FYS-MENA3110 Problem set #4

Problem 4.1:

Show that the kinetic energy of a three-dimensional gas of N free electrons (with periodic boundary conditions) at $T = 0$ K is $E_0 = \frac{3}{5}NE_F$ where E_F is the Fermi energy of the gas.

Problem 4.2:

a) Calculate the Fermi energy (in electron volts, eV) of copper assuming a density of 8.96 g/cm^3, atomic weight of 63.5 g/mole and $q = 1$ free electron per atom.
b) What is the corresponding electron velocity v_F?
Is it safe to assume that the electrons are nonrelativistic?
c) Calculate the Fermi temperature T_F of copper.
Is T_F small or large compared to the melting temperature of copper.

Problem 4.3:

The bulk modulus B of a substance is the ratio of a small decrease in the pressure to the resulting fractional increase in volume

$$B = \frac{(-\mathrm{d}P)}{\frac{\mathrm{d}V}{V}} = -V\frac{\mathrm{d}P}{\mathrm{d}V}\,.$$

Show that $B = \frac{5}{3}P$ in the free electron gas model and use this to estimate the bulk modulus of copper. Compare the result to the experimentally measured value of $13.4 \cdot 10^{10}\,\mathrm{N/m^2}$.

Problem 4.4:

a) Show for a simple square lattice in two dimensions that the kinetic energy of a free electron at the corner of the first Brillouin zone is higher than that of an electron at the midpoint of a side face of the zone by a factor 2.
b) What is the corresponding factor for a simple cubic lattice in three dimensions?