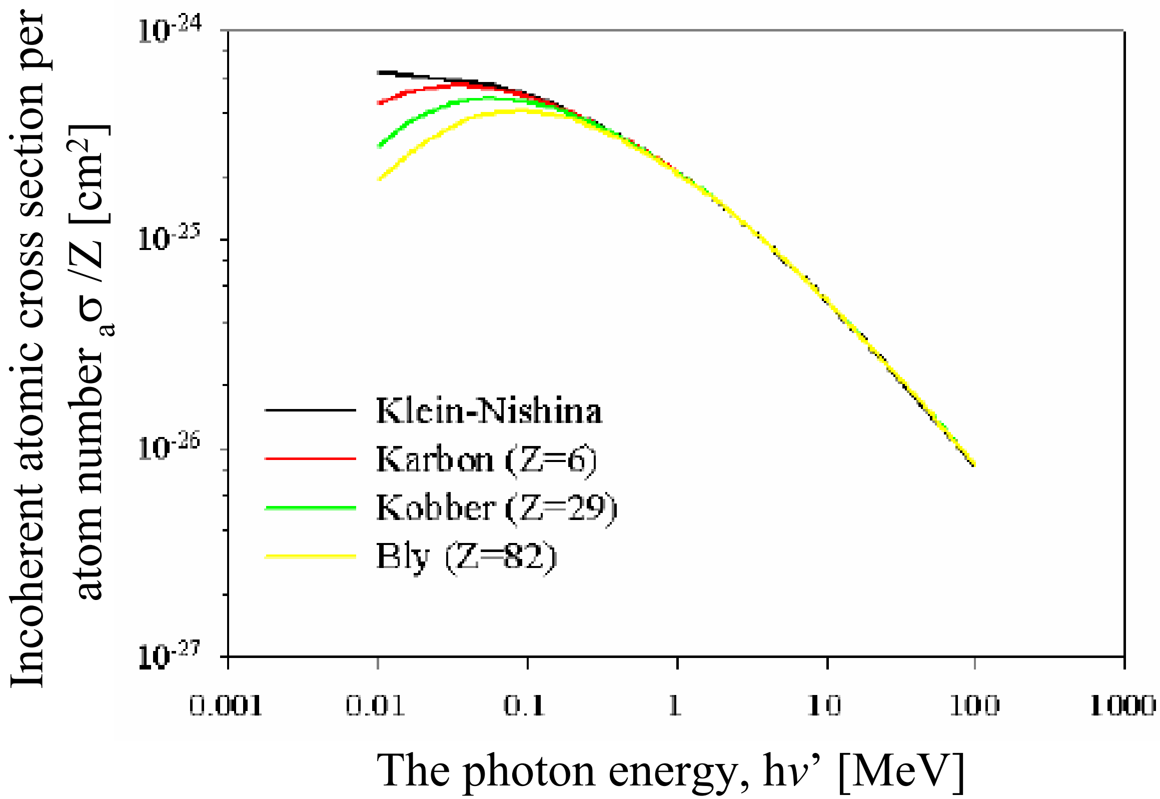
Compton scattering-Cross section(5)

- The cylinder symmetry gives:



Compton scattering-Cross section(6)

- More correct treatment of the cross section gives a small atom number dependence:



