Fasit til eksamensoppgaver 1983-1995

1983

Oppgave 1a)

$$Z = \left[\frac{2Isinh(\beta\mu B)}{(\hbar\beta)^2\mu B}\right]^N$$

$$G = -Nk_BTln\left[\frac{2Isinh(\beta\mu B)}{(\hbar\beta)^2\mu B}\right]$$

Oppgave 1b)

$$S = -\frac{G}{T} + Nk_B\left(2 - \beta\mu B coth(\beta\mu B)\right)$$

Oppgave 1c)

$$M = N\mu\left(coth(\beta\mu B) - \frac{1}{\beta\mu B}\right)$$

Oppgave 1d)

$$H = Nk_BT\left(2 - \beta\mu B coth(\beta\mu B)\right)$$

$$U = Nk_BT$$

Oppgave 2a)

$$Z_{MB} = \frac{1}{3!(1 - e^{-\beta\hbar\omega})^3}$$

Oppgave 2c)

$$Z_{BE} = \frac{1}{(1 - e^{-\beta\hbar\omega})}\frac{1}{(1 - e^{-2\beta\hbar\omega})}\frac{1}{(1 - e^{-3\beta\hbar\omega})}$$

Oppgave 2d)

$$\overline{E} = \frac{\hbar\omega}{(e^{\beta\hbar\omega} - 1)} + \frac{2\hbar\omega}{(e^{2\beta\hbar\omega} - 1)} + \frac{3\hbar\omega}{(e^{3\beta\hbar\omega} - 1)}$$

Oppgave 3a)

$$\overline{n}_k = \frac{2}{(1 + e^{\beta\epsilon_k})}$$

Oppgave 3b)

$$S = k_B \beta U + \frac{k_B V}{3\pi^2 (\beta \hbar c)^3} \zeta(3)$$

hvor

$$\zeta(3) = \int_0^\infty dx \frac{x^3}{1 + e^x}$$

Oppgave 3c)

$$p = \frac{7\pi^2 k_B^4}{360 \hbar^3 c^3} T^4$$

Oppgave 3d)

$$122 \text{ nøytrinoer}/cm^3$$

Oppgave 4b)

$$5/24$$

Oppgave 4c)

$$P_\infty = (3/26, 3/13, 4/13, 3/13, 3/26)$$

1984

Oppgave 1b)

$$Z = 4\left(cosh(2J\beta) + 1\right)$$

$$C_V = \frac{2J^2}{k_B T^2 cosh^2(\beta J)}$$

Oppgave 1c)

$$< \sigma_2 >= 0 \qquad < \sigma_1 \sigma_3 >= tanh^2 \beta$$

Oppgave 2a)

$$N_\pm = \frac{N}{2cosh(\beta \mu B)} e^{\pm \beta \mu B}$$

Oppgave 2b)

$$M = N \mu tanh(\beta \mu B)$$

Oppgave 2d)

$$< N_\pm > \approx \frac{4}{3} \pi \frac{V}{(2\pi\hbar)^3} (2m\epsilon_F)^{3/2} \left(1 \pm \frac{3\mu B}{2\epsilon_F}\right)$$

$$\chi \approx \frac{3N\mu^2}{2\epsilon_F}$$

Oppgave 3a)

$$189K$$

Oppgave 3b)

$$S = k_B \left[N ln \frac{N}{N-n} + n ln \frac{N-n}{n} \right]$$

Oppgave 3c)

$$\overline{n} = \frac{N}{1 + e^{\beta \epsilon}}$$

1985

Oppgave 1a)

$$< E > /N = \frac{3}{10m} \left(3\pi^2 \hbar^3 N/V \right)^{2/3}$$

Oppgave 1b)

$$p = \frac{\hbar^2}{5m} (3\pi^2)^{2/3} (N/V)^{5/3}$$

Oppgave 1c)

$$\rho^* \sim \frac{1}{3\pi^2} \left(\frac{mc}{\hbar} \right)^3$$

$$p = \frac{1}{4} \hbar c (3\pi^2)^{1/3} (N/V)^{4/3}$$

Oppgave 2a)

$$G = -N k_B T ln(2 cosh \beta B)$$

$$m = tanh(\beta B)$$

$$F = -N k_B T ln 2 + \frac{N k_B T}{2} \left[(1+m) ln(1+m) + (1-m) ln(1-m) \right]$$

Oppgave 2b)

$$S = N k_B \left(ln(2 cosh \beta B) - \beta B tanh \beta B \right)$$

Oppgave 2c)

$$< \sigma_i \sigma_j >= tanh^2 \beta B$$

Oppgave 2d)

$$C_B = N k_B \left(\frac{\beta B}{cosh \beta B} \right)^2$$

Oppgave 3b)

$$< n(t) >= e^{\lambda t}$$

$$< \Delta n^2 >= e^{\lambda t} \left(e^{\lambda t} - 1 \right)$$

Oppgave 3c)

$$P_1(t) = e^{-\lambda t}$$

$$P_2(t) = e^{-\lambda t} \left(1 - e^{-\lambda t} \right)$$

1986

Oppgave 1a)

$$Z = \frac{1}{N!}\frac{V^N}{\Lambda^{3N}}\frac{(1+e^{-\beta b})^N}{(1-e^{-\beta b})^{2N}}$$

hvor Λ er den termiske deBroiglie bølgelengde.

Oppgave 1b)

$$pV = Nk_BT$$

Oppgave 1c)

$$\mu = k_BTln\frac{p\Lambda^3}{k_BTZ_{vib}}$$

med

$$Z_{vib} = \frac{1+e^{-\beta b}}{(1-e^{-\beta b})^2}$$

Oppgave 1d)

$$\overline{E} = N\left[\frac{3}{2}k_BT + b\frac{3x+x^2}{1-x^2}\right]$$

med $x = e^{-\beta b}$.

Oppgave 2a)

$$M = 2\frac{e^{\beta B} - e^{-2\beta B}}{2e^{\beta B} + e^{-2\beta B}}$$

Oppgave 2b)

$$\frac{k_BT_c}{J} \approx 12$$

Oppgave 3a)

$$p = \frac{m(k_BT)^2}{2\pi\hbar^2}\left(\lambda + \frac{\lambda^2}{4}\right)$$

Oppgave 3b)

$$\mu = k_BTln(1 - e^{\frac{-2\pi\hbar^2}{mk_BT}\rho})$$

Oppgave 3c)

$$p \approx \rho k_BT - \frac{\pi\hbar^2\rho^2}{2m} + \delta(\rho^3)$$

1987

Oppgave 1c)

$$B_2 = \frac{2\pi}{3}\sqrt{\pi\alpha\beta}$$

$$pV = Nk_BT\left(1 + \frac{N}{V}\frac{2\pi}{3}\sqrt{\frac{\pi\alpha}{k_BT}}\right)$$