FYS-MENA3110 Problem set #1

Problem 1.1:

Show that $zz*$ and $z + z*$ are real when $z = a + bi$ where a and b are real (* means complex conjugate).

Problem 1.2:

Show that the eigenvalues of a Hermitian operator are real.

Problem 1.3:

Show that the eigenvectors that correspond to different eigenvalues of a Hermitian operator are orthogonal.

Problem 1.4:

Consider a particle of mass m confined in an infinite one-dimensional potential well of width a with

$$V(x) = 0 \quad \text{for } -\frac{a}{2} \leq x \leq \frac{a}{2}$$

$$V(x) = \infty \quad \text{otherwise}$$

Find the energy eigenstates and energy levels.

Problem 1.5:

Let $|\psi_1\rangle$ and $|\psi_2\rangle$ be two orthonormal states of a physical system and let Q be an observable of the system. Consider an eigenvalue q_n(nondegenerate) of Q with corresponding normalized state $|\varphi_n\rangle$, and define $P_i(q_n) = |\langle\varphi_n|\,\psi_i\rangle|^2$.
a) What is the interpretation of $P_i(q_n)$?
b) A particle is in a state $3\,|\psi_1\rangle - 4i\,|\psi_2\rangle$. What is the probability of obtaining q_n when Q is measured?

Problem 1.6:

About representations. Consider a two-dimensional physical system. The kets $|\psi_1\rangle$ and $|\psi_2\rangle$ form an orthonormal basis of the state space. We now define a new basis $|\varphi_1\rangle$ and $|\varphi_2\rangle$ given by

$$|\varphi_1\rangle = \tfrac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right) \qquad \text{and}$$
$$|\varphi_2\rangle = \tfrac{1}{\sqrt{2}}\left(|\psi_1\rangle - |\psi_2\rangle\right)$$

An operator P is represented in the $|\psi_i\rangle$-basis by the matrix $(p_{ij}) = \begin{bmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{bmatrix}$.
Find the representation of P in the $|\varphi_i\rangle$-basis, i.e., the matrix with elements $p_{ij} = \langle\varphi_i| P |\varphi_j\rangle$.
(Hint: in this special case you do not need to find the general transformation matrix.)

Problem 1.7:

Consider a physical system with a three-dimensional state space. An orthonormal basis of the state space is chosen, and in this basis the Hamiltonian is represented by the matrix

$$H = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

a) What are the possible results when the energy is measured?

b) Find $\langle H \rangle = \langle\psi| H |\psi\rangle$ for a particle in a state $|\psi\rangle$ represented by $\frac{1}{\sqrt{3}} \begin{bmatrix} i \\ -i \\ i \end{bmatrix}$ in this basis.