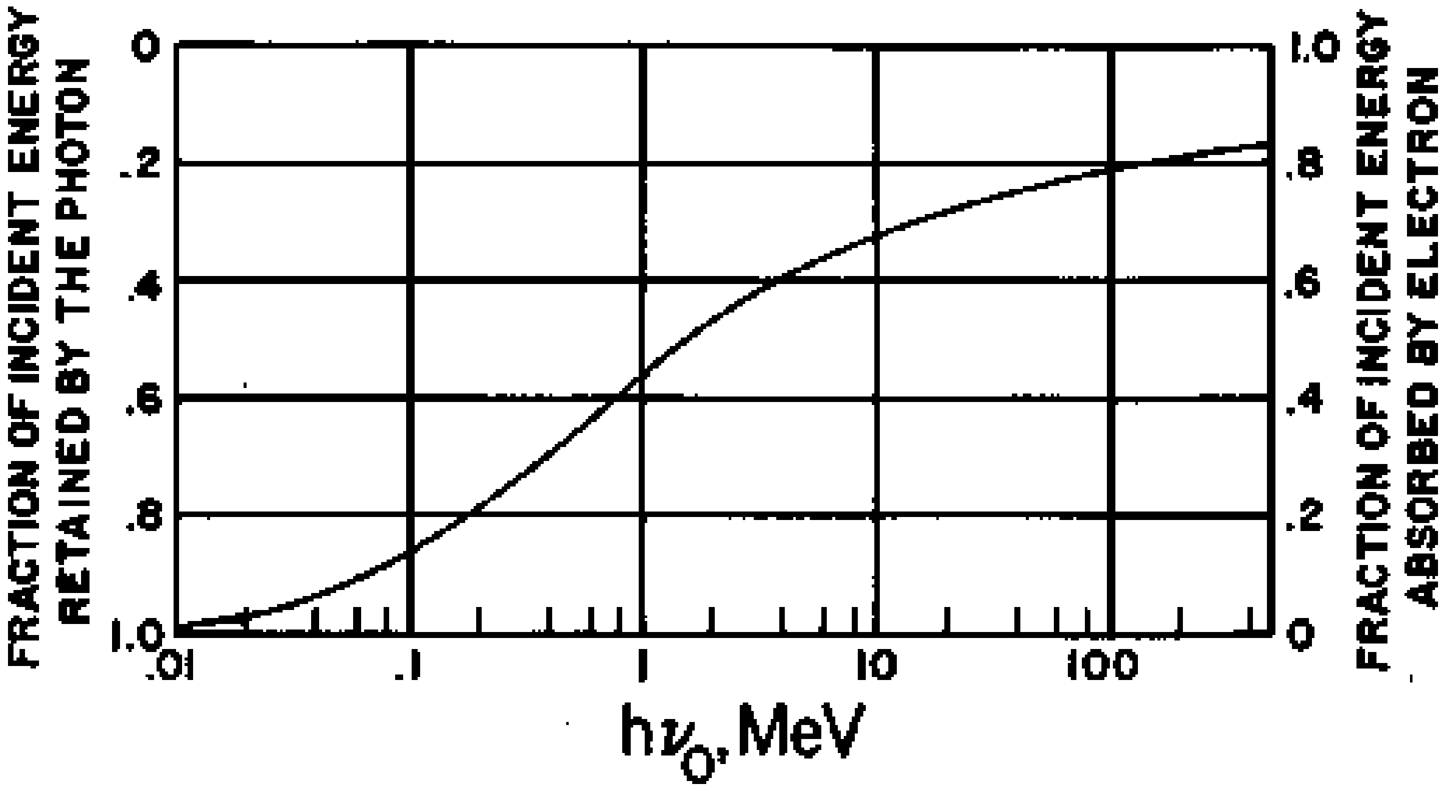
Energy transferred

- The energy transferred to the electron in a Compton process:
  $T = h\nu - h\nu'$
- The energy-transfer cross section:
  $$\frac{d_e\sigma_{tr}}{d\Omega_\theta} = \frac{d_e\sigma}{d\Omega_\theta}\frac{T}{h\nu} = \frac{d_e\sigma}{d\Omega_\theta}\frac{h\nu - h\nu'}{h\nu}$$
- The average fraction of transferred energy:

$$\bar{T} = h\nu \frac{{}_e\sigma_{tr}}{{}_e\sigma}$$

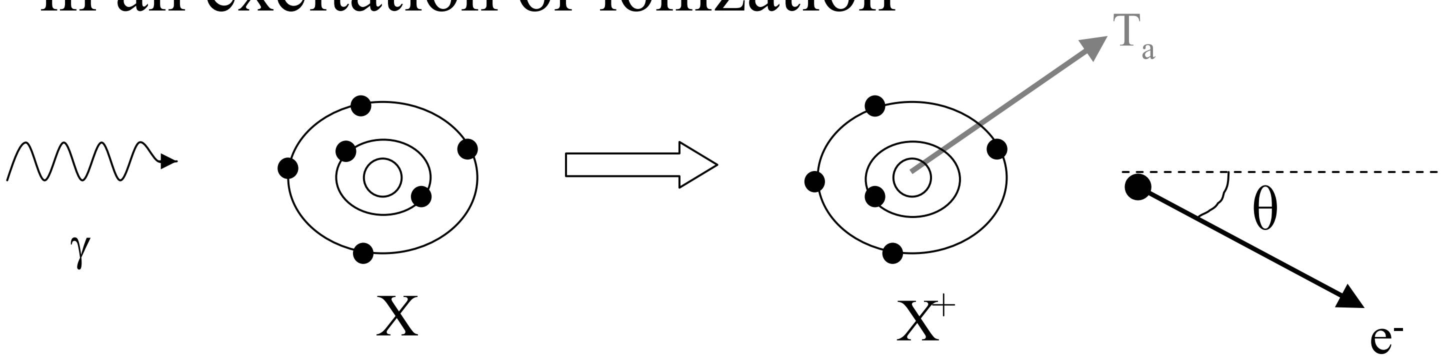
Energy transferred(2)

- Mean fraction of energy transferred to the electron:

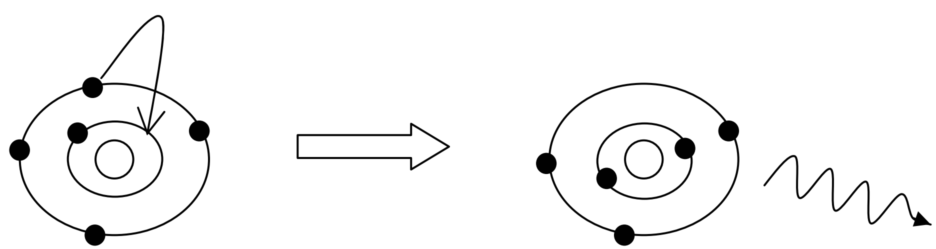


Photoelectric effect

- Photon is absorbed by an atom/molecule; resulting in an excitation or ionization



- The vacancy is filled by an electron from an outer orbit and characteristic radiation is emitted



Photoelectric effect (2)

- The binding energy of the electron E_b most be accounted for:

$$T = h\nu - E_b - T_a \simeq h\nu - E_b$$

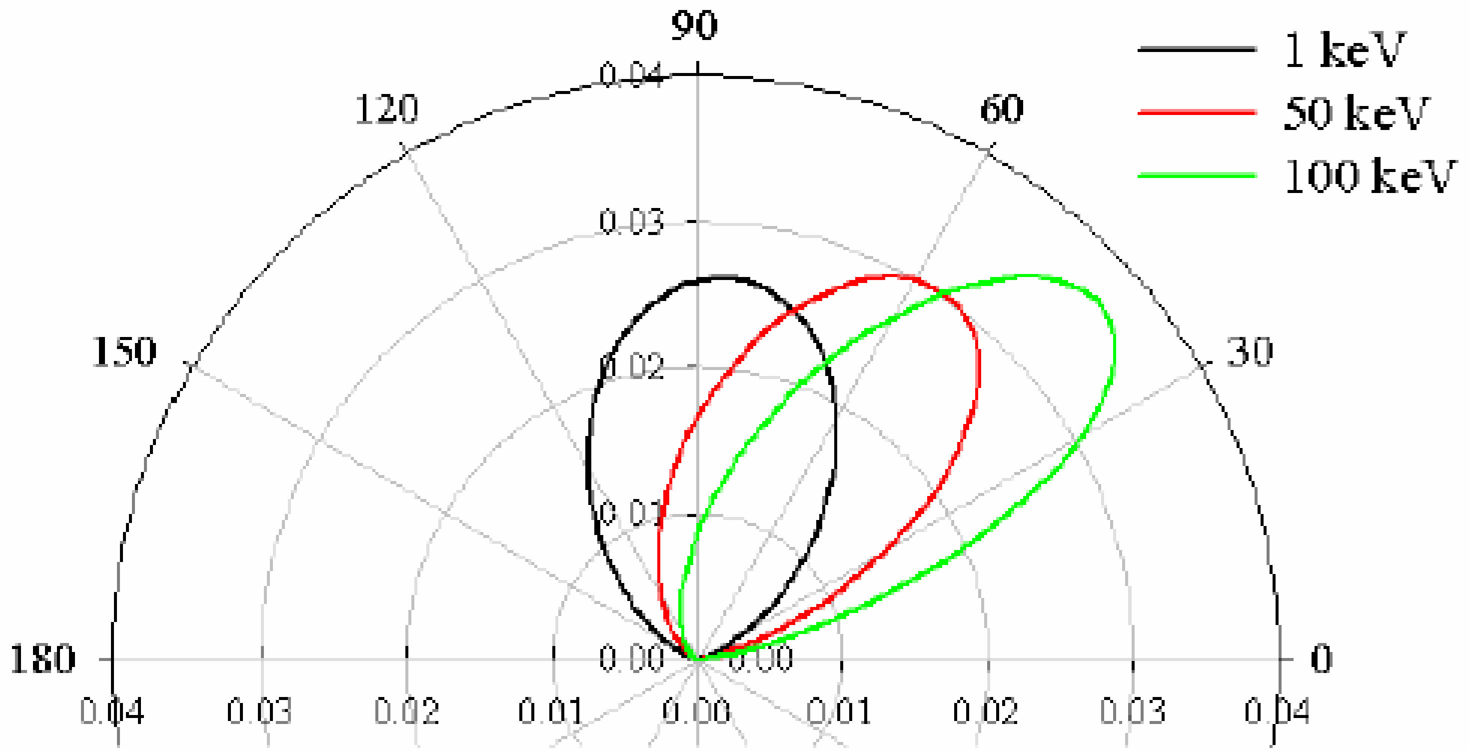
- The atomic cross section τ is approximately when E_b=0 assumed:

$$\frac{d\tau}{d\Omega} = 2\sqrt{2} r_e^2 \alpha^4 Z^5 \left(\frac{m_e c^2}{h\nu}\right)^{7/2} \sin^2\theta \left(1 + 4\sqrt{\frac{2h\nu}{m_e c^2}}\cos\theta\right)$$

α: Fine structure constant
Ω: Points to the emitted electron

Photoelectron – distribution angle

Photoelectric cross section $(d\tau/d\theta)/\tau$



Characteristic radiation

- The energy of characteristic radiation depend on the electron structure- and transitions
- The K- and L-shell vacancies: photons with energy $h\nu_K$ and $h\nu_L$ are emitted after de-excitation
- Emitted photons are isotropic distributed
- The fraction of events that occur in the K- or L-shell: $P_K\ [h\nu>(E_b)_K]$ and $P_L\ [h\nu>(E_b)_L]$
- The probability of c.r. being emitted: Y_K and Y_L
- The energy transport away from the atom by c.r.: $P_K Y_K h\nu_K + P_L Y_L h\nu_L$

Auger effect

- Alternative path which the ionized atom dispose energy
- Shallow outer-shell vacancies are emitted from the atom with kinetic energy corresponding to its excess energy
- Low Z: most Auger
- High Z: most characteristic radiation
- Auger electrons are low-energetic

Photoelectric cross section

- It is observed: $\tau \propto \frac{Z^n}{(h\nu)^m}, 4 < n < 5, 1 < m < 3$
- The fraction of energy transferred to the photoelectron

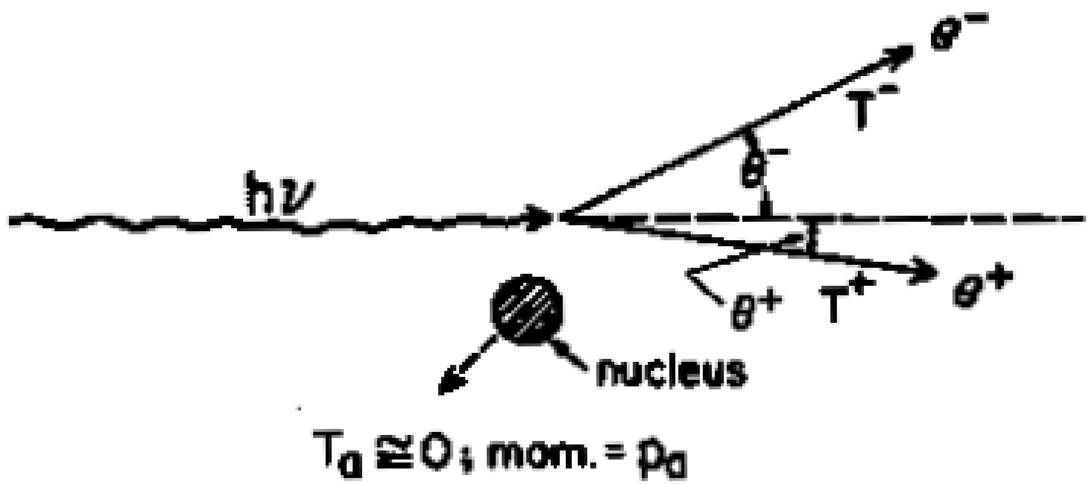
$$\frac{T}{h\nu} = \frac{h\nu - E_b}{h\nu}$$

- But: Auger electrons are also given energy
- The energy-transfer cross section of the electron:

$$\tau_{tr} = \tau \frac{\left(h\nu - P_K Y_K h\nu_K - \left(1 - P_K\right) P_L Y_L h\nu_L\right)}{h\nu}$$

Pair production

- Photon absorption when an electron-positron pair is created
- Occurs in a Coulomb force field from an atom nucleus or atomic electron (triplet production)



Pair production (2)

- Conservation of energy:
  $$h\nu = 2m_e c^2 + T^+ + T^-$$
- Average kinetic energy after absorption:
  $$\bar{T} = \frac{h\nu - 2m_e c^2}{2}$$
- Estimate of scatter angle of electron/positron:
  $$\bar{\theta} \approx \frac{m_e c^2}{\bar{T}}$$
- Total cross section:
  $$\kappa \approx \alpha r_0^2 Z^2 \bar{P}$$ $\bar{P}$ increases with $h\nu$

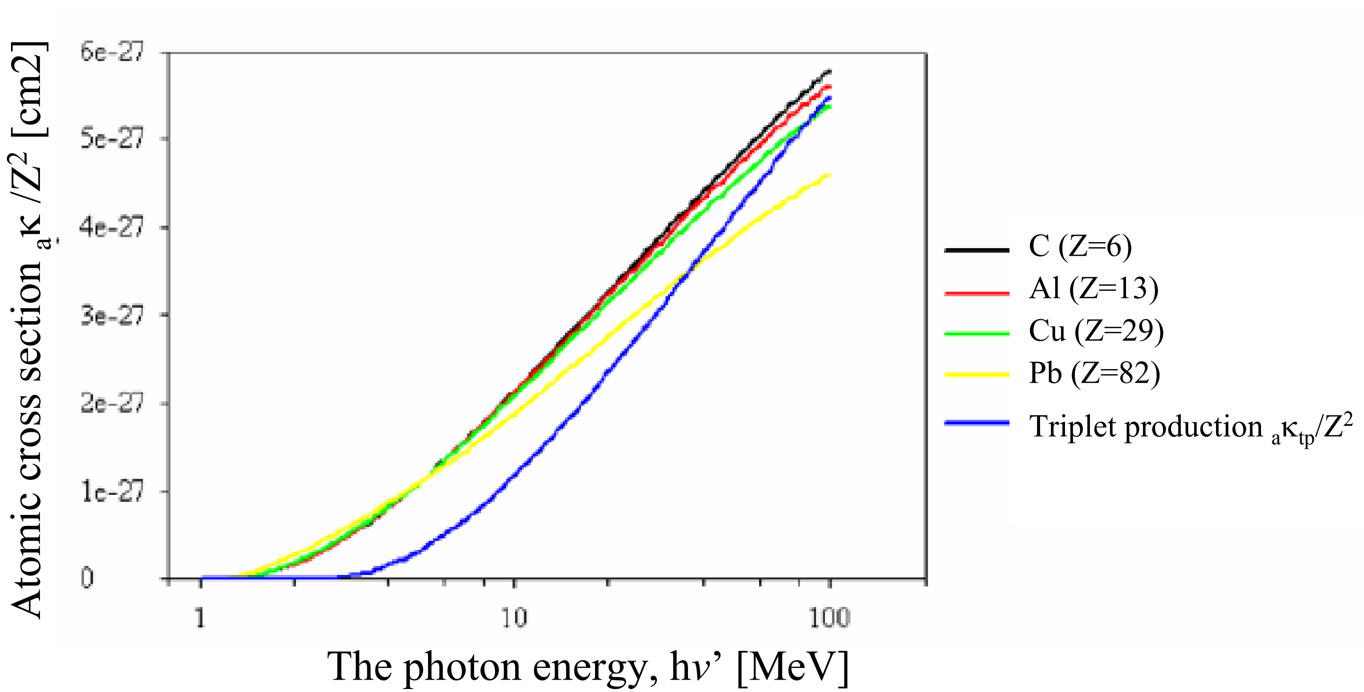
Triplet production

- An electron-positron pair is created in a field from an electron – conservation of energy:
  $h\nu = 2m_e c^2 + T^+ + T_1^- + T_2^-$
- Note that the atomic electron can gain significant kinetic energy
- Average kinetic energy after absorption:
  $$\bar{T} = \frac{h\nu - 2m_e c^2}{3}$$
- Threshold photon energy: h$\nu = 4m_0c^2$

Pair- and triplet production

- Pair production most important:

$$\frac{\kappa(triplet)}{\kappa(pair)} \approx \frac{1}{CZ} \;,\; 1 < C < 2, \text{ depending only on } h\nu$$



Photonuclear interactions

- Photon (energy above a few MeV) excites a nucleus
- Proton or neutron is emitted
- (γ, n) interactions may lead to radiation protection problems
- Example: Tungsten W (γ, n)
- Not important in dosimetry

Attenuation coefficients

- Total coefficient of photon interaction:

$$\frac{\mu}{\rho} = \frac{\tau}{\rho} + \frac{\sigma}{\rho} + \frac{\kappa}{\rho} + \frac{\sigma_R}{\rho}$$

- Coefficient of energy transfer to electrons:

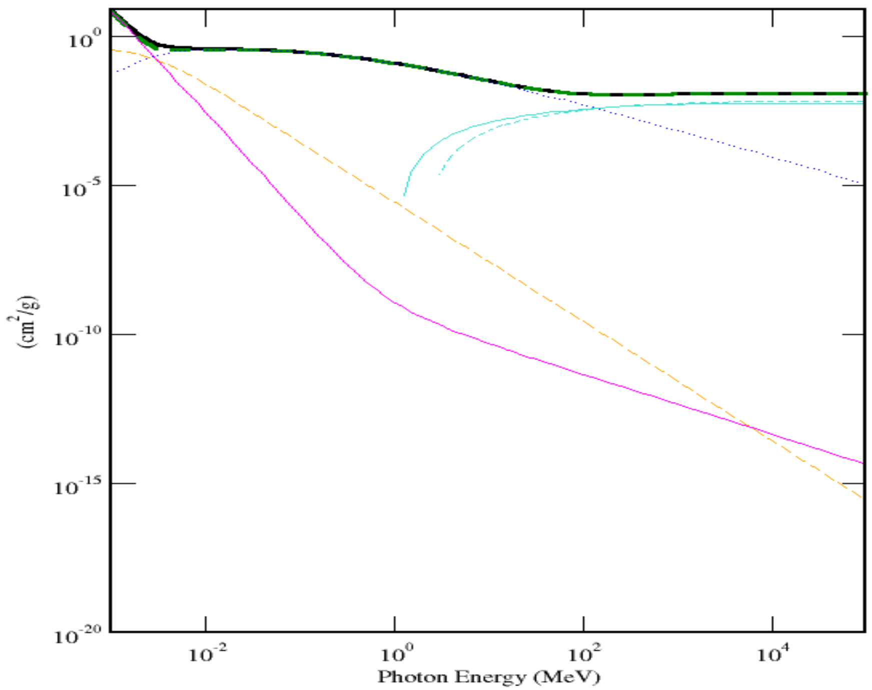
$$\frac{\mu_{tr}}{\rho} = \frac{\mu}{\rho} \frac{\overline{T}}{h\nu}$$

- Braggs rule for mixtures of n-atoms/elements:

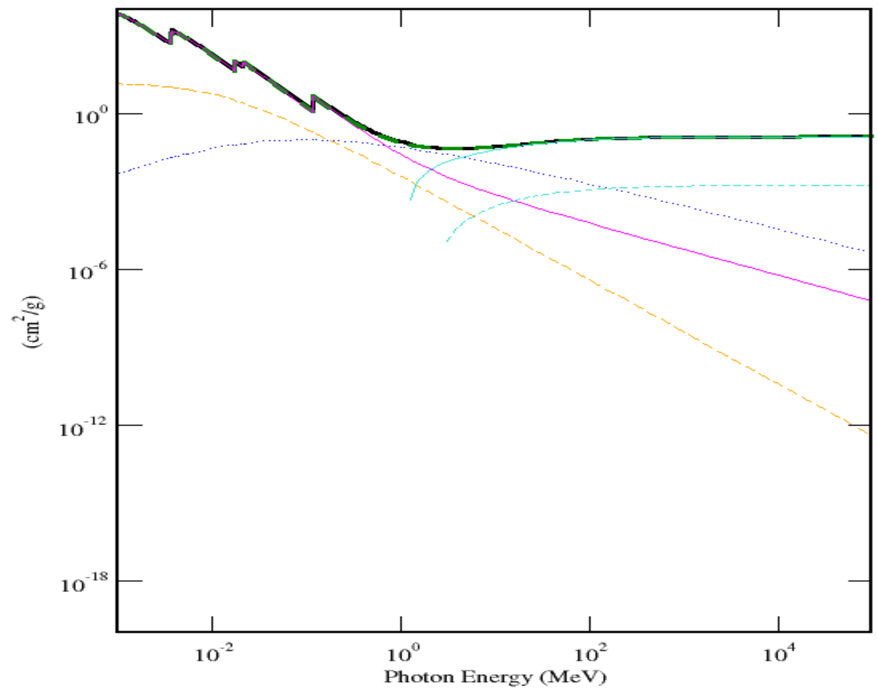
$$\left(\frac{\mu}{\rho}\right)_{mix} = \sum_{i=1}^{n} f_i \left(\frac{\mu}{\rho}\right)_i, \qquad f_i = \frac{m_i}{\sum_{i=1}^{n} m_i}$$

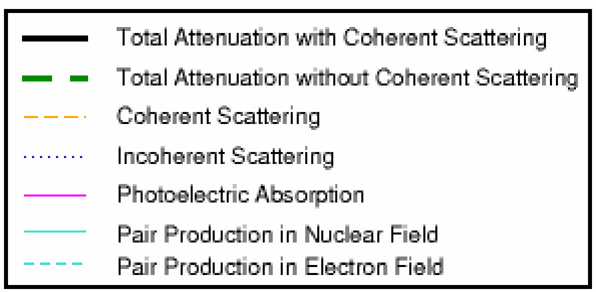
Attenuation coefficients (2)

Hydrogen



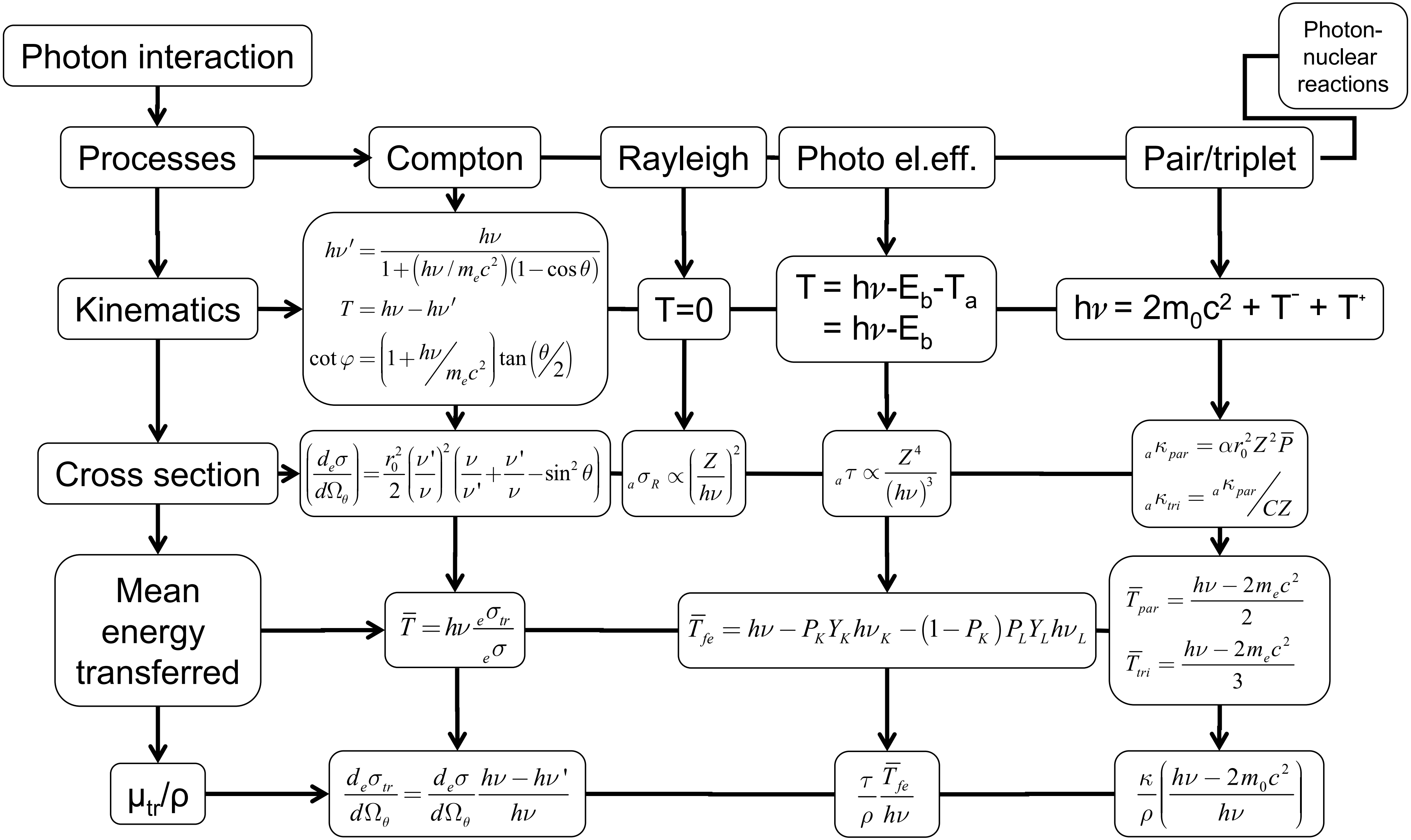
Uranium



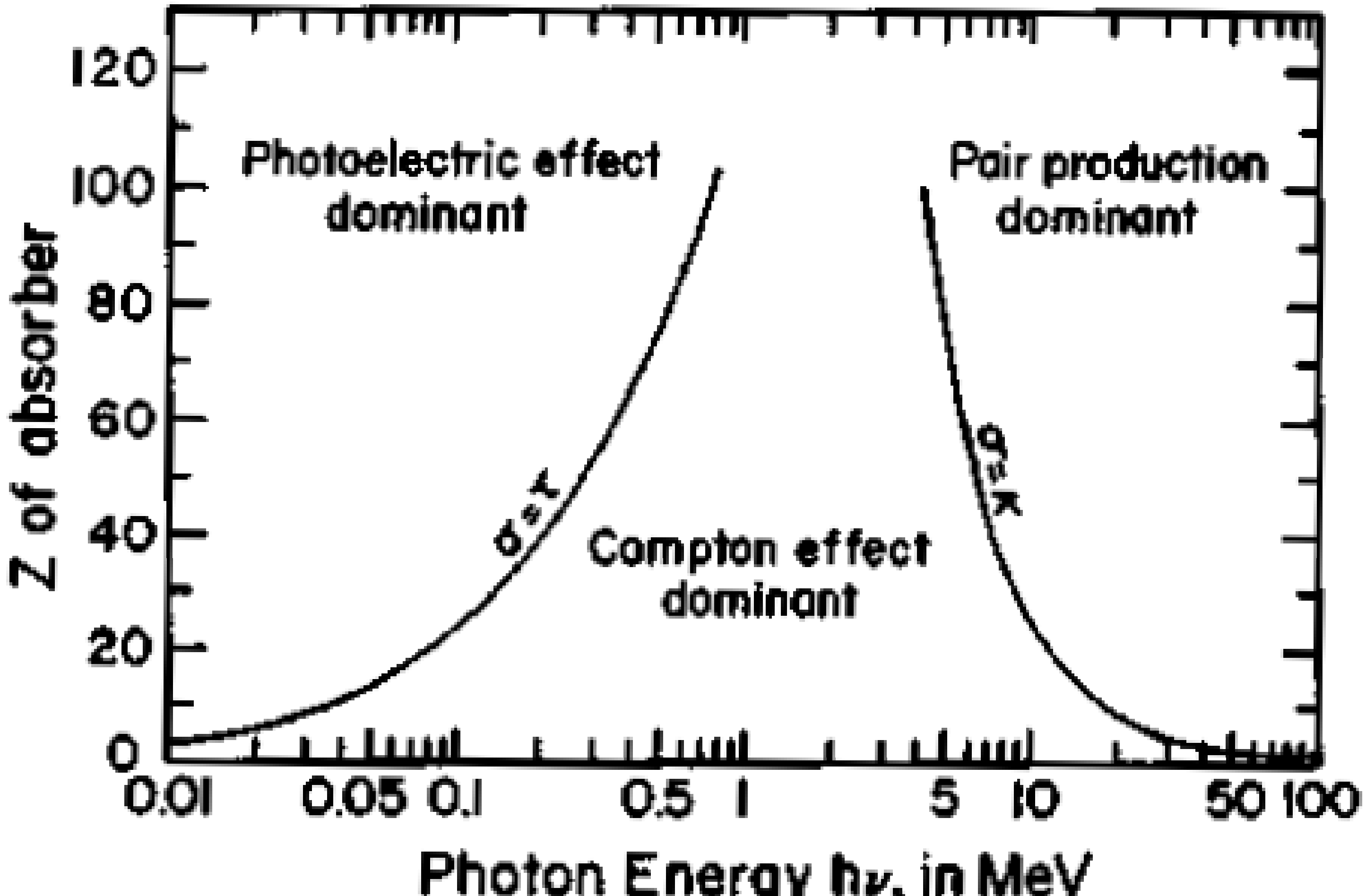


http://physics.nist.gov/PhysRefData/Xcom/Text/XCOM.html

Photon Interaction Summary



Summary


Photoelectric effect dominant
Pair production dominant
Compton effect dominant
σ = τ
σ = κ
Z of absorber
0
20
40
60
80
100
120
0.01
0.05
0.1
0.5
1
5
10
50
100
Photon Energy hν, in MeV