FYS-MENA3110 Problem set #7

Problem 7.1:

Close to the Brillouin zone boundary an electron in a weak, periodic potential has the following energy

$$E = \left(\frac{E^0_{\mathbf{k}} + E^0_{\mathbf{k}-\mathbf{G}}}{2}\right) \pm \sqrt{\left(\frac{E^0_{\mathbf{k}} - E^0_{\mathbf{k}-\mathbf{G}}}{2}\right)^2 + |V_{\mathbf{G}}|^2}$$

where $E^0_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m}$ is the free-electron energy corresponding to wavevector $\mathbf{k}$, $\mathbf{G}$ is a reciprocal lattice vector, $V_{\mathbf{G}}$ is the appropriate coefficient from

$$V(\mathbf{r}) = \sum_{\mathbf{G}} V_{\mathbf{G}} \mathrm{e}^{i\mathbf{G}\cdot\mathbf{r}}$$

and $V(\mathbf{r})$ is the potential. Consider a simple cubic lattice with primitive lattice translation vectors $a\mathbf{e}_1$, $a\mathbf{e}_2$, and $a\mathbf{e}_3$ where $\mathbf{e}_i$, $i = 1, 2, 3$ are the Cartesian unit vectors along the x, y, and z axis respectively.
a) Show that the effective masses for motion parallel to $\mathbf{e}_1$ with $\mathbf{k}$ close to the point $(\frac{\pi}{a}, 0, 0)$ are smaller than m and directly proportional to $|V_{\mathbf{G}}|$.
b) Why do you obtain two masses and what does their relative size tell you about the bands close to the zone boundary?
c) Show that the effective mass for motion parallel to $\mathbf{e}_2$ with $\mathbf{k}$ close to the $(\frac{\pi}{a}, 0, 0)$ is m.

Problem 7.2:

Find the Fermi surface for a triangular lattice in two dimensions with six noninteracting electrons per lattice site.
a) Calculate the size of the Fermi sphere.
b) Draw the reciprocal lattice of the triangular lattice and indicate the first three Brillouin zones. (Remember: $\mathbf{a}_i \cdot \mathbf{b}_j = 2\pi\delta_{ij}$, with $\mathbf{a}_i$ a real lattice vector and $\mathbf{b}_j$ a reciprocal lattice vector.)
c) Draw the Fermi sphere on a separate piece of semi-transparent paper (or plastic slide) at the same scale as used for the Brillouin zone. By moving this slide over the reciprocal lattice, find the Fermi surface